MULTI-DEGREE OF FREEDOM RESPONSE SPECTRA FOR ELASTIC SYSTEMS

by

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SYNOPSIS

This paper introduces the concept of the multi-degree of freedom (MDOF) response spectrum for elastic systems. The use of such spectra permits the rapid determination of many of the significant maximum response parameters necessary for the seismic design of MDOF systems such as highrise buildings. Upon assuming or delineating the mode shapes and frequency ratios of a building (for example those of a shear beam), it is possible to integrate the response of "n" modes of vibration simultaneously, and to scan the algebraic sum of these "n" modal responses to obtain a true maximum value of the total combined response. Response parameters thus expressed are those of roof acceleration, velocity and displacement as well as base shear and base overturning moment. These MDOF response spectra may be generated for any number of modes and be plotted as functions of the damping ratio in each mode and of the natural fundamental period of the structure.

INTRODUCTION

Response spectra for elastic single-degree of freedom oscillators have been widely accepted by the engineering profession. Not only do they clearly illustrate the frequency content of earthquake ground motions, but they are also used with advantage for the dynamic design of simple structures. For multi-degree of freedom structures, various schemes have been developed in order to estimate the effect of higher modes. However, for the seismic design of most highrise buildings, more accurate methods are desirable. One such method is outlined below.

Results based upon such MDOF response spectra are presented for several ground motions and are compared with: a) results based on single-degree of freedom (SDOF) response spectra for the same ground motions; b) results from time-dependent elastic dynamic simulations; c) results derived from SDOF response spectra using the root mean square (RMS) method; d) peak response values recorded during the recent San Fernando earthquake, February 9, 1971.

In conclusion, the merits and limitations of using MDOF spectra techniques in highrise building design are discussed and evaluated. Attention is focused on their use as a preliminary design-aid tool, and as a guide in comparing different ground motions.

MDOF RESPONSE SPECTRUM CONCEPT

After studying tall buildings, Jennings(1) has shown that tall buildings generally have constant story heights and masses. Also, their frequency ratios and mode shapes, ϕ, for "n" modes are similar to those of a shear beam i.e.,

$$\omega_i = \omega_1 \ (2i-1) \qquad i = 1,2,\ldots,n \tag{1}$$

$$\phi_i = \sin \frac{(2i-1)\pi x}{2h} \qquad i = 1,2,\ldots,n \tag{2}$$

where ω_1 is the fundamental frequency, and h the building height.

These findings are used to construct a simplified analytical building model as follows; The i'th uncoupled normal equation for a structural system having n degrees of freedom reads,

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$$\ddot{Y}_i(t)+2\xi\omega_i\dot{Y}_i(t)+\omega_i^2 Y_i(t) = -\Gamma_i \ddot{y}_g(t) \tag{3}$$

where the participation factor is defined by

$$\Gamma_i = \frac{\int_0^h \phi(x)m(x)dx}{\int_0^h \phi^2(x)m(x)dx} \tag{4}$$

Substituting Eq. (2) into (4) leads, for m(x) = constant, to

$$\Gamma_i = \frac{4}{\pi(2i-1)} \tag{4a}$$

The solution of Eq. (3) is the well-known Duhamel integral

$$Y_i(t) = \frac{\Gamma_i}{\omega_{id}} \int_0^t \ddot{y}_g(\tau) e^{-\xi\omega_i(t-\tau)} \sin \omega_{id}(t-\tau)d\tau \tag{5}$$

where $\omega_{id}=\omega_i\sqrt{1-\xi^2}$, ω_i= i'th frequency, ξ = damping ratio, and $\ddot{y}_g$ = ground acceleration. With the traditional definition of SDOF response spectra, maximum displacements, velocities, and accelerations in the i'th mode of vibration follow as;

$$Y_i(x) = \Gamma_i\phi_i(x)S_d(\omega_i,\xi); \quad \dot{Y}_i(x) = \Gamma_i\phi_i(x)S_v(\omega_i,\xi); \quad \ddot{Y}_i(x) = \Gamma_i\phi_i(x)S_a(\omega_i,\xi)$$

respectively, while shears and overturning moments are expressed as;

$$V_i(x) = \Gamma_i S_a(\omega_i,\xi)\int_x^h m(\zeta)\phi_i(\zeta)d\zeta$$

$$M_i(x) = \Gamma_i S_a(\omega_i,\xi)\int_x^h m(\zeta)(h-x-\zeta)\phi_i(\zeta)d\zeta$$

Defining the following maxima of certain combined modal responses as;

$$SA_1^{(k)}(\omega_1,\xi) = \max_t\left[\ddot{Y}_1(t,\omega_1,\xi)+\frac{\ddot{Y}_2(t,\omega_2,\xi)}{(\omega_2/\omega_1)}+\frac{\ddot{Y}_3(t,\omega_3,\xi)}{(\omega_3/\omega_1)}+\dots+\frac{\ddot{Y}_k(t,\omega_k,\xi)}{(\omega_k/\omega_1)}\right]$$

$$SA_2^{(k)}(\omega_1,\xi) = \max_t\left[\ddot{Y}_1(t,\omega_1,\xi)+\frac{\ddot{Y}_2(t,\omega_2,\xi)}{(\omega_2/\omega_1)^2}+\frac{\ddot{Y}_3(t,\omega_3,\xi)}{(\omega_3/\omega_1)^2}+\dots+\frac{\ddot{Y}_k(t,\omega_k,\xi)}{(\omega_k/\omega_1)^2}\right]$$

$$SA_3^{(k)}(\omega_1,\xi) = \max_t\left[\ddot{Y}_1(t,\omega_1,\xi)+\frac{\ddot{Y}_2(t,\omega_2,\xi)}{(\omega_2/\omega_1)^3}+\frac{\ddot{Y}_3(t,\omega_3,\xi)}{(\omega_3/\omega_1)^3}+\dots+\frac{\ddot{Y}_k(t,\omega_k,\xi)}{(\omega_k/\omega_1)^3}\right] \tag{8}$$

$$SV^{(k)}(\omega_1,\xi) = \max_t\left[\dot{Y}_1(t,\omega_1,\xi)+\frac{\dot{Y}_2(t,\omega_2,\xi)}{(\omega_2/\omega_1)}+\frac{\dot{Y}_3(t,\omega_3,\xi)}{(\omega_3/\omega_1)}+\dots+\frac{\dot{Y}_k(t,\omega_k,\xi)}{(\omega_k/\omega_1)}\right]$$

$$SD^{(k)}(\omega_1,\xi) = \max_t\left[Y_1(t,\omega_1,\xi)+\frac{Y_2(t,\omega_2,\xi)}{(\omega_2/\omega_1)}+\frac{Y_3(t,\omega_3,\xi)}{(\omega_3/\omega_1)}+\dots+\frac{Y_k(t,\omega_k,\xi)}{(\omega_k/\omega_1)}\right]$$

in which $Y_i(t,\omega_i\xi)$; $\dot{Y}_i(t,\omega_i,\xi)$; $\ddot{Y}_i(t,\omega_i\xi)$ are the relative displacement, velocity, and absolute acceleration, respectively, for the i'th mode with frequency ω_1 and damping ξ at time t. The superscript k indicates the number of modes considered. With the definitions of Eq. (8), and Eq. (4a), it can be shown that the combined maxima for displacement, velocity, and acceleration at the roof (x=h) and shear and overturning moment at the base (x=0), respectively, are given by;

$$S_d(\omega_1,\xi) = \frac{4}{\pi} SD^{(k)}(\omega_1,\xi)$$

$$S_v(\omega_1,\xi) = \frac{4}{\pi} SV^{(k)}(\omega_1,\xi)$$

$$S_a(\omega_1,\xi) = \frac{4}{\pi} SA_1^{(k)}(\omega_1,\xi) \tag{9}$$

$$S_V(\omega_1,\xi) = \frac{8W}{g\pi^2} SA_2^{(k)}(\omega_1,\xi)$$

$$S_M(\omega_1,\xi) = \frac{16Wh}{g\pi^3} SA_3^{(k)}(\omega_1,\xi)$$

where $W = g\int_0^h m(x)dx$ is the building weight, ω_1 the fundamental building frequency, and g the gravity constant.

Jennings(1) has shown that the first mode of a tall building, generally may be described as a straignt line, x/h, thus falling between the shear beam approximation, $(x/h)^{\frac{1}{2}}$, and the bending beam approximation, $(x/h)^{\frac{3}{2}}$. Hence, the results of Eq. (9) for the first mode contributions need to be adjusted accordingly. hus, displacements and accelerations are increased by 18%, while base shears and overturning moments are reduced by 7% and 3% respectively. It may be noted that the formula ion of Eq. (8) permits the substitution of varying damping ratios, as well as alternative sets of frequency ratios and mode shapes to replace Eqs. (1) and (2), if structures other than tall buildings are considered.

RESULTS

Fig. 1 compares the 5% damped SDOF and an 8 mode, 5% constantly damped MDOF velocity spectrum curve for the N-S component of the May, 1940, El Centro, earthquake (Berg Version). Fig. 2 illustrates 5% damped, 8 mode MDOF response spectra for several ground motions. Thus, the intensity of ground shaking for a particular building experiencing a particular ground motion can readily be assessed in both relative and absolute terms. Fig. 3 displays the results for maximum roof displacement and for the dimensionless base shear coefficient, S_V/W, for the N-S component of the San Fernando earthquake of February 9, 1971, as recorded at 8244 Orion Blvd. A comparison of results obtained by using several methods of analysis is included in Table 1. It is seen that the MDOF response curves for the 5 multi-storied buildings are in reasonably good agreement with those obtained by an "exact" elastic time-dependent dynamic analysis and those measured during the February 9, 1971 San Fernando earthquake. Generally speaking, it has been found that the MDOF curves predict maximum roof displacements and base overturning moments within 5% to 10% and base shears within 15% of those determined by an "exact" elastic time-dependent dynamic analysis. It is seen that the results obtained by the RMS spectral modal analysis do not compare as well. The "inaccuracies" of the MDOF results as noted for the 5 buildings listed in Table 1, are attributed to non-uniformity of mass and story heights and assumed mode shapes of those structures. However, the magnitude of such errors are perfectly acceptable, expecially when used for preliminary design purposes.

The intent of this paper has been to introduce the concept of the MDOF response spectrum and to illustrate how such spectral curves can be derived for any number of modes and any values of modal damping ratios. Furthermore, it has been shown that for multi-storied moment frame structures the reliability of such curves is satisfactory, possibly even for final design purposes. Similar MDOF spectral curves can be generated readily for other building types that can be represented by a relatively simple analytical model.

BIBLIOGRAPHY

(1) Jennings, P. C., "Spectrum Techniques for Tall Buildings", 4th World Conference on Earthquake Engineering, Santiago, Chile 1969.

COMPARISON OF MAXIMUM RESPONSE PARAMETERS FOR MULTI-STORIED MOMENT FRAME BUILDINGS

BUILDING NUMBER (DIRECTION)	NUMBER OF STORIES	NUMBER OF MODES	EARTHQUAKE RECORD	ROOF ACCELERATION, G				ROOF DISPLACEMENT, INCH				BASE SHEAR COEFFICIENT		OVERTURNING MOMENT COEFFICIENT	
				A	B	C	D	A	B	C	D	A	B	A	B
#1 (TRANSV.)	42	8	SAN FERNANDO FEB. 9, 1971 RECORDED AT BUILDING SITE	.150	.167	.157		14.2	13.3	11.7		.041	.034	.021	.020
#1 (LONGTL.)	42	8		.129	.135	.126		16.8	17.1	14.8		.052	.054	.034	.034
#2 (TRANSV.)	17	6		.157	.188	.132	.123	8.3	8.9	7.5	8.2	.049	.051	.026	.030
#2 (LONGTL.)	17	6		.161	.160	.149	.129	11.5	12.4	10.2	11.7	.068	.072	.040	.045
#3	60	8	#1	.334	.289	.246		19.3	16.0	14.8		.035	.032	.021	.016
			#2	.224	.500	.396		43.2	49.3	63.4		.089	.101	.065	.052
			#4	.203	.279	.239		20.7	26.0	21.1		.059	.056	.031	.027
#4	29	6	#1	.270	.321	.282		19.8	19.6	17.1		.072	.075	.045	.042
			#4	.416	.427	.269		27.1	27.7	22.9		.099	.099	.059	.059
#5 (TRANSV.)	11	6	#1	.540	.557	.555		6.0	6.5	5.7		.206	.144	.083	.074
#5 (LONGTL.)	11	6		.445	.611	.575		4.9	6.4	5.3		.213	.189	.129	.115

A) FROM DYNAMIC TIME HISTORY ANALYSIS, USING NORMAL MODE METHOD.
B) FROM MDOF RESPONSE SPECTRA, USING ACTUALLY COMPUTED FREQUENCY RATIOS.
C) FROM SDOF RESPONSE SPECTRA, USING RMS METHOD.
D) FROM ACTUAL RECORDING MADE DURING THE SAN FERNANDO EARTHQUAKE, FEBRUARY 9, 1971.

EARTHQUAKE #1 EL CENTRO, MAY 1940, N-S
#2 SIMULATED EARTHQUAKE 'A-1'
#3 SIMULATED EARTHQUAKE 'B-1'
#4 SAN FERNANDO, FEB. 9, 1971, N-S (8244 ORION BLVD., HOLIDAY INN)

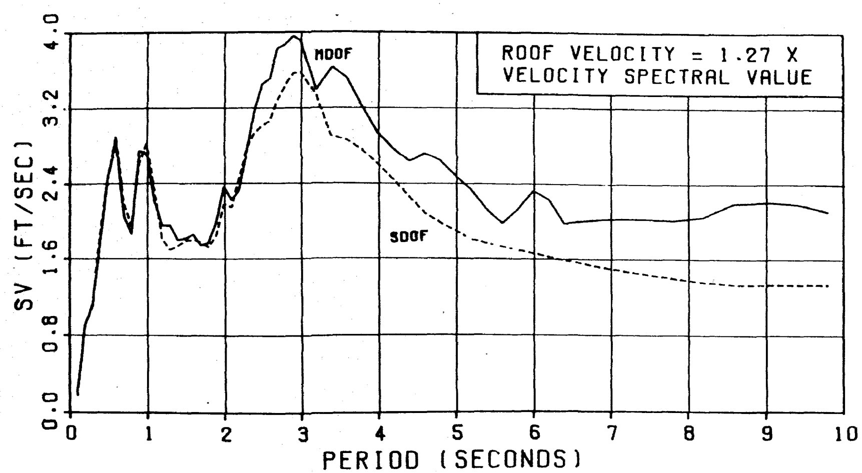


FIG. 1 COMPARISON OF SDOF AND MDOF VELOCITY RESPONSE SPECTRUM FOR MAY 1940 EL CENTRO EARTHQUAKE, N-S COMPONENT

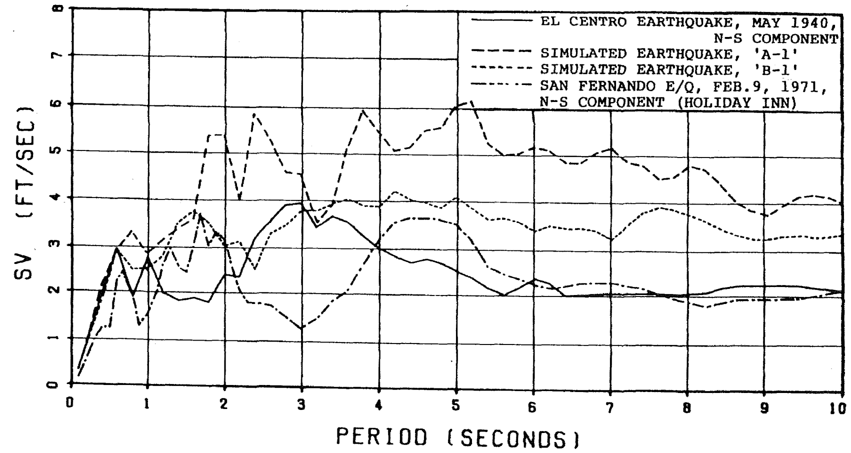
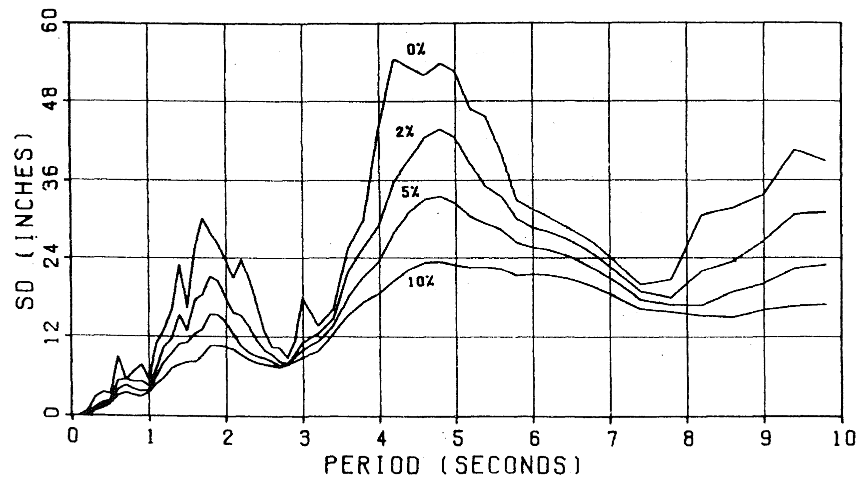
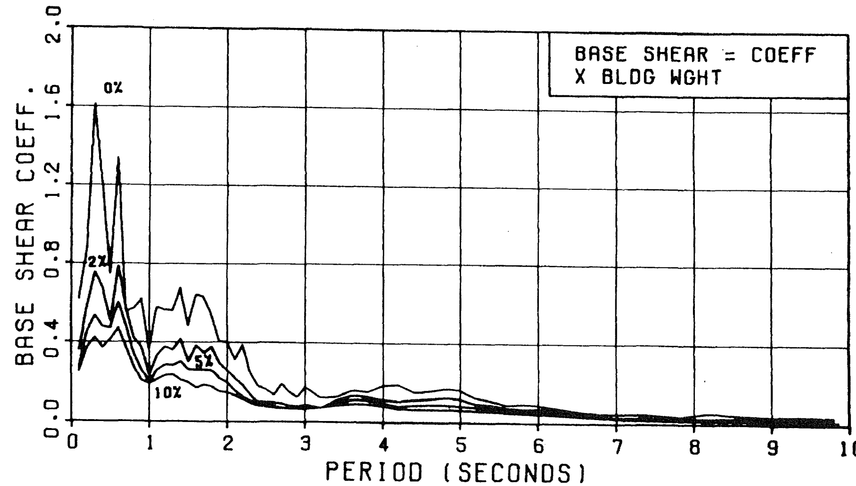


FIG. 2 MDOF VELOCITY RESPONSE SPECTRA FOR VARIOUS GROUND MOTIONS



a) ROOF DISPLACEMENT SPECTRUM



b) BASE SHEAR COEFFICIENT SPECTRUM

FIG. 3 MDOF RESPONSE SPECTRA FOR SAN FERNANDO EARTHQUAKE, FEB. 9, 1971, RECORDED AT HOLIDAY INN (8244 ORION BLVD.), N-S COMPONENT

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by

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DISCUSSION by: Sigmund A. Freeman[I]

The MDOF presented by the authors uses an idealized building model to represent a real building. The example in Figure 1 illustrates little difference between the MDOF and the SDOF spectra for fundamental periods less than 2 seconds. Therefore, it appears that the usefulness of the MDOF is most significant for large highrise structures over 20 stories. For buildings of this size it will usually be economically feasible to develop a mathematical model that more accurately describes the actual building rather than use an idealized model. However, as the authors state, the MDOF approach may be useful for preliminary design purposes.

In reviewing the Table I roof accelerations, columns a) and b) agree fairly well except in one case the difference is over 100% and in some others the differences are 20% to 40%. These differences represent variations between the calculated mathematical model and the idealized mathematical model as indicated by the authors. However, the comparison of roof accelerations in column c) and column a) appear to be in unreasonably poor agreement. I suspect that there may be some error in the values listed in column c). If the same mathematical models were used for a) and c), on the average, the agreement between the two methods should be fairly good. My own experience has shown that the SDOF response spectra using the square root of the sum of squares (which the author refers to as the RMS Method) usually gives good agreement with time-history analysis methods (1).

REFERENCE

(1) "San Fernando, California, Earthquake of February 9, 1971," Leonard Murphy Scientific Coordinator, ERL, NOAA. Department of Commerce, to be published December 1973. Buildings 29 and 30, Tables 8 and 9.

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Reply to Discussion by Sigmund A. Freeman

As Mr. Freeman noted, spectrum values computed by the root mean square method seemed to be in unreasonably poor agreement with results from time history analysis, as they were listed in Table I. Unfortunately, erroneous values had been recorded in Table I for the RMS method (Columns "C"). These values have now been corrected. The comparison shows much better agreement, but the authors believe none of the statements made in the paper are affected.