EVALUATION OF DYNAMICAL PROPERTIES OF PILE FOUNDATION BASED ON WAVE DISSIPATION THEORY

by

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SYNOPSIS

The author obtained the spring constants of a pile foundation from the force displacement relation in case of placing one point excitation in an elastic half space and evaluated numerically the resonance curve of the pile, utilizing these obtained complex spring constants. The numerical results and field tests results maintain comparatively good agreements.

INTRODUCTION

In Japan, especially in the field of public works, a question how much equivalent mass for the pile or caisson foundation should be considered, has been being arised about these ten years and many researches concerning this problem were performed. At present, those experiments in laboratories or fields give the complicated or confused results.

Around ten years ago, Prof. Penzien proposed the method of dynamical analysis of a pile foundation, which replaces a pile to a lumped mass system and gives a calculation formula for the equivalent masses of the surrounding soil, too. And being based on the assumption that the damping factor to be considered would be mainly caused from the internal consumption of energy of soil, therefore, the factor is decided by the results of soil dynamic test in a laboratory.

The author has an opinion that the vibration properties of a pile could be described completely by the reaction force of the surrounding soil to a pile when the stress wave propagates from the vibrating pile into the ground very similar to the phenomenon in the case of a flat surface foundation.

The author made a vibration test of a pile due to an exciter placed at a top of the pile and was informed how much part of the surrounding soil vibrates in the same phase as the pile motion. The test results show that the volume of the surrounding soil vibrating with together the pile is much more a cross section of the pile which is frequently adopted as the equivalent mass volume in the case of a vibrating body in the water. The author considered that the reason why the volume of the vibrating soil is so much, comes from the fact that soil has the shear rigidity, however, the shear displacement can not propagate in the water. If we adopt the vibrating volume of the surrounding soil with the pile as the definition of the equivalent mass, the volume of the equivalent mass is to be numerous. After all, the auther confirmed that the vibration wave of soil starts from the pile, and the vibration properties could be

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analyzed by the wave propagation theory.

VIBRATION TEST OF A PILE

Dynamic test of a single pile was performed in the river site of a new bridge construction. The details of the pile and the exciter are indicated as follows,

Pile;	Steel, 60 m length, 16 mm breadth, 60 cm diameter.
Exciter;	Eccentric Mass Type. Frequency Range, 1~12 Hz. Maximum Force 40 ton (at 12 Hz).
Soil condition;	Fine sand, one layer. Water level is 20 meter deep, N value in a penetration test is shown in Fig. 2. Test site is located in the flood level of a river.
Location of vibration pick ups;	Accelerometers of semi-conductor were distributed in the boring holes as shown in Fig. 1 to disclose the vibration behavior of the ground soil near the vibrating pile.

A boring hole was made corresponding to each pick-up and pick-ups were set at the bottom of holes firmly in order to avoid the inclination or the settlement of pick-ups.

Dynamic Test

Resonance curves were obtained by changing the rotating frequencies of the exciter from 1 Hz to 10 Hz. Mainly the frequency change was made continuously by an automatic controller, therefore, this test is a kind of a sweep test and as a result two resonance peaks were obtained corresponding to increasing or decreasing of the exciting frequencies. Three kinds of weithts of rotar mass were adopted to make clear the influence of nonlinearity of the pile-soil system. The dynamic forces due to these different rotar's weight are 4 ton, 20 ton and 40 ton at the frequency of 12 Hz.

Test Results

Several resonance curves are shown in Fig. 2~4. These curves are normalized by a unit dynamic force. Fig. 3 indicates the resonance phenomenon at the site 20 m deep and 4 m apart from the pile. Particular characteristics of the vibration of this point are to have two peaks of resonance whereas a single peak at the top of the pile or at the surface of the ground. The frequency of a peak at the top of the pile is same as the one of two peaks at the underground. The author considers this phenomenon to be owing to the existence of many different vibration modes of the ground, therefore, at one peak the pile-ground system shows the resonance as a whole, however, at another peak the resonance phenomenon of the all system does'nt occur and the motion at a few deep points shows a large amplitude.

Fig. 5 indicates the vibration amplitude at same shase of the pile and its surrounding ground in the plane of the excitation of the pile top. From this figure, the author recognized that a diameter of the ground soil vibrating in phase with the pile is larger than ten meters, e.g. almost eight times of the pile's diameter.

Only on the surface of the ground, the author placed a pick-up 100 m away from the pile and the vibration amplitude there is small, but it has still some amplitude components of same phase with the pile.

The reason why the ground soil so far from the vibrating pile has the component of the same motion as the pile is considered to prove the elastic wave propagation from the pile.

If the author estimates an equivalent mass of the vibrating pile under the definition that the equivalent mass is the soil mass which vibrates in the same phase motion as the vibrating pile, the equivalent mass would be twenty or thirty times of the pile's mass itself. However, the "equivalent" mass is only a "convenient" mass for the dynamic analysis, and so it has no physical meanings. The author considers that it is not reasonable to adopt the equivalent mass which has been adopted to explain the elongation phenomena of the resonance periods of the structure supported by a soft ground, in order to estimate the earthquake force.

From the previous reason, the author computed the dynamic properties of a pile which is reduced to the lumped mass system without the equivalent mass and buried in a half elastic space.

DYNAMIC ANALYSIS OF PILE BASED ON WAVE DISSIPATION THEORY

The usual method to make the dynamical analysis of a pile foundation is to replace the pile as a lumped mass system as indicated in Fig. 6. In this case, there are several questions left in respect of the way to obtain the spring constants or the damping coefficients. The present method is to decide the spring constants from the Mindlin's static solution and the damping coefficients from the internal dissipation value at the laboratory test, however, the spring constants might be changed with frequency and the actual damping coefficients could include a part caused from the energy consumption by the wave dissipation.

Equation of Motion of a Pile

Equation of motion of a pile foundation is expressed as eq. (1) with Kmn of the stiffness matrix of the ground.

$$\rho \frac{\partial^2 y_m}{\partial t^2} + EI \frac{\partial^4 y_m}{\partial x^4} + \sum K_{mn} y_n = f(t) \qquad \cdots\cdots(1)$$

where, ρ: mass per unit length of the pile,
EI: bending stiffness of the pile,
f(t): external force

This equation can be solved by replacing it to a difference equation. In case of f(t) being equal to $F e^{i\omega t}$,

$$-\omega^2 \rho y_m + \frac{EI}{\Delta x^4}\left(y_{m+2} - 4y_{m+1} + 6y_m - 4y_{m-1} + y_{m-2} \right) + \sum k_{mn} y_n = F \qquad \cdots\cdots(2)$$

Since Kmn is a function of frequencies in general, Eigen value of eq. (2) cannot be obtained by placing the value of F to be zero.

Dynamic Spring Constants Kmn of Ground

Dynamic stiffness matrix $\mathbb{K}$ is obtained from the force displacement relation between the concentrated sinusoidal force acted in an elastic half space and its produced displacement.

Concentrated Force Acting in an Elastic Infinite Space

The method to obtain the solution contains three processes as below.

Fig. 7 (a) shows the state of a force applied in the half space, however, the solution corresponding to this state can not be obtained directly. Fig. 7 (a) is divided into two stages Fig. 7 (b), (c), and the solution of Fig. (a) is equal to the summation of the solution of Fig. (b) and Fig. (c). Fig. (b) shows the stress field in case of placing the two horizontal sinusoidal forces in an infinite elastic space at same distance from the assumed free surface and there is no shear stress produced on the assumed surface but the normal stress exists. The stress field in Fig. (a) is derived by extracting the stress field (Fig. (c)) which is produced by the surface normal stress of Fig. (b) acted on the surface of a half elastic space from the stress field of Fig. (b).

Stage 1) One point sinusoidal force in an infinite elastic space. This solution was obtained by H. Lamb in 1903 as the following expression.

$$w = \frac{R}{2\pi\omega^2\rho}\left[\frac{e^{-ihr}}{r^2}\left\{ih + \frac{1}{r}(1+h^2z^2) - \frac{3ihz^3}{r^2} - \frac{3z^2}{r^3}\right\} - \frac{e^{-ikr}}{r^2}\left\{ik + \frac{1}{r}(1+k^2z^2) - \frac{3ikz^3}{r^2} - \frac{3z^2}{r^3} - rk^2\right\}\right] \quad \cdots\cdots(3)$$

where, w : Displacement in direction of exiting force
R : Exciting force = $Re^{i\omega t}$
r : Distance from exciting force
z : Distance from and in direction of force
h : h = p/Vp
k : k = p/Vs
p : Circular frequencies

This is the solution in displacement representation and we can easily calculate the stress produced on the assumed surface from this solution, but the author doesn't write the stress expression here because of its length.

Stage 2) Vertical sinusoidal excitation on the surface of an elastic half space. The horizontal displacement Ur produced by the vertical excitation on the surface of an elastic half space is obtained as below.

$$U_r = \frac{Pe^{i\omega t}}{2\pi\mu}\int_0^\infty\left\{(2\xi^2 - j^2)e^{-\alpha z} - 2\alpha\beta e^{-\beta z}\right\}\frac{\xi^2 J_1(\xi r)}{F(\xi)}d\xi \quad \cdots\cdots(4)$$

where, μ : Shear rigidity
α : $\sqrt{\xi^2 - \omega^2/V_p^2}$

$\beta : \sqrt{\xi^2 - \omega^2/V_s^2}$
$j : \omega/V_s$

Then, the horizontal displacement in the half space when the normal stress obtained from stage 1) acts on the surface, can be obtained through the numerical integration of the above equation (4).

Stage 3) The author obtained the vibration displacement at each place of the pile by means of superposing the solution from stages 1) and 2).

Stiffness Matrix Kmn

The author evaluates the displacement matrix $\mathbb{U}mn$ at the position n with the pile radii distant from the pile axis in case of the acting position m of sinusoidal force. Stiffness matrix $\mathbb{K}mn$ is obtained as the inverse of the displacement matrix $\mathbb{U}mn$ and is of complex form since eq.s (3) and (4) are complex.

Solution of an Equation of Motion of a Pile

Stiffness matrix $\mathbb{K}mn$ is a complex spring coefficient corresponding to a concentrated sinusoidal force $P e^{i\omega t}$, therefore, its value variates with the frequencies of the force. Therefore, if the external force is not the sinusoidal one but of a complicated form such as the earthquake, the force must be solved into Fourier components and the last solution is obtained as the summation of each solution by the resolved Fourier components.

In case of sinusoidal force, eq. (1) will be

$$EI \frac{\partial^4 y_m}{\partial x^4} - \rho \omega^2 y_m + \sum K_{mn} y_n = A$$

This equation is expressed as the difference equation in eq. (5)

$$\frac{EI}{\Delta x^4}(y_{m-2} - 3y_{m-1} + 4y_m - 3y_{m+1} + y_{m+2}) - \rho\omega^2 y_m + \sum K_{mn} y_n = A \qquad \cdots\cdots(5)$$

This equation is a simultaneous equation of y_m and the solution y_m is a complex since the value of Kmn is of complex.

Numerical Results

1) One point excitation in the infinite elastic medium. Examples of numerical results for the Lamb's displacement solution are indicated in Fig.s 8, 9. The imaginary parts of the solution are not of so large in the frequency of 2 Hz, but greatly increased and changed the deformation form in 10 Hz.

2) Fig. 10 is a results for the stage 2) and shows the vibration displacement in the vicinity of the sinusoidal exciting forces which are placed in the symmetrical position against the assumed surface. The displacement shapes and the ratio of the imaginary parts variate with the change of frequencies of excitation.

3) The compressive stress destribution produced on the assumed surface in the stress field of stage 2), (Fig. 7) are indicated in Fig. 11. The stress value is of smaller with more distant excitation from the assumed surface.

4) Fig.s 12, 13 show the horizontal displacement in the half space with the one point excitation over the surface of the half space. Fig. 12 shows the relation between the horizontal displacement of the half space medium and the horizontal distance from the excitation position at same depth and Fig. 13 shows the displacement distribution form in the vertical direction at the constant horizontal distance from the excitation point.

5) By summing up the previous results, the author obtained the horizontal vibrational displacement at internal point of half space with the horizontal excitation placed in the arbitrary position. The contribution of the displacement obtained from the stage 2) to the total displacement is at most of several percentages. This fact tells that the displacement from the stage 2) which is the influence of the surface wave does not have the important role in the horizontal vibration problem of a pile foundation.

An example of real diagonal parts of the computed stiffness matrix $\mathbb{K}mn$ from the previously obtained vibration displacement matrix is shown in Fig. 14. It is pointed out that the values of stiffness are almost constant for the various depth, and this characteristics are different from the familiar tendency.

6) The frequency response curve in Fig. 16 and the vibration shape in Fig. 15 of the pile are obtained by solving the difference equation (5) against the top excitation of the pile utilizing the above obtained stiffness matrix $\mathbb{K}$. The following boundary condition is adopted.

at the top of the pile,

Moment = 0 , Shear force = $A\,e^{i\omega t}$

at the bottom of the pile,

Moment = 0 , Shear force = 0

The damping coefficient obtained from the computed resonance curve in Fig. 16 is approximately 5 percent.

Consideration

1) The author must take the value of Vs less than 50 m/s in order to make the theoretical resonance frequency agree with the results from the field test. The actual value of Vs at the site of test field is considered to be around 100 m/s. The difference between the actual value and the numerical results seems to be caused from the decrease of the spring constants by the slight fracture of the soil contact with the pile which was observed at time of the field test.

2) The values of damping coefficient are 5 percent in numerical results and 7 percent in field tests.

3) The influence of stage 2) that is the influence of the surface wave on the stiffness matrix is very small, so it can be neglected.

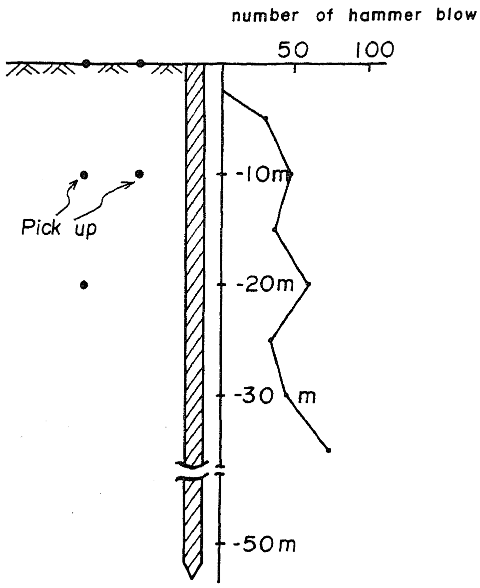

Fig. 1 Soil Condition of Pile Foundation

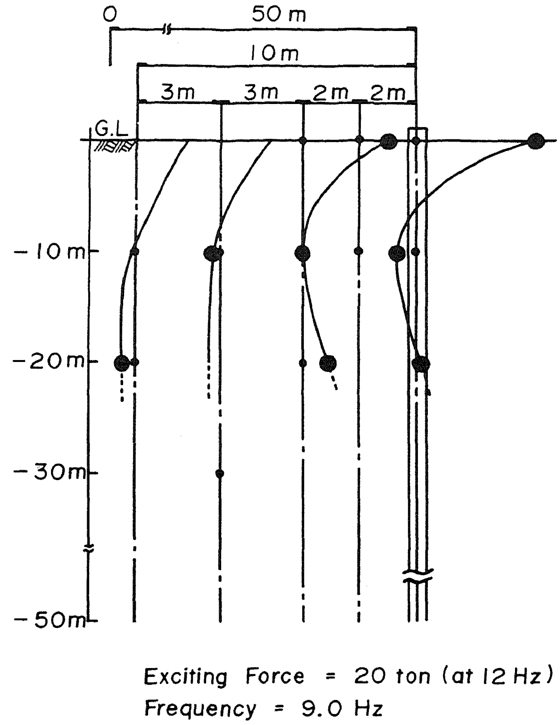

Fig. 5 Vibration Amplitude of Surrounding Soil of Pile

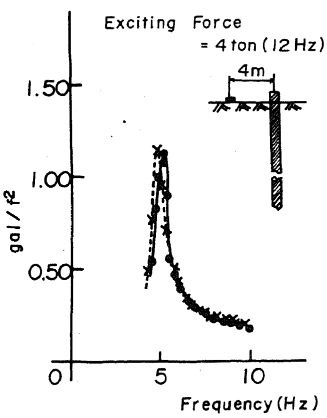

Fig. 2 Resonance Curve of Pile Foundation at Field Tests

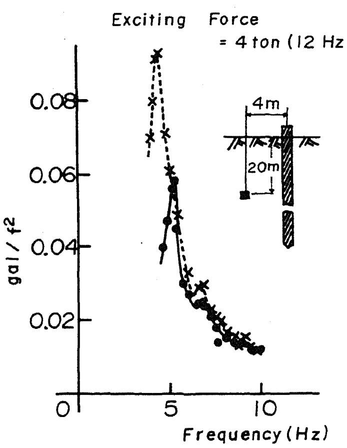

Fig. 3 Resonance Curve of Pile Foundation at Field Tests

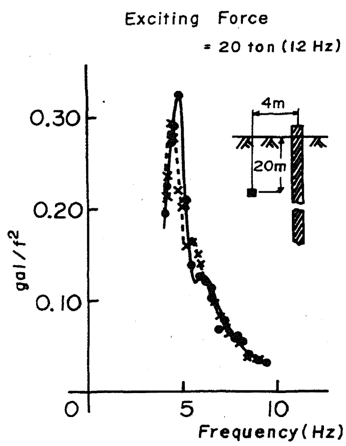

Fig. 4 Resonance Curve of Pile Foundation of Field Tests

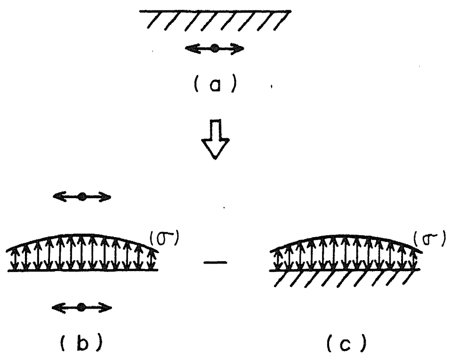

Fig. 7 Two Stages for Obtaining Solution of One Point Excitation in Elastic Half Space

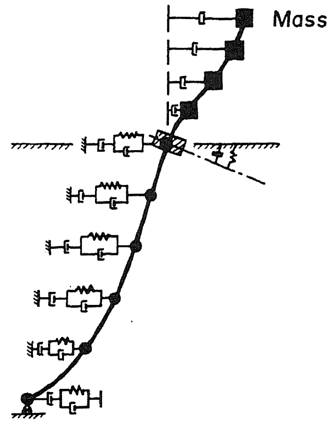

Fig. 6 Lumped Mass System for a Pile Foundation

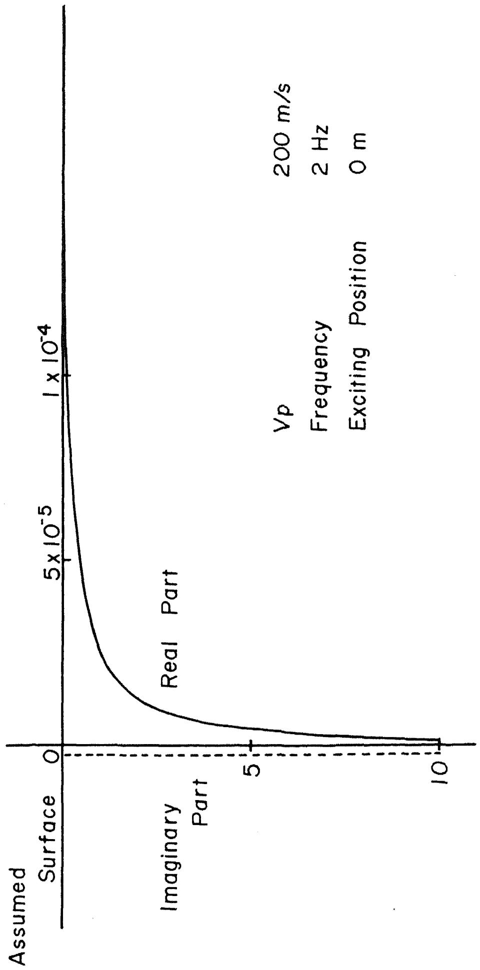

Fig. 8 Displacement in the Infinite Elastic Medium due to One Point Excitation

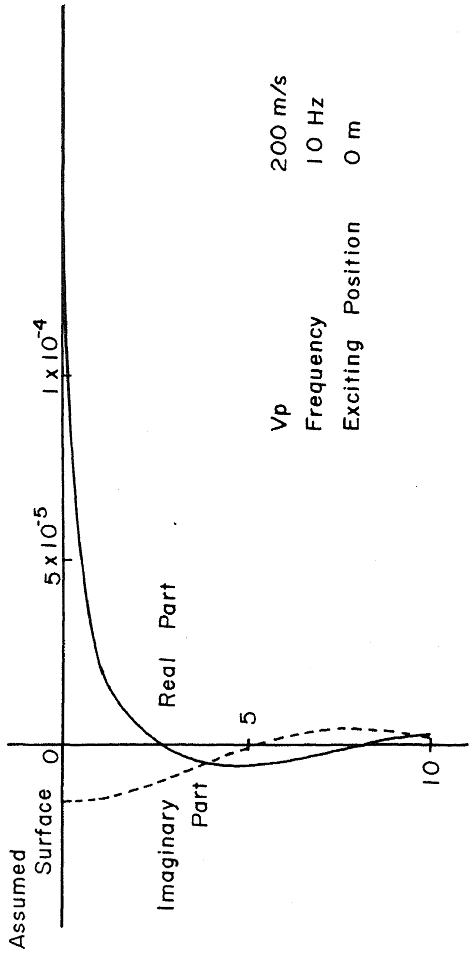

Fig. 9 Displacement in the Infinite Elastic Medium due to One Point Excitation

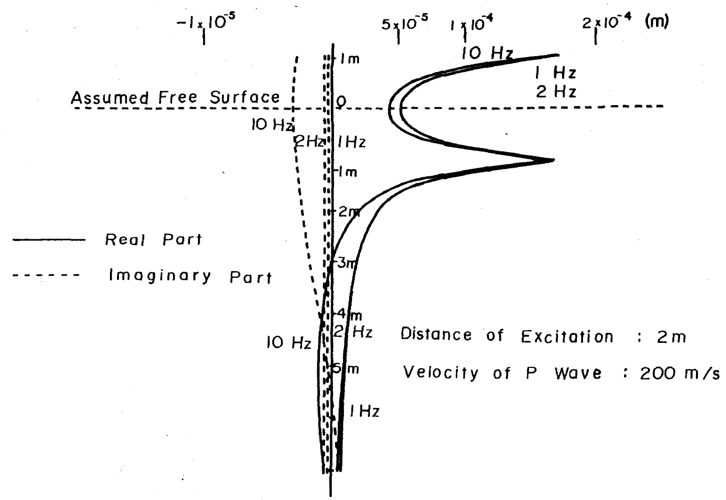

Fig. 10 Two Point Excitation in Infinite Elastic Space

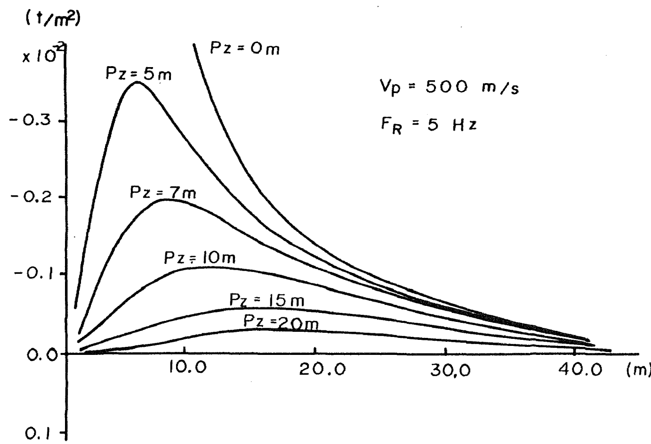

Fig. 11 Normal Stress Distribution Produced on the Assumed Surface (Fig. 7(b))

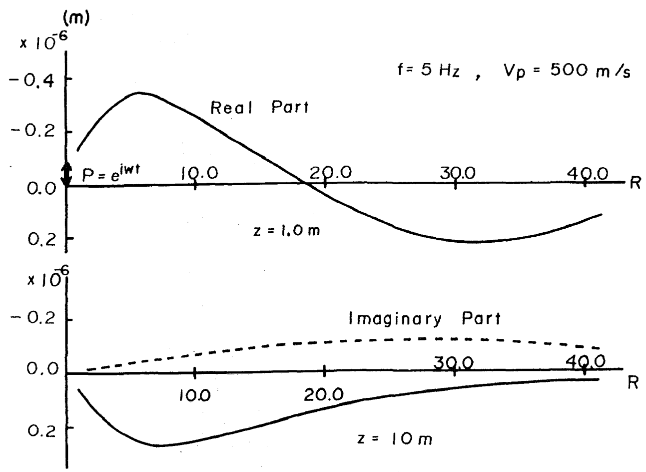

Fig. 12 Horizontal Response of the Ground to Surface Excitation

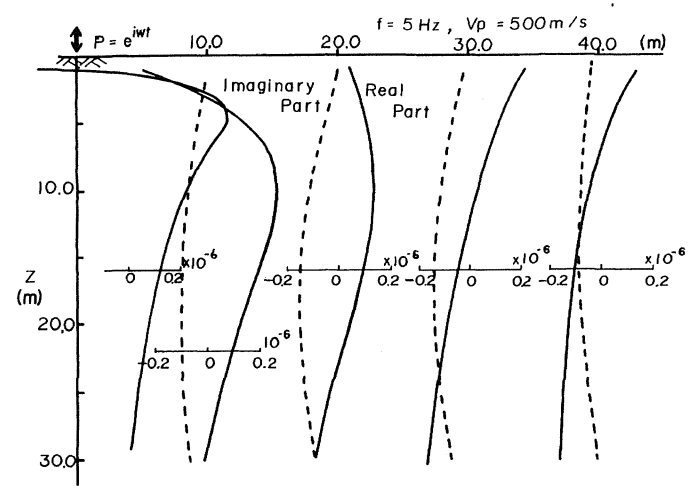

Fig. 13 Displacement in the Elastic Half Medium due to One Point Excitation on the Surface

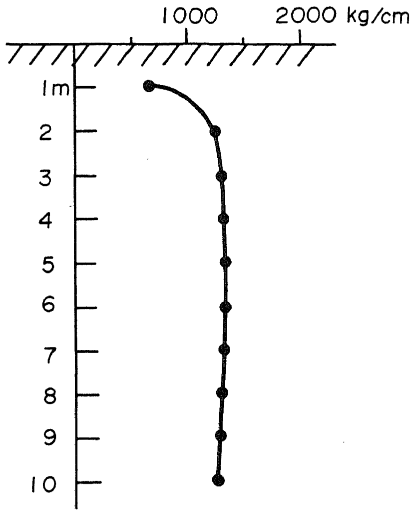

Fig. 14 Dynamic Spring Constants of the Ground

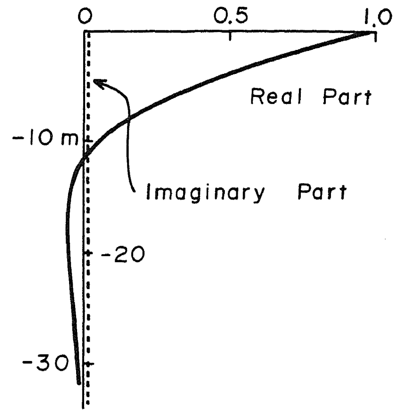

Fig. 15 Vibration Mode

Pile Length 40 m

EI 2 x 10 tm^2

Vs 50 m/s

σ 0.4

h (from Fig.) 0.05

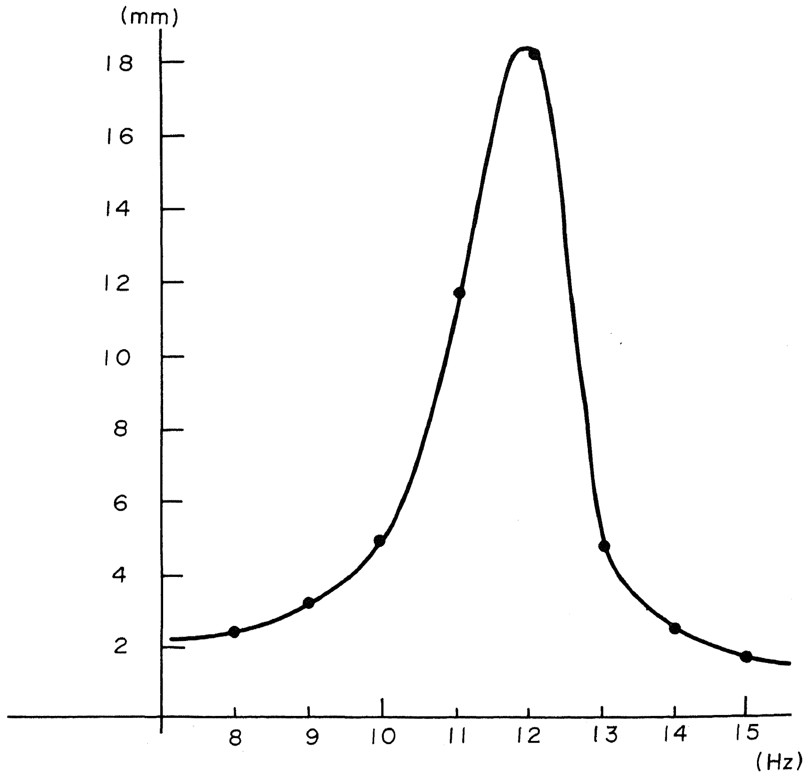

Fig. 16 Computed Resonance Curve of a Pile Foundation