Sway-Rocking Vibration of Rigid Structure
Embedded in an Elastic Stratum

by

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SYNOPSIS

The present paper discusses on dynamic characteristics of rigid structure supported on hard soil layer at its base plane and surrounded by soft surface soil layer. In the present paper, after H. Tajimi (1969, Proc. 4-WCEE), not only rotation around the axis through the center of gravity of rigid structure but also swaying are taken into consideration. To obtain the information of fundamental characteristics as first step, using the values represent the dynamic effects of surface layer vibration to the rigid body, two numerical example calculations are conducted.

1. Introduction

The present study is concerning the dynamic effects of surrounding soft soil layer to the embedded rigid structure which is supported on the lower hard soil layer. Professor H. Tajimi presented analytical solution applying three dimensional elastic wave theory in case of rigid cylindrical structure supported on lower hard soil layer, and having rocking vibration freedom around the fixed axis through the center line of bottom circular plane. In the present paper, similar solution is presented in case of vibrational freedom of not only the rotation around the axis through the center of gravity of the structure but also the translation of c.g. Analytical method is completely same as Tajimi's method, and treated under following assumptions.

1) Surface soil layer consists only single layer and is homogeneous elastic stratum.

2) Damping is treated as internal damping characteristics.

3) Vertical vibration is neglected.

4) Input motion at lower hard layer is horizontal sinusoidal wave of $U_g \cdot e^{i\omega t}$.

2. Moment and horizontal force acting at side wall of embedded part of the structure due to interaction between surface layer and the structure

Fig. 1 indicates the dynamical model used for the analysis.

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The displacement U(z) of the vertical center line of rigid structure relative to lower hard soil layer is given as following equation under the notation $U_\zeta e^{i\omega t}$ as displacement of c.g. relative to the base layer, $\varphi \cdot e^{i\omega t}$ as angular displacement around c.g.

$$U(z) = (\varphi \cdot z - \varphi \cdot \zeta + U_\zeta) \cdot e^{i\omega t}$$

expanding this U(z) by $\sin(n\pi z/2H)$ (n = 1,3,5......)

$$U(z) = \sum_{n=1,3,5}^{\infty} \left[\varphi \left\{ \frac{8H}{n^2\pi^2}(-1)^{\frac{n-1}{2}} - \frac{4\zeta}{n\pi} \right\} + U_\zeta \frac{4}{n\pi} \right] \sin\frac{n\pi z}{2H} \cdot e^{i\omega t}$$

Moment M and horizontal force P at side wall due to interaction are obtained as follows using Tajimi's solution.

$$M = -\alpha_1 k_R (\tilde{M}_{\varphi 1} + i\tilde{M}_{\varphi 2}) \cdot \varphi - \alpha_2 k_H \zeta^2 (\tilde{M}_{\zeta 1} + i\tilde{M}_{\zeta 2}) \left(\frac{U_\zeta}{\zeta}\right) - \gamma_1 I_a (\tilde{I}_{E1} + i\tilde{I}_{E2}) \left(\frac{-U_g \omega^2}{\zeta}\right)$$

$$P = -\beta_2 k_H (\tilde{P}_{\varphi 1} + i\tilde{P}_{\varphi 2}) \cdot (\zeta\varphi) - \beta_1 k_H (\tilde{P}_{\zeta 1} + i\tilde{P}_{\zeta 2}) \cdot U_\zeta - \gamma_2 m_a (\tilde{m}_{E1} + i\tilde{m}_{E2})(-U_g \omega^2)$$

where

$$\tilde{M}_{\varphi 1} + i\tilde{M}_{\varphi 2} = \frac{\sum_{n=1,3,5}^{\infty} \xi_n^2 \frac{\Omega_n}{n^2} \left\{ \left(\frac{a}{\zeta}\right)\left(\frac{2H}{\pi a}\right)\frac{(-1)^{\frac{n-1}{2}}}{n} - 1 \right\}^2}{\sum_{n=1,3,5}^{\infty} \Omega_{n\zeta} \left\{ \left(\frac{a}{\zeta}\right)\left(\frac{2H}{\pi a}\right)\frac{(-1)^{\frac{n-1}{2}}}{n} - 1 \right\}^2} , \quad \tilde{P}_{\zeta 1} + i\tilde{P}_{\zeta 2} = \frac{\sum_{n=1,3,5}^{\infty} \xi_n^2 \frac{\Omega_n}{n^2}}{\sum_{n=1,3,5}^{\infty} \Omega_{n\zeta}}$$

$$\tilde{M}_{\zeta 1} + i\tilde{M}_{\zeta 2} = \tilde{P}_{\varphi 1} + i\tilde{P}_{\varphi 2} = \frac{\sum_{n=1,3,5}^{\infty} \xi_n^2 \frac{\Omega_n}{n^2} \left\{ \left(\frac{a}{\zeta}\right)\left(\frac{2H}{\pi a}\right)\frac{(-1)^{\frac{n-1}{2}}}{n} - 1 \right\}}{\sum_{n=1,3,5}^{\infty} \Omega_{n\zeta} \left\{ \left(\frac{a}{\zeta}\right)\left(\frac{2H}{\pi a}\right)\frac{(-1)^{\frac{n-1}{2}}}{n} - 1 \right\}}$$

$$\tilde{I}_{E1} + i\tilde{I}_{E2} = \frac{\sum_{n=1,3,5}^{\infty} \frac{\Omega_n}{n^2} \left\{ \left(\frac{a}{\zeta}\right)\left(\frac{2H}{\pi a}\right)\frac{(-1)^{\frac{n-1}{2}}}{n} - 1 \right\}}{\sum_{n=1,3,5}^{\infty} \frac{\Omega_{n\zeta}}{n^2} \left\{ \left(\frac{a}{\zeta}\right)\left(\frac{2H}{\pi a}\right)\frac{(-1)^{\frac{n-1}{2}}}{n} - 1 \right\}} , \quad \tilde{m}_{E1} + i\tilde{m}_{E2} = \frac{\sum_{n=1,3,5}^{\infty} \frac{\Omega_n}{n^2}}{\sum_{n=1,3,5}^{\infty} \frac{\Omega_{n\zeta}}{n^2}}$$

$$\alpha_1 = \frac{a^2 \mu\pi \left(\frac{\pi a}{2H}\right)\left(\frac{\zeta}{a}\right)\left(\frac{4\zeta}{\pi}\right)}{k_R} \sum_{n=1,3,5}^{\infty} \Omega_{n\zeta} \left\{ \left(\frac{a}{\zeta}\right)\left(\frac{2H}{\pi a}\right)\frac{(-1)^{\frac{n-1}{2}}}{n} - 1 \right\}^2 , \quad \beta_1 = \frac{a\mu\pi\left(\frac{\pi a}{2H}\right)\frac{4}{\pi}}{k_H} \sum_{n=1,3,5}^{\infty} \Omega_{n\zeta}$$

$$\alpha_2 = \beta_2 = \frac{a^2 \mu\pi \left(\frac{\pi a}{2H}\right)\left(\frac{\zeta}{a}\right)\left(\frac{4\zeta}{\pi}\right)}{k_H \cdot \zeta^2} \sum_{n=1,3,5}^{\infty} \Omega_{n\zeta} \left\{ \left(\frac{a}{\zeta}\right)\left(\frac{2H}{\pi a}\right)\frac{(-1)^{\frac{n-1}{2}}}{n} - 1 \right\}$$

$$\gamma_1 = \frac{a^2 \mu\pi \left(\frac{\pi a}{2H}\right)\left(\frac{\zeta}{a}\right)\left(\frac{4\zeta}{\pi}\right)\frac{1}{\omega_g^2}}{I_a} \sum_{n=1,3,5}^{\infty} \frac{\Omega_{n\zeta}}{n^2} \left\{ \left(\frac{a}{\zeta}\right)\left(\frac{2H}{\pi a}\right)\frac{(-1)^{\frac{n-1}{2}}}{n} - 1 \right\} , \quad \gamma_2 = \frac{a\mu\pi\left(\frac{\pi a}{2H}\right)\frac{4}{\pi}\frac{1}{\omega_g^2}}{m_a} \sum_{n=1,3,5}^{\infty} \frac{\Omega_{n\zeta}}{n^2}$$

$$\Omega_n = \frac{4K_1(\eta_{Ln})K_1(\eta_{Tn}) + \eta_{Tn}K_1(\eta_{Ln})K_0(\eta_{Tn}) + \eta_{Ln}K_1(\eta_{Tn})K_0(\eta_{Ln})}{\{K_1(\eta_{Ln}) + \eta_{Ln}K_0(\eta_{Ln})\}\{K_1(\eta_{Tn}) + \eta_{Tn}K_0(\eta_{Tn})\} - K_1(\eta_{Ln})K_1(\eta_{Tn})} , \quad \Omega_{n\zeta} = \Omega_n \Big|_{\omega=0}$$

$$\eta_{Tn} = \xi_n \frac{\pi a}{2H} , \quad \eta_{Ln} = \eta_{Tn}\frac{c_T}{c_L} , \quad \xi_n^2 = n^2 - \left(\frac{\omega}{\omega_g}\right)^2 + i\,2h_g\frac{\omega}{\omega_g} , \quad i = \sqrt{-1}$$

ω = angular frequency
ω_g = $\pi c_T/2H$ (fundamental natural frequency of surface layer)
c_T = $\sqrt{\mu/\rho}$ (transverse wave velocity in surface layer)
c_L = $\sqrt{(\lambda+2\mu)/\rho}$ (longitudinal wave velocity in surface layer)
h_g = equivalent fraction of critical damping in surface layer
ρ = mass density of soil, λ, μ = Lame's constant
H = thickness of surface layer
k_H = $2a\pi\mu_b/(2-\nu_b)$ $\quad k_R = \pi a^3\mu_b/(2-2\nu_b)$

μ' = shear rigidity of lower hard layer
m_a = mass of removed soil replaced by rigid structure
I_a = moment of inertia of removed soil around the axis through its c.g.
H_0 = height of the top of rigid structure from the surface of lower hard soil layer
s = height of the c.g. of rigid structure from the surface of lower hard soil layer.
a = radius of rigid structure.
$K_i(z)$ = second kind modified Bessel functions of i-th order

3. Equations of sway-rocking motion of a cylindrical rigid body embedded in the elastic stratum

The sinusoidal response of the rigid structure is obtained by solving following equations using M and P given in section 2.

$$-\omega^2 m U_s = -k_H(U_s - s\cdot\varphi) - i\omega c_H(U_s - s\cdot\varphi) + P + \omega^2 m u_g$$
$$-\omega^2 I\varphi = -k_R\varphi + k_H(U_s - s\varphi)\cdot s - i\omega c_R\varphi + i\omega c_H(U_s - s\cdot\varphi)\cdot s + M$$

where, m and I are the mass and moment of inertia around axis through c.g. of rigid structure respectively.
k_H and k_R are the spring constants of horizontal and rotation at the Base of rigid structure, and c_H , c_R are the damping constants corresponding respective springs.

4. Example calculations

Computed results for the case of a model shown Fig. 2(a) under the parametric change of thickness H of surface layer are given in Fig.3 to Fig.8. Fig. 3 to 5 indicate the relation between the dynamic effects of spring rigidity due to side wall pressure from surface layer and ω/ω_g for the parameter of thickness H. Fig. 6 and 7 indicate the dynamic effects to the mass or moment of inertia due to the vibration of surface layer. The calculated values of nondimensional constants α_i(i = 1,2), β_i (i = 1,2), γ_i (i = 1,2) in these cases are given in Table 1 which contains those constants about model-2 shown in Fig. 2(b). The absolute displacement amplification factors in response at the top of rigid structure are presented in Fig. 8 and Fig. 9. For the purpose of comparison, calculated results for the case of rocking freedom only are also given in the figures by thin lines. In these figures ω_1 and ω_2 are the fundamental and the second natural frequencies of sway-rocking vibration, and ω_s is the natural frequency of single rocking freedom vibration in case of no surface layer. The assumption of $c_H = c_R = 0$ is adopted for the above example calculations.

BIBLIOGRAPHY

H. Tajimi: Dynamic Analysis of a Structure Embedded in an Elastic Stratum: 1969 Proc. 4WCEE

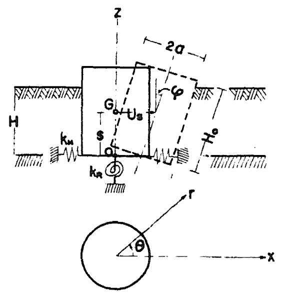

Fig. 1

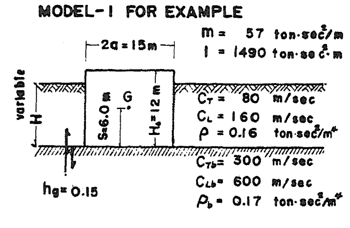

Fig. 2 (a)

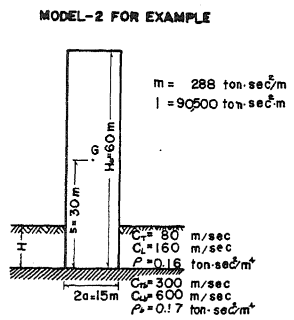

Fig. 2 (b)

MODEL NO.	H (m)	COEFFICIENTS CONNECT STATIC CONST. WITH DYNAMIC FACTORS				
		α_1	β_1	$\frac{\alpha_2}{\beta_2}$	γ_1	γ_2
MODE-1	12.0	.672	.863	-.504	1.07	3.66
	6.0	.596	.779	-.626	-1.32	1.58
	3.0	.578	.677	-.608	-1.23	.72
	1.2	.524	.560	-.534	-.61	.27
MODE-2	12.0	18.9	.863	-.791	-95.9	3.66
	6.0	18.5	.779	-.748	-73.3	1.58
	3.0	16.7	.677	-.663	-41.3	.72
	1.2	14.1	.560	-.555	-16.9	.27

TABLE-1

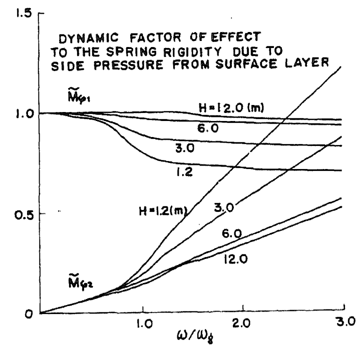

Fig. 3

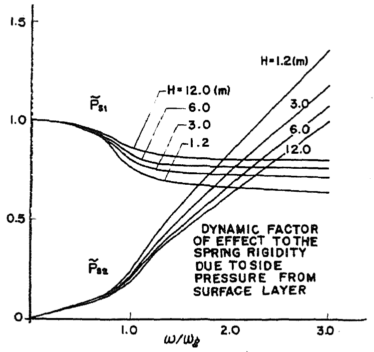

Fig. 4

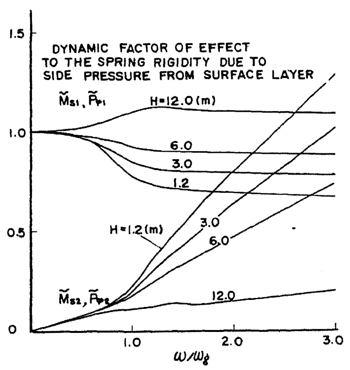

Fig. 5

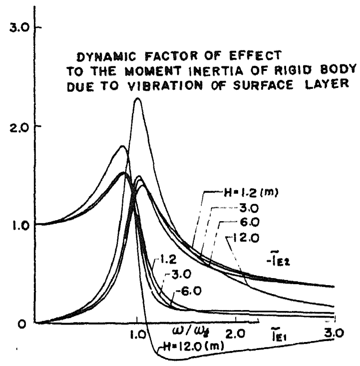

Fig. 6

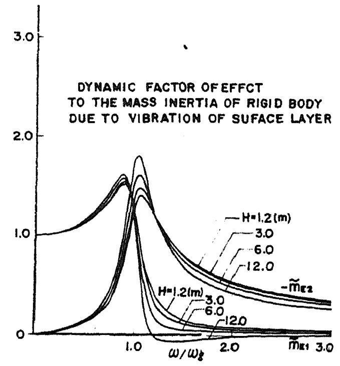

Fig. 7

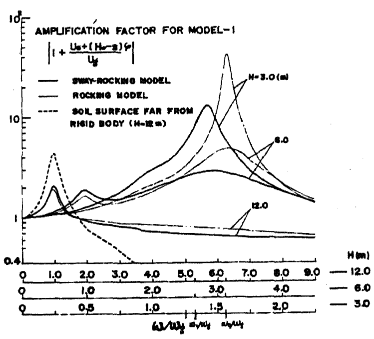

Fig. 8

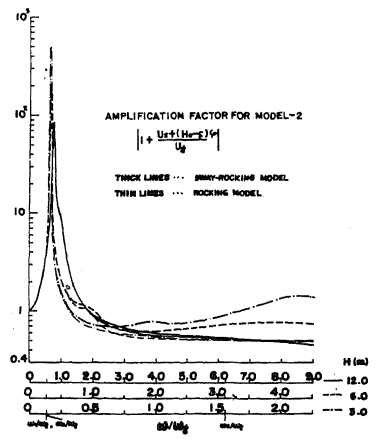

Fig. 9