MEASUREMENTS AND STATISTICAL EVALUATION
OF DYNAMIC CHARACTERISTICS OF FULL-SCALE STRUCTURES

by

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SYNOPSIS

This paper presents the measurements of the dynamic characteristics of full-scale structures utilizing Fourier analysis of pseudo-ambient vibration records. The types of structures upon which measurements have been taken include:

(1) Multistory steel and concrete buildings with both moment resisting and shear/type frames.
(2) Bridges
(3) Internal components of structures (e.g. floor vibrations)
(4) Support structures used in high voltage electrical switchyards.

The nature of the analysis performed is somewhat unique in that it may be done entirely at the site of the structure. On site analysis is useful since it eliminates having to measure and save huge amounts of raw time series data. It is also convenient to view the results of the on site analysis to determine whether the vibration record is truly representative of the natural modes of vibration of the structure. Viewing of the results of the analysis also gives the researchers a better idea of what further analyses he wishes to perform.

Necessary to efficient analysis done with small computers is the utilization of an algorithm for the quick determination of the discrete Fourier transform (DFT). This has been accomplished by the development of the fast Fourier transform (FFT) (ref. 1). The FFT provides the same informational content as the DFT but in a much shorter calculation time when the number of data points used in the vibration record is a power of 2.

Analysis of a typical structure begins by first selecting an appropriate location to sample the vibrations of the structure. Enough different locations must be used so as to determine what modes of vibration are significant and what the nature of the mode shapes are. Good engineering sense is of importance here. Either a Kistler Servo accelerometer

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is used to produce an acceleration time record or a Hewlett-Packard laser interferometer system is used to produce a displacement time record. It has been found that the accelerometer is easier to use and is the most commom source for the vibration record. With the accelerometer in place at some location on the structure the analog time series data is transmitted via a coaxial cable to the location of the Fourier analysis equipment. The fourier analyzer unit consists essentially of an 8K computer, an analog to digital converter with a display unit, and the appropriate input and output devices including a teletype terminal and an X-Y recorder. An additional piece of equipment, a Khronhite filter, is used to "alias" or eliminate contaminating frequencies.

When the analog signal has been properly aliased it may be sampled according to our selection of F_{max} (Nyquist frequency) and the number of samples. The resulting discretized record is stored in the computer's memory. The FFT can then be called to transform the time series data into the frequency domain. In general the frequency data will be both imaginary and real in content. To make the data more useful the Fourier transform of the time series data is multiplied by its complex conjugate producing the energy density spectrum for the record of length, T. The inverse Fourier transform of this will produce the auto-correlation function of which the value at the origin is equivalent to the total power in the signal. The square root of the total power in the signal is better known as the root mean square (r.m.s.) level of acceleration (Figures 1 and 2). The usefulness of the power spectral density plot depends on our engineering sense and liberal use of good assumptions. Assuming that our structure is a linear, time invariant system and that the record analyzed is ergodic in nature, the power spectral density plot will be proportional to the transfer function for the structure. Typically, the record is stationary but not ergodic. To overcome the lack of ergodicity ensemble averaging is used where a number of power spectral density functions are added together until the equivalent length of data analyzed could be considered to be ergodic. Assuming that the above assumptions of stationarity and an essentially bandwidth limited excitation force are true, the resulting power spectral density function will contain the frequency and damping information for the linear structure. The frequency is easily determined since it will be measured as a fraction of the maximum frequency of the plot. The damping, as determined from ambient analysis, is always very small and this makes its calculation difficult.

There are many methods available for calculating damping, however, one of the easiest methods for use in the field is to measure the width of the log magnitude of the power density spectrum at the 3 dB level. (This corresponds to the half-power points for the linear scale.) This width divided by twice the resonant frequency is equivalent to the percentage of critical damping for that particular mode of vibration. There are many sources for error in all of these calculations. By understanding these sources the investigator will, hopefully, avoid any misinterpretations. We may classify these errors into two groups (1) errors due to manipulation of ambient analysis and (2) application of ambient analysis to non-ambient force levels. The first error in interpretation of ambient data can be due to modulation. Structures which transfer energy between one another periodically will exhibit a modulation of their natural frequency of vibration. Fourier analysis of the periodic modulation of a simple one degree freedom oscillator will produce, in general a central frequency located at the natural frequency of the oscillator and two side frequencies symmetrically spaced around the center frequency and separated by the modulation frequency. These side frequencies contain information about the nature of vibration but must not be misinterpreted as natural frequencies.
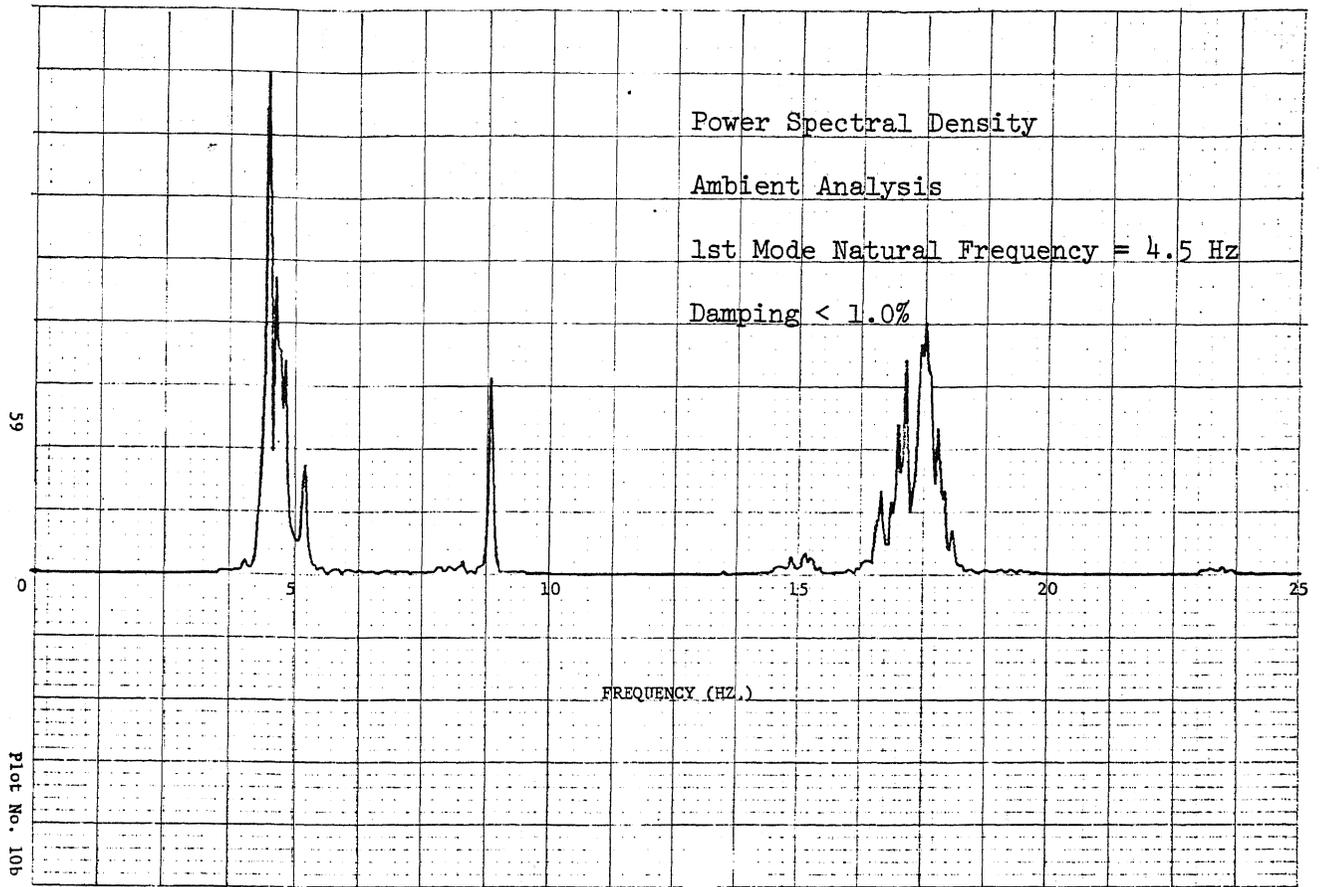
The determination of the natural frequency, excluding misinterpretations, is quite accurate, however the damping calculation is subject to great error. This is due largely to the fact that structural damping is always very small and the calculations are very sensitive to small errors in the shape of the resonance peak of the power spectrum. One common source of error is due to analysis of non-periodic data. The DFT or FFT will not correctly resolve the frequency components of a signal which is not periodic within the time interval, T. For non-periodic data the Hanning function should reduce leakage effects, helping to eliminate erroneous contributions to neighboring frequencies. The Hanning function should not be applied to non-stationary or non-stochastic data such as a damped sine wave, or pseudo-random noise. If Hanning were applied to the damped sine wave the damping calculated would be reduced by almost 50 percent.

There is yet another basic source of error in the measurement of damping which is always a problem. This is referred to as noise. When only a few data points define the peak at a resonant frequency each point carries a very significant percent of the damping information. Hence, small variations in the shape of the spectrum due to noise can greatly affect the damping calculated. The only solution to this is to average as many power spectrums as possible to improve the quality of the resonance peak. Thus, the damping calculated under ambient conditions may not be very reliable. Typically the value calculated is around 1-2% of critical damping. Clearly this value is consistently lower than the damping experienced during high level, non-ambient force levels such as that due to earthquake loads. This has been graphically demonstrated by work performed on electrical switchyard equipment associated with the pumping of water along the California aqueduct (Figs. 1 and 2). Here, the structures were of such a size and nature that they could be measured ambiently as well as non-ambiently. Typically the damping increased from approximately 1 percent at the ambient force level to about 3 to 5 percent when the peak force level was about 1 g. at the top of the structure. It should also be noted that the frequency dropped usually, often as much as 10 percent. An example of this behavior between ambient and non-ambient force levels is included.

For detailed discussions and further results including statistical analysis of data, refer to ref. 2. In conclusion it can be said that measurement of dynamic characteristics of full-scale structures under ambient conditions can be made and the results made available in real time. However proper engineering judgment should be used in interpreting the ambient results for strong motion data.

BIBLIOGRAPHY

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Power Spectral Density

Ambient Analysis

1st Mode Natural Frequency = 4.5 Hz

Damping < 1.0%

59

0   5   10   15   20   25

FREQUENCY (HZ.)

Plot No. 10b

519-135

ACCELERATION

Non-Ambient Time Series Data T = 102.4 Seconds

TIME (SECS)

.5g —

0   102

Power Spectral Density

Non-Ambient Analysis

60

1st Mode Natural Freqency = 4.4Hz

Damping = 3.3%

0   2   4   6   8   10

Plot No. 10c