EVALUATING ULTIMATE STATE OF BUILDINGS UNDER EARTHQUAKE DISTURBANCE

by

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Plastic deformations are expexted in a building during earthquake disturbance; this leads to decrease of stiffness. At the ultimate state, i.e. at the moment a buikding becomes a mechanism, the inertial loads induced by the ground motions, do not increase further under greater deformations. At any state approaching the ultimate state, the safety factor may be evaluated by the allowable displacements, not horizontal loads. The design based on these considerations consists of the following steps:

1. The ultimate state of a structure is determined.
2. A system of critical inertial loads is established and their work at corresponding displacements is determined.
3. An equvalent initial vibratory impulse and corresponding initial velocity are determined to simulate the speciefied earthquake. The maximum of kinetic energy is calculated. Equating this kinetic energy to the work of external loads, the actual stress-and-strain state of the building.

Designations.

M_i - ultimate moments at the base of the shear wall;

Q_{ki} - ultimate transverse force of floor "k", frame "i";

L - length of buiding;

r_i - distance between i-axis and I-axis;

$P_{ki} = \alpha_{ki} . P$ - external point load applied to the intorsection of wall N"i" and floor N"k", determined by the load factor α_{ki} and the load P ;

H - height of buiding;

h_k - heigth of floor N"k";

H_k - altitude of floor N"k";

$\mu = \frac{\varphi_n}{\varphi_1}$ = torsional characteristic of the buiding;

φ_1 and φ_n - slopes of shear walls N"1" and N"n" at their bases

m_i - mass of point "i";

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p_j, T_j - natural frequency and period, respectively, of the "Nj" natural mode;
α_j - dissipation parameter;
X_{ij} - ordinate of point "i" in mode "Nj";
γ - completeness factor of the indicating curve;
n - number of lumped masses of the structure;
y_o - movements of the foundation;
q_j - fundamental function "Nj".

I. Ultimate state of braced systems under horizontal disturbances is established to appear at the moment of plastic hinging of buttom sections of shear walls and beams of the bracing (1). In a pure frame the ultimate state is reached after first hinge is formed in the beam.

Virtual displacements of the examined structural system do not cause additional in-plane bending of floors, because the interaction forces between the walls and the floor slabs will not increase after complete hinging of the building.

The largest possible internal forces allowable by the given cross-sections, are assumed to be given as well. External horizontal loads are assumed to be distributed in a certain way and determined by the load parameter "P". The building is considered a three-dimentional structure. The parameter "P" is determined from the equality of virtual external and internal works; this parameter characterized the ultimate state of the structure:

$$P = \frac{A\mu + B}{C\mu + D} \qquad \text{I}$$

wher A,B,C and D are positive quantities determined by the following formulas:

$$A = \sum_i r_i (M_i + \sum_k Q_{ik} h_k);$$
$$B = \sum_i (L - r_i) \cdot (M_i + \sum_k Q_{ik} h_k);$$
$$C = \sum_i r_i \sum_k H_k \cdot \alpha_{ki}; \quad D = \sum_i (L - r_i) \sum_k H_k \cdot \alpha_{ki};$$

Indices "k" and "i" under the signs of sums ($\sum$) designate summarizing over all the floors and walls, respectively. The sums of moments are evaluated over the shear walls; the sums of transverse forces are evaluated over the frames.

Expression (I) is a hyperbola; the lowest possible values of P are given by $\mu \geq 0$

$$P = \frac{A}{C} \quad \text{or} \quad P = \frac{B}{D} \quad (\text{at } \mu \geq 0) \text{ for } \mu = \infty \text{ and } \mu = 0,$$

respectively (μ_i=0 means that the building undergoes pure rotation about one of the walls, without translational movements). In case the buiding is symmetrical in plane and the load is symmetrical too, μ is assumed equal to 1 in formula (I). The "P" parameter may be determined for pure frames according to formula (I), in which the sum is evaluated over all the frames (M_i=0) and $H_k=h_k$.

II. A structure oscillates during earthquake; its deformed state corresponds with the displacements caused by statically applied inertial horizontal loads induced by movements of masses.

Expanding the movements of a building to basic functions and assuming that the inertial forces of masses located on the structure are in proportion to the ordinater of the basic functions, the critical loading parameter "P" may be determined for each natural mode and an indicating curve "P-f" may be plotted after calculation of appropriate deflections "f".

Thus, if $P_j x_{1j}$ and $f_j x_{1j}$ are an inertial load and displacement of mass "N1" ~~and~~ at the mode Nj, respectively, then the same quantities for mass Ni will be designated by $P_j x_{ij}$ and $f_j X_{ij}$. An area of the indicating diagram P_j f_j represents the work of this force during displacements of the structure. Then the total work of the inertial loads will be expressed by

$$A_j = \gamma_j \cdot \bar{P}_j \, \bar{f}_j \sum_{i=1}^{n} X_{ij}^2 \qquad (2)$$

If the work A_j may be determined and the indicating curve is plotted, then the unknown deflection of the structure may be calculated.

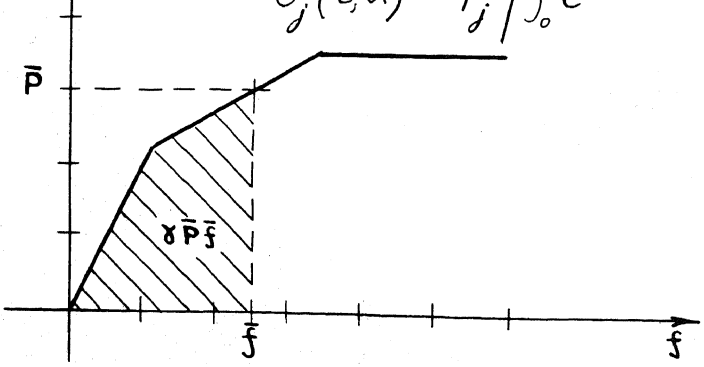
III. The work A_j depends on the intencity of the external disturbance.

A system of differential equations of displacements is written in terms of the basic coordinates the following way:

$$\ddot{q}_j + q_j P_j^2 \cdot e^{i\alpha_i} = - \delta_j \cdot y_0'' ;$$

in which
$$\delta_j = \frac{\sum_{i=1}^{n} X_{ij} m_i}{\sum_{i=1}^{n} X_{ij}^2 m_i}$$
hence
$$q_j = \frac{\tau_j(t,\alpha) \cdot \delta_j}{P_j^2};$$

where
$$\tau_j(t,\alpha) = P_j \left| \int_0^t e^{-\frac{\alpha}{2} P_j (t-\xi)} y_0'' \sin P_j (t-\xi) d\xi \right|_{max}$$



is determined from the recording of the ground motions during various earthquakes.

The amplitude of the acceleration of mass N„i" is determined by

$$W_{ij} = -q_j'' X_{ij} = p_j^2 q X_{ij} = \tau_j(T,\alpha) X_{ij} \delta_j \qquad (3)$$

Consider a vibratory impulse, which will induce the same accelerations W_{ij} and, therefore, the same inertial loads. The corresponding equation of motions is formulated as

$$q_j = A \cdot \sin p_j t + B \cos p_j t$$

where A and B are the intergration constants determined from the initial conditions. As $y_j = x_{ij} \cdot q_j = 0$ at the initial moment, B=0.

The other intergration constant (A) is determined from the equalence of the given vibratory impulse and the actual external disturbance:

$$y_i''{}_{max} = W_i$$

According to eq.: $y_i = A\, x_{ij} \sin p_j t$; $y_i''{}_{max} = A \cdot p_j^2 X_{ij}$; taking W_i from (3) gives:

$$A = \frac{\tau_j(T,\alpha)\delta_j}{p_j^2}$$

hence,

$$V_{i_0} = y_i' = \frac{\tau_i(T,\alpha)}{p_j} X_{ij} \frac{\sum_{i=1}^{n} X_{ij} m_i}{\sum_{i=1}^{n} X_{ij}^2 m_i}$$

The maximum kinetic energy at the initial moment $T=0$ is given by

$$E = \frac{1}{2}\sum_{i=1}^{n} V_{i_0}^2 m = \frac{1}{2}\left[\frac{\tau_j(T,\alpha)}{p_j}\right]^2 \cdot \frac{\left(\sum_{i=1}^{n} X_i m_i\right)^2}{\sum_{i=1}^{n} X_i^2 m_i} \qquad (4)$$

The stress and sfrain parameters ($\bar{f}$ and $\bar{P}$) are determined by equating„E" from (4) and„A" from (2).

In case several successive vibratory impulses are to be accounted for, the kinetic energy is represented by a sum of kinetic energies of each impulse and the result is equated with the work of inertial loads determined from the indicating curve; the stepwise unloading must be accounted for in the latter case: the unloading follows the elastic characteristic of the structure.

References:

1. S.M. Sojbelman. On design of braced systems of buildings under horizontal loads with plastic strains taken into account. Collected articles: Buildings Construction, design and erection". Shtiintza, Kishinev, 1972.
2. A.G. Nazarov. An engineering method of examining earthquake loads. Academy of Science, Armenia, Erevan, 1959