CUMULATIVE DAMAGE OF PLASTIC FRAMES SUBJECTED TO EARTHQUAKES: A PROBABILISTIC APPROACH[I]

by

Giuliano Augusti[II] and Alessandro Baratta[III]

SYNOPSIS

A ductile frame subjected to constant vertical loads and a horizontal ground shaking large enough to cause plastic deformations, is weakened by the unstabilizing effect of the residual eccentricity of the vertical loads. The accumulation of this damage up to the complete collapse of the frame is studied in detail under simplifying assumptions by a Montecarlo sampling of histories of quake arrivals in a site of known seismicity. The curves of the probability of survival of the structure and of the expected capital profit versus time are calculated and plotted for the assumed numerical data. Procedures and results are to be regarded as a first tentative with a view to more sophisticated developments.

1. INTRODUCTION AND SUMMARY

In his well-known book, Richter underlines the relevance of "the weakening effects of repeated shaking on commonplace construction" and remarks that "some spectacular failures of old buildings are attributable to progressive weakening in successive minor shakings ..." (Ref. 1).

The danger of such a progressive weakening should definitely be taken into account whenever, as is indeed commonplace nowadays, the designer intends to allow some plastic deformations to develop during strong-motion earthquakes. In these cases it would be logical not only to check the survival (or calculate the risk of failure) of the structure with respect to a single, albeit strong, earthquake, but also to consider a situation in which a structure is subjected to earthquakes, each strong enough to cause plastic deformations but not enough to bring the original structure to collapse. In fact, the structure is "weakened" by such a quake with respect to successive quakes. Two main types of "damage" can be recognized: a) "material" damage, i.e. deterioration of the material characteristics due to the repeated cycles of alternating stresses; b) "geometrical" damage due to the residual displacements that can cause unstabilizing load effects in some usual types of construction.

Some authors have already tackled the probabilistic analysis of seismic damage but have suggested a linear (additive) law for the cumulative structural deterioration (Ref. 2). On the contrary, the accumulation of plastic damage (of either type) is highly non-linear, because each "damaging" quake lowers the threshold intensity below which a quake does not cause any plastic deformation.

This paper investigates the practical possibility of a probabilistic treatment of this accumulation: a procedure of general validity is devel-

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[I] Supported by the Italian National Research Council (C.N.R.).

[II] Associate Professor, Università di Firenze, Florence, Italy.

[III] Research Fellow, Università di Napoli, Naples, Italy.

oped with reference to the specific example of a typical structure.

Attention is focussed on the stochastic uncertainty in the prevision of quake arrivals rather than in the calculation of the response to a single quake. Therefore, the evaluation of the damage of the considered structure when subjected to a quake of known characteristics is simplified drastically. In particular, only effects of the geometrical/unstabilizing type are considered for simplicity's sake. The introduction of material deterioration into the treatment (which should not be a major difficulty) is postponed to the knowledge of the many new researches on this subject that will be reported in 1973 at the 5WCEE and a special Symposium (Ref. 3).

Moreover, the actual earthquake motion is simplified into a periodic oscillation with a triangular acceleration vs. time diagram. Thus, each quake is fully described by a limited number of (stochastic) parameters, whose (joint) probability distribution law is assumed to be known in the given site of construction: its effect on the considered structure, at a given stage of "damage", is easily determined. However, the lack of a simple law for the accumulation of damage has suggested the use of a Montecarlo procedure to sample the "seismic history" of the structure, i.e. the succession of quake arrivals and the damage they cause, up to final collapse.

In particular, from a sufficiently large sample, the probability of the structure surviving the n-th quake can be calculated. Then, if the (stochastic) process of quake arrivals is also known, it is possible to transform this probability into the probability of survival R(t) (reliability) as a function of time t. Further, introducing appropriate laws for estimating costs and income, the knowledge of R(t) allows the calculation of curves of expected net profit vs. time during the life of the structure, from which useful information on an economic maintenance policy can be derived.

A few qualitative but interesting indications are derived from the numerical results, and are noted at the relevant places in the body of the paper. However, the main scope of this work is to open a first trail throughout the whole seismic analysis of a ductile structure, down to the conclusive reliability and economic estimates, using only mathematical tools within the average engineer's grasp. This scope appears to have been fully reached; introduction of more exact assumptions and more sophisticated computing techniques is left to future research.

2. THE STRUCTURE: RESPONSE TO THE IDEALIZED QUAKE

The structure considered in this paper is the one-degree-of-freedom rigid-plastic plane portal frame of Fig. 1, already studied in previous works by one of the present writers (Refs. 4,5,6). Some recent examples have confirmed that this may be a realistic model for the response of whole buildings to strong-motion earthquakes (Ref. 7).

The frame is subjected to a horizontal ground shaking of acceleration $-a(t)$, and is then loaded by the inertia force $m(a-\ddot{u})$, where $u=u(t)$ is the (horizontal) relative displacement of the mass m (Fig. 1). The effective current shear strength of the frame, measured as an acceleration, is

$$k_t = k - Gu \tag{1}$$

where

$$k = \frac{\Sigma Mo}{mh} \quad \text{and} \quad G = \frac{\Sigma W - \gamma}{mh} \tag{2}$$

are respectively the original strength and the unstabilizing parameter. In the present paper, G is assumed positive (i.e. the unstabilizing effect of the vertical loads ΣW prevails on the stabilizing strain-hardening measured by γ). Therefore, the current strength k_t "deteriorates" with increasing u, as qualitatively shown in Fig. 2; the frame collapses if and when k_t vanishes, i.e. when the displacement u reaches the "critical" value

$$u = u_c = \frac{k}{G} \tag{3}$$

(As anticipated, other forms of strength deterioration are not considered in this paper, but could easily be introduced in the treatment in successive works. In the numerical calculations, $G = 3.27 \text{ sec}^{-2}$.)

The "idealized quake" is further specified as a periodic-symmetric triangular a(t) diagram with peak values $\pm a_o$ and period T (Fig. 3).

Then, the procedures developed in Refs. 4-6 and straightforward numerical integration allow the calculation of the response u=u(t). The thin lines in Fig. 4 are a family of response curves, calculated for a given ratio a_o/k and several values of the ratio k'/k of the initial strength in the two directions (Ref. 8). It is easily noted that when the asymmetry of strength increases, the response curve u/u_c tends rapidly to the limit heavy line, which is reached as soon as $a_o/k = k'/k$. For this and larger values of k'/k, velocity $\dot{u}$ and displacement u have always the same sign. An asymmetry of loading (different positive and negative peak a) would have a similar effect.

Therefore, a simplified closed-form response law is used in this paper, which neglects all negative velocities (Ref. 8): the increment of displacement $(u_\tau - u_o)$ due to an idealized quake of peak acceleration a_o, duration τ and period T, is given by the equation

$$\frac{1}{\left[a_o - k_o\right]^2} - \frac{1}{\left[a_o - k_o + G(u_\tau - u_o)\right]^2} = \frac{1}{a_o^2}\,\frac{GT\tau}{4} \tag{4}$$

provided that

$$a_o > k_o = k - Gu_o \tag{5}$$

i.e. that the peak ground acceleration is larger than the frame strength at the beginning of the quake; otherwise

$$u_\tau = u_o \quad \text{if} \quad a_o \le k_o \tag{6}$$

As anticipated, the "threshold" acceleration k_o decreases with increasing u_o, which leads to a marked non-linearity in the accumulation, because the damage due to a single quake $(u_\tau - u_o)$ depends strongly on previous history. The "critical" quake is therefore obtained by repeated application of Eqs. (4-6), until

$$u_o < u_c \leq u_\tau \tag{7}$$

(It is also worth noting that the over-simplified treatment proposed in Ref. 5 cannot be used in this context, because a very short quake above the threshold intensity would appear to lead the frame to collapse, so that any tentative consideration of "accumulation" would be grossly mistaken.)

3. STOCHASTIC PROPERTIES OF IDEALIZED QUAKE

In order to obtain significant results in the numerical examples, the characteristic quantities of each idealized quake (τ, T and a_o) have been obtained from those of an actual earthquake as follows.

The peak acceleration a_o has been taken equal for both quakes; the period T has been assumed known for a given site; the duration of the idealized quake is either taken equal to the duration τ of the actual earthquake, or "reduced" to

$$\tau^* = \tau \cdot \text{Prob}\{a_m \geq k_o\} \tag{8}$$

multiplying τ by the probability that the local maximum acceleration is larger than the strength at the beginning of the quake. Assuming further that the actual quake is a Gaussian stationary process,

$$\text{Prob}\{a_m \geq k_o\} = 1 - \exp(-k_o^2/2\overline{a^2}) \tag{8'}$$

where $\overline{a^2}$ is the mean square value of the acceleration (average power of the earthquake). In the calculation, in accord with other authors (Ref. 9), $\overline{a^2}$ has been obtained from a_o through the linear transformation

$$\overline{a^2} = 0.0364\ a_o^2 \tag{9}$$

Fig. 5 shows the resulting relation between τ and τ^* for some different values of a_o/k_o, while Fig. 6 compares the values of the residual displacements calculated with the above formulae for idealized quakes of duration τ and τ^* (continuous lines) and the maximum displacements obtained by numerical integration of the equations of motion, introducing as $a=a(t)$ the digitalized records of some well-known earthquakes, every time assuming that $a_o/k_o=2$. Inspection of Fig. 6 shows that the residual displacements (hence, the "damages") calculated by the approximate treatment just described with reduced quake duration τ^*, and calculated from actual records are in an accord that is satisfactory for the purposes of the present paper.

The stochastic properties of the variables τ and a_o are obtained from those of the Richter magnitude m and the focal distance D.

Assuming that the quakes at the given site originate from a single linear fault of length L, at a distance d from the site, the distribution of D is (Ref. 10)

$$F_D(D) = \text{Prob}\ (\tilde{D} \leq D) = \frac{\sqrt{D^2 - d^2}}{L/2} \qquad (10)$$

The distribution of quakes of magnitude $m \geq m_1$ is usually assumed to be given by (Refs. 2,10)

$$F_m(m) = 1 - \exp\left[-\beta\ (m - m_1)\right] \qquad (11)$$

where m_1 is the smallest magnitude of engineering significance.

Then, using an attenuation law of the type (Ref. 11)

$$a_o = b_1\ e^{b_2 m}\ D^{-b_3} \qquad (12)$$

the distribution of a_o is obtained. In our numerical calculations, the following typical values have been introduced:

$$\begin{array}{llll} \beta = 1.48 & b_2 = 0.8 & L = 650\ \text{km} & T = 1/3\ \text{sec} \\ b_1 = 2000 & b_3 = 2.0 & d = 45\ \text{km} & m_1 = 5 \end{array} \qquad (13)$$

Finally, it is well known that the duration τ is related to the quake magnitude m. Housner proposed a few years ago (Ref. 12) the linear correlation law plotted in Fig. 7; more recently the same Author has published a table of typical durations (Ref. 13) from which the relation

$$\tau = -0.33m^3 + 8.5\ m^2 - 56\ m + 111 \qquad (14)$$

plotted as a dashed line in Fig. 7, can be obtained by interpolation. In the present paper, Eq. (14) is used as a deterministic law to obtain τ from m.

4. CALCULATION OF RELIABILITY AND EXPECTED PROFIT FUNCTION

To investigate the seismic history of the structure, sequences of values of m and D are chosen by random sampling associated with the distribution laws (10) and (11) and the numerical values (13); the corresponding sequences of a_o and τ (or τ^*) are obtained from Eqs. (12-14). At each stage, the increment of damage $(u_\tau - u_o)$ is given by Eqs. (4-6); an "element" of the sample is completed when the critical condition (7) is reached.

Figs. 8 and 9 show the histograms, calculated from two such samples, of the probability of failure at the n-th quake $(m > m_1)$

$$P_n(n) = \text{Prob}\ \{u_o < u_c \leq u_\tau \mid \tilde{n} = n\} \qquad (15)$$

for a frame with $G = 3.27\ \text{sec}^{-2}$ and initial limit acceleration $k = 100\ \text{cm.sec}^{-2}$, using respectively the total duration τ (14) and the reduced duration τ^* (8). Fig. 10 shows the corresponding histograms of the percent probability of survival to $\bar{n}$ "damaging" quakes $(a_o > k_o)$.

Then, in accord with most researchers (cf. Ref. 2) the arrival of quakes is assumed to be a Poisson stochastic process: i.e. the probability of n quakes $(m > m_1)$ arriving in a time interval t is given by

$$\text{Prob}\ \{\tilde{n} = n \mid t\} = e^{-\nu t}\ (\nu t)^n\ /\ n! \qquad (16)$$

The return period $1/\nu$ has been taken equal to 10 years in the calculations. Then, Eq. (16) and the numerically calculated $P_n(n)$ (15) give the reliability $R(t)$, i.e. the probability of survival of the structure as a function of time:

$$1 - R(t) = \sum_{n=1}^{\infty} P_n(n).\text{Prob}\ \{\tilde{n} \geq n|t\} = \sum_{n=1}^{\infty} P_n(n) \left[\sum_{i=n}^{\infty} \text{Prob}\ \{\tilde{n} = i|t\}\right] \qquad (17)$$

The two solid lines in Fig. 11 are the plots of the reliability $R(t)$ (which turns out to be a function of the non-dimensional time νt), introducing $P_n(n)$ respectively from Fig. 8 (lower curve) and Fig. 9. The dashed curves in the same Fig. 11 represent respectively, R_1 the probability that no damaging quake ($a_o > k$) arrives on the structure (i.e. the reliability of a structure unable to survive any plastic deformation), and R_s the reliability calculated neglecting the accumulation of damages (i.e. $(1-R_s)$ is the probability of arrival of a quake that by itself alone would bring the undeformed structure ($u_o=0$) in the critical condition $u_\tau \geq u_c$).

Inspection of Fig. 11 shows that R_1 is much smaller than R, but that R and R_s are rather close, i.e. that accumulation of damage is not very relevant under the set hypotheses, as far as reliability is concerned. However, the validity of these conclusions should be tested after removal of the many limitations to which the present calculations are subjected.

It is also to be noted that Fig. 8 was based on a sample of 5120 elements, but the calculated reliability was practically the same when derived from much smaller samples. Therefore, the sample for Fig. 9 has been limited to 1280 elements.

Figs. 12 and 13 show the results of the analogous calculations for other limit accelerations, using only the reduced duration τ^*. Note that for the Poisson process (16) with return period $1/\nu = 10$ years, the return periods of quakes with $a_o \geq 50$, 100 and 150 cm. sec^{-2} are respectively 66, 238 and 503 years.

The reliability curves of Fig. 13 are easily transformed into the hazard functions plotted in Fig. 14

$$\lambda(t) = -\frac{1}{R}\ \frac{dR}{dt}\ ; \qquad (18)$$

$\lambda(t)dt$ measures the probability that a structure, surviving at time t, fails in the subsequent time increment dt. A statistical interpretation of the shape of these curves would be that, given a sufficiently large number of nominally identical structures in equivalent sites, the expected number of failures per year increases rapidly in the first years, then stabilizes for a long time on approximately constant values. Note that these curves are qualitatively different from those obtained in other fields of engineering, in which there is an initial large value of $\lambda(t)$, connected with the elimination of the defective elements. In fact, in this paper the structural strength is a deterministic quantity, and the randomness is limited to the arrival and intensity of the loading.

Finally, on the basis of a very rough economic function, some calculations on the "expected profit" have been developed. Let V be the annual income from the structure (assumed constant), C_f the cost of failure, and r the discount rate. The expected profit $\overline{Q}(t)$, assumed nil at the end of construction, varies in time according to the law

$$\overline{Q}(t) = V \frac{r^t - 1}{r - 1} - \left[1 - R(t)\right] C_f \; r^t \qquad . \quad (19)$$

where the expected cost of failure $[1-R]\, C_f$ is revaluated according to the discount rate r.

With the positions

$$\alpha = \frac{C_f}{V} \qquad Q = \frac{\overline{Q}}{V} \qquad (20)$$

Eq. (19) becomes

$$Q(t) = \frac{r^t - 1}{r - 1} - \alpha \left[1 - R(t)\right] r^t \qquad (21)$$

Introducing into (21) different values of α and the function R(t) plotted in Fig. 13 (k = 100 cm. sec^{-2}), the curves in Fig. 15 are obtained.

The shape of these curves is very interesting: the expected profit for a structure increases approximately at an exponential rate (in a constant general economic situation) for some time, then falls very rapidly at a "critical" time, when the expected cost of failure prevails. If this qualitative behaviour remains valid when the expected profit is calculated from some more exact and sophisticated treatments of the technical and economic aspects, the determination of the "critical time" is paramount in an overall view of the problem of building in a seismic site: demolition or thorough revision of each structure should be planned before the "critical time" is reached.

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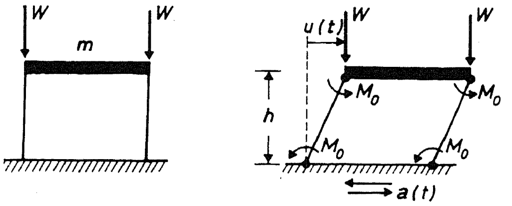

Fig. 1 : Model frame.

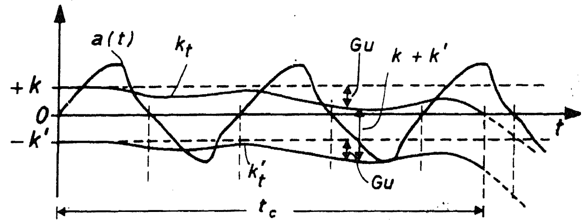

Fig. 2 : Qualitative response to repeated dynamic loading.

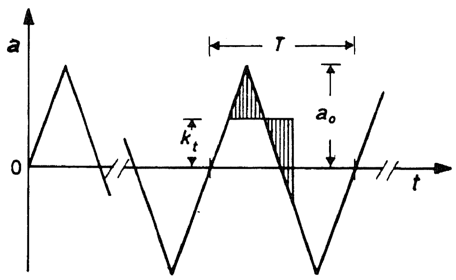

Fig. 3 : Idealized quake.

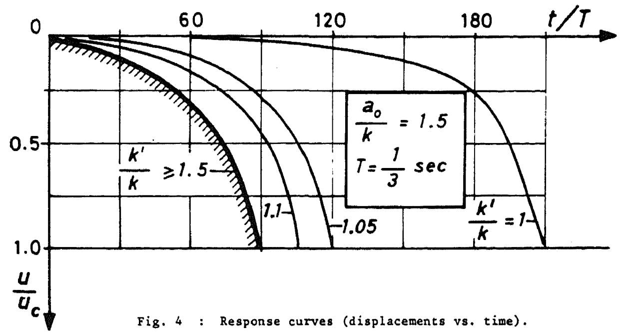

Fig. 4 : Response curves (displacements vs. time).

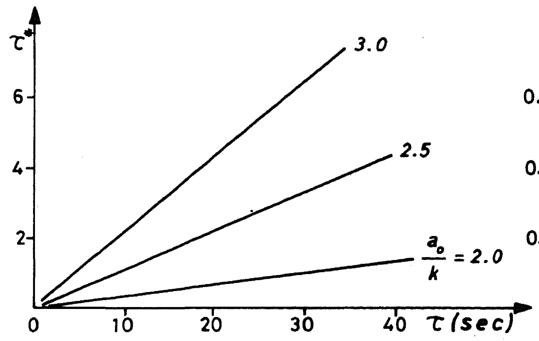

Fig. 5 : "Reduced" vs. total quake duration.

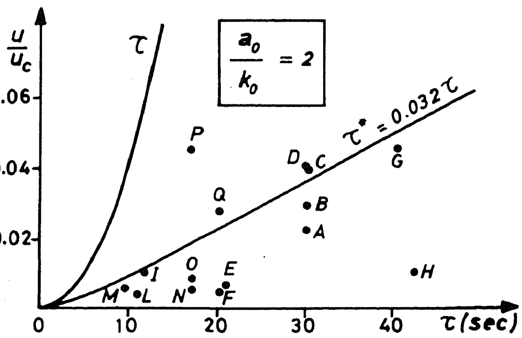

Fig. 6 : Comparison of damages of idealized and actual earthquakes.

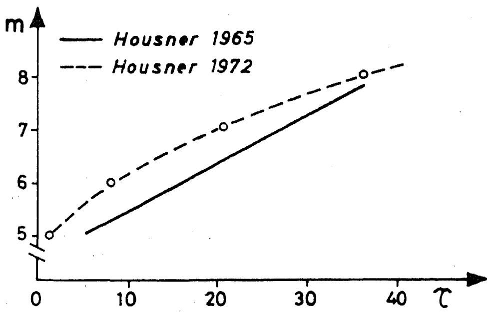

Fig. 7 : Quake duration vs. magnitude.

Site	Date	Component	Site	Date	Component
A) El Centro	19/5/40	N00S	I) Vernon	2/10/33	S82E
B) El Centro	19/5/40	E00W	L) Helena	31/10/35	N00S
C) Olympia	13/4/49	N10W	M) Helena	31/10/35	E00W
D) Taft	21/7/52	N69W	N) Ferndale	3/10/41	N45E
E) Ferndale	11/9/38	S45E	O) Ferndale	3/10/41	S45E
F) Ferndale	11/9/38	N45E	P) Hollister	9/3/49	S01W
G) Vernon	10/3/33	N08E	Q) Seattle	13/4/49	N88W
H) Vernon	10/3/33	S82E			

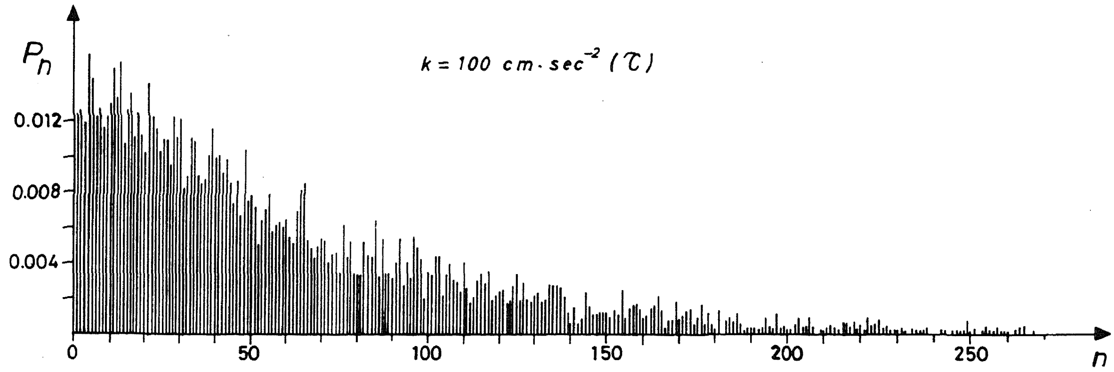

Fig. 8 : Probability of failure at n-th quake; total duration τ.

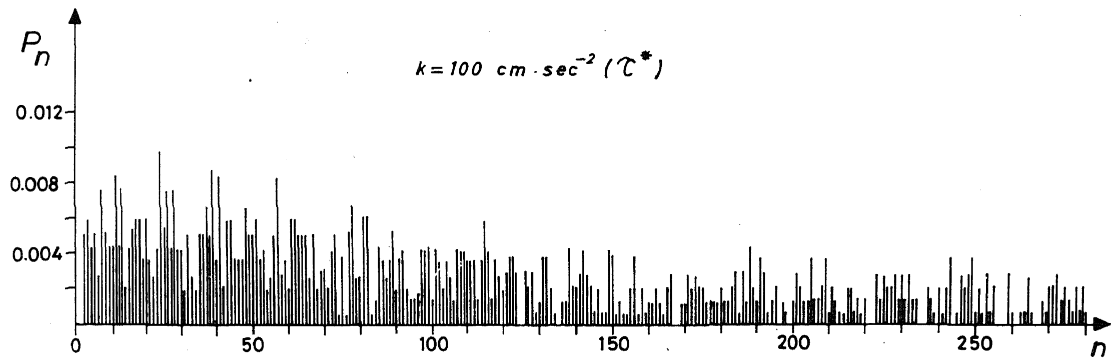

Fig. 9 : Probability of failure at n-th quake; reduced duration τ*.

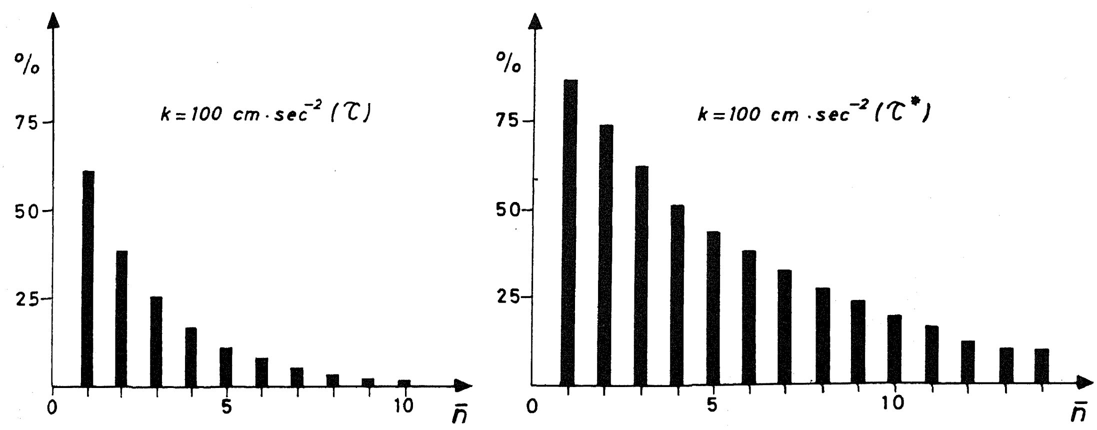

Fig. 10 : Probability of survival to $\bar{n}$ damaging quakes.

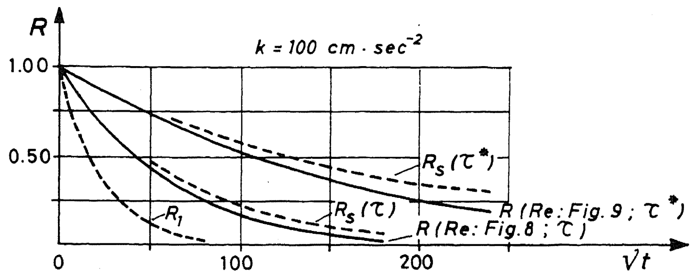

Fig. 11 : Reliability under different assumptions.

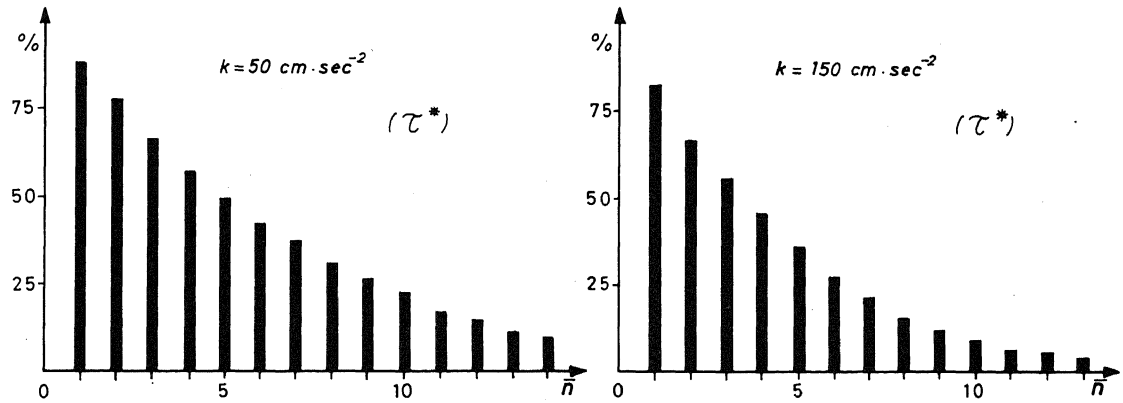

Fig. 12 : Probability of survival to n̄ damaging quakes.

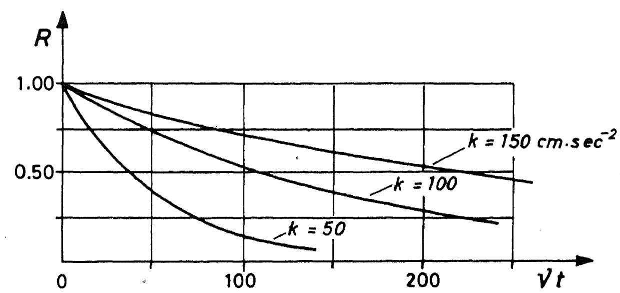

Fig. 13 : Reliability for different design strengths.

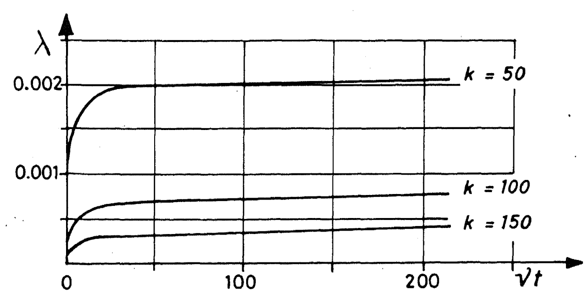

Fig. 14 : Hazard functions, Eq. 18.

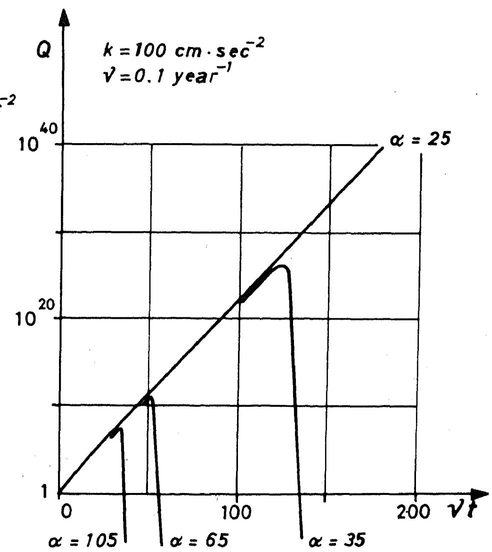

Fig. 15 : Non-dimensional expected profit, Eq. 21.