HYSTERETIC DAMPING SYSTEMS APPLIED TO SOIL-STRUCTURE INTERACTION BY FINITE ELEMENT METHOD

by

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SYNOPSIS

The problem of soil-structure interaction during earthquakes is investigated applying the finite element method to the hysteretic damping systems. The complex stiffness matrices are formed with different damping values of soil and structures, then complex eigen value problems are solved, from which natural frequencies and modal dampings are evaluated. Transfer functions are computed by the modal superposition. Response quantities due to the earthquake motion which is assumed as the band-limited white noise are estimated in a frequency domain.

INTRODUCTION

The soil-structure interaction during earthquakes has recently attracted the public attention from the aseismic point of view and being analysed by many researchers with different kinds of approaches. However the question always lies, regardless of sorts of techniques, in how to take into account the damping properties of materials in case that the system is composed of quite different materials as a building and soil. The traditional method where damping ratios are given directly to each mode of vibration of the complete system is likely to lead to unrealistic results, because each part has different influence on each mode of vibration. Hence it will be more reasonable to solve the composed system in such a way that damping properties are given to each part beforehand according to the characteristics of each material. The technique where the properties of each element can be completely arbitrary as the finite element method is quite powerful for solving this sort of problem, because damping characteristics have only to be given to elements like others. In this case the hysteretic damping is conveniently adopted not only because it will be physically realistic but because it has the advantages not to waste computer time and core memory so much. Under these circumstances, soil-structure interaction during earthquakes was investigated herein applying the finite element method to the hysteretically damped system with the complex stiffness.

HYSTERETIC DAMPING SYSTEM

In the hysteretic damping system the stiffness matrix is written as $\underline{K} + i\underline{C}$ where i is an imaginary unit. The complex eigen vector $\underline{u}$ and complex eigen value ω satisfy the following equation.

$$[\underline{K} + i\underline{C}]\underline{u} = \omega^2 \underline{M}\,\underline{u} \tag{1}$$

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in which $\underline{M}$ is a real mass matrix. Eq.(1) can be solved in the same manner as the real eigen value problem. Natural angular frequency $\bar{\omega}$ and damping ratio h would be ω_R and ω_I/ω_R respectively by analogy with viscous damping where ω_R and ω_I are real and imaginary part of ω respectively. When the damping ratio of each element is known as h_e, the imaginary part of element stiffness matrix, $\underline{C_e}$, may be written as $2h_e\underline{K_e}$ where $\underline{K_e}$ is a real part of element stiffness matrix. The superposition of element stiffness matrix to the complete system can be achieved in a similar fashion as the usual technique in a finite element method. When the system is forced by a harmonic ground motion $\ddot{X}_g = e^{i\omega t}$, the response absolute acceleration at the j-th point, $\ddot{X}_j + \ddot{X}_g$, is given by $\sum_r \beta_r u_{rj} S_r(i\omega)e^{i\omega t}$ in which β_r is the complex participation factor, u_{rj} is the complex mode shape and $S_r(i\omega)$ is the complex frequency response function which is written as $[1-(\omega/\omega_r)^2]^{-1}$, of the r-th mode. The transfer function of acceleration, $H(i\omega)$, is defined by $\sum_r \beta_r u_{rj} S_r(i\omega)$, and the amplification factor of frequency response is equal to $\sqrt{|H(i\omega)|^2}$. The mean square value of response acceleration $\overline{(\ddot{X}_j+\ddot{X}_g)^2}$ is given by $\int_0^\infty G(\omega)|H(i\omega)|^2 d\omega$ in which $G(\omega)$ is the power spectral density function of ground acceleration. If the ground acceleration is assumed to be a band-limited white noise whose intensity has the constant value, c, over the frequency range $0 \sim \omega_{max}$, then the ratio of output power to input power, R_j, becomes

$$R_j = \frac{\overline{(\ddot{X}_j + \ddot{X}_g)^2}}{c\,\omega_{max}} \doteq \frac{1}{\omega_{max}} \sum_{\omega=0}^{\omega_{max}} |H(i\omega)|^2 \Delta\omega \qquad (2)$$

ANALYSES AND RESULTS

The simple system shown in Fig.1 has been supposed in analyses. The building is 30 meters high above the ground and soil uniformly expands 30 meters deep. The lower boundaries of soil are assumed to be fixed, where the horizontal earthquake motion is introduced. The building as well as soil is transformed into the equivalent homogeneous continuum for simplification. Damping ratios of each element in a building, h_B, and in soil, h_S, are taken as different values 2 % and 10 % respectively. Of course no trouble occurs even if arbitrarily different damping values are given for every element. The following two cases associated with the fundamental natural frequency of the building with the fixed base, f_B, and that of soil without a building, f_S, have been analysed.

1) The case that f_B = 1 cps and f_S = 2 cps —— Fig.2 shows the relationship between natural frequencies and modal dampings for the first ten modes. A solid line is for the hysteretic damping system discussed herein and a dotted line is for Rayleigh viscous damping system in which the damping ratio for the fundamental mode is taken as 2 % and minimum. Every modal damping ratio for hysteretically damped system lies between h_B and h_S. The first, third and nineth modal damping ratios approximate to 2 % that is the damping of building alone, which shows that a building mostly vibrates and soil little deforms in these modes. On the other hand, except these modes h's are nearly equal to 10 % that is the damping of soil only, which shows that vibration of soil has a great influence on the composite system in those modes. In this way the hysteretic damping system markedly contrasts with the classical viscous damping

system. Hence in the analysis of a composite system with different kinds of materials, it might be more reasonable to deal with it as the former system. Magnification factors of acceleration at point ① through ⑤ are shown in Fig.3. Peaks of the first and third mode which appear only in a building and peaks of the second mode which commonly appear in each point can be clearly recognized. Effects of higher modes except them are negligible. Fig.4 shows $\sqrt{R_j}$ evaluated by eq.(2) with $\omega_{max} = 20\pi$ rps. It is recognized that the absolute acceleration would be amplified about 1.8 times on a ground surface and besides twice its surface value at the top of building.

2) The case that f_B = 2 cps and f_S = 1 cps —— Fig.5 shows the relationship between f and h for the first sixteen modes. h's are nearly equal to the damping of soil except the second mode, which tells that the deflection of soil has great influence over these modes of vibration. The second mode corresponds to the fundamental mode shape of building alone and the fact that its frequency is 50 % lower than f_B shows the influence of soil is remarkable to that extent. The high value of damping ratio, around 7 %, also shows the pronounced soil-structure interaction as is apparent comparing with the previous case. Any modes which correspond to the higher modes of building do not exist in the first sixteen modes, which shows, to be interesting, that a building can not vibrate in higher modes if soil is soft enough. Fig.6 shows the magnification factor. Attention should be called to the fact that even the response of building forms the highest peak in the first mode which corresponds to the first mode shape of soil alone. This also indicates the building tends to translate like a rigid body according to the movement of soil.

CONCLUSIONS

It has been proved that results of finite element-hysteretically damped systems applied to soil-structure interaction seem to be quite reasonable and that even only the values of modal damping can powerfully help us to understand the phenomena. Of course it can not be necessarily guaranteed that the hysteretic damping which is independent of the frequency is close to the actual behavior. Probably the combination of the hysteresis, viscous and radiation damping will be closer to real conditions. The author is intending to deal with such combination for future study.

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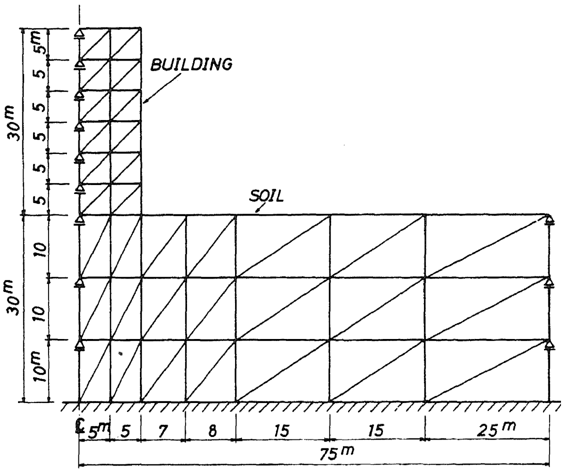

Fig.1 Finite element idealization

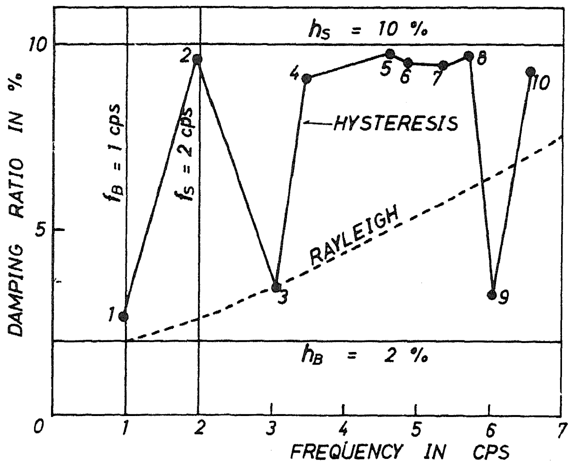

Fig.2 Relationship between frequency and damping ratio

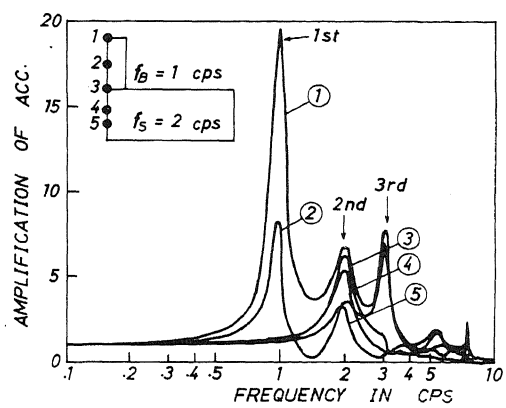

Fig.3 Magnification factor of acceleration

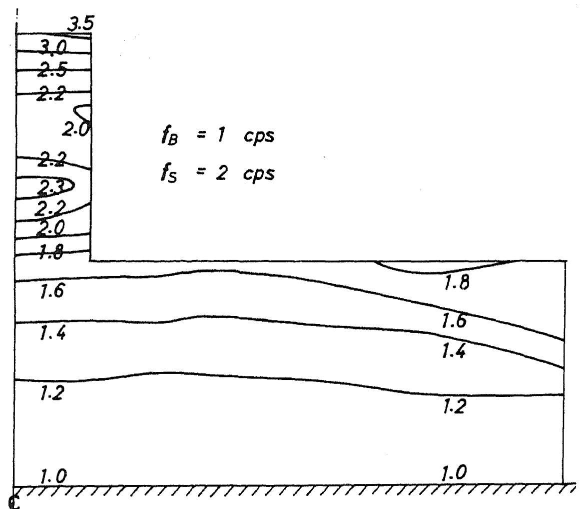

Fig.4 Root mean square value ratio

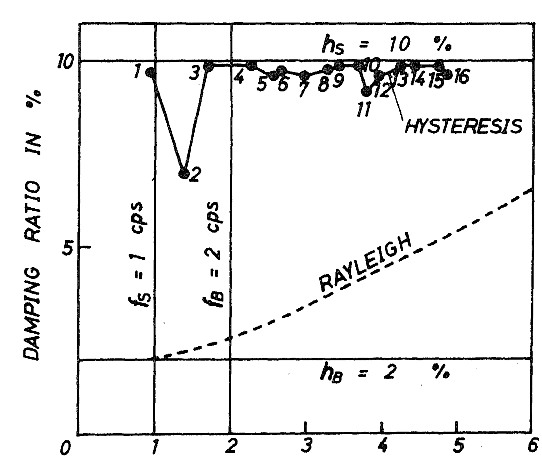

Fig.5 Relationship between frequency and damping ratio

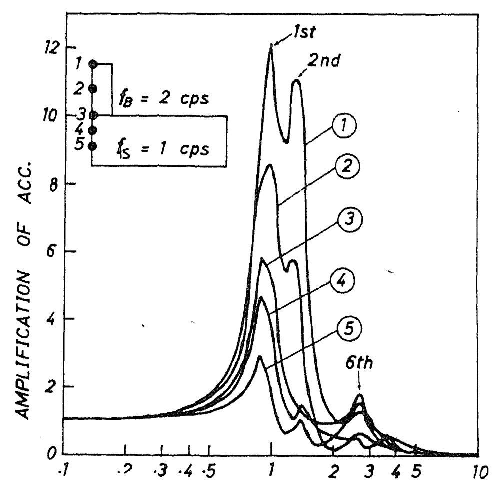

Fig.6 Magnification factor of acceleration