A STOCHASTIC MODEL FOR PREDICTING SEISMIC RESPONSE OF LIGHT SECONDARY SYSTEMS

by

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SYNOPSIS

A stochastic model is presented for predicting the earthquake response of light secondary systems attached at one or several points of a primary system. The proposed model is evaluated on the basis of time-history solutions, and discussed relative to conventional response spectrum methods.

INTRODUCTION

The earthquake response of equipment, piping systems and other light secondary systems mounted on walls or floors of power plant buildings may be determined by time-history solutions or by the response spectrum method (2). Time-history solutions are expensive. Moreover, the maximum responses from earthquakes of the same intensity can vary considerably; thus, a single time-history solution is not sufficient to predict the maximum response of interest in aseismic design. On the other hand, the response spectrum method is not reliable when applied to the prediction of secondary systems response. Summarized herein is a random vibration method for calculating the response of light secondary systems, representing some "maximum" from a population of possible earthquakes.

RANDOM VIBRATION MODEL

The basic purpose of random vibration analysis in engineering is to predict structural response at a given probability of exceedance. This problem is closely related to the first passage problem in the theory of probability. Figure 1 shows one record of a process r(t) and $\pm b$ are the given barrier levels. Assuming that (i) the up-crossing of $+b$ and the down-crossing of $-b$ are independent events and that the crossings constitute a Poisson process, (ii) r(t) and its derivative $\dot{r}(t)$ are jointly Gausian with a zero mean, and (iii) the structural response is stationary; the response level corresponding to a specified probability of exceedance p_e is [see Ref. (1)].

$$b(p_e) = \sigma_r \left[2\ln \left\{ \frac{\sigma_{\dot{r}}\, t_d}{\pi\, \sigma_r\, \ln\left(\frac{1}{1-p_e}\right)} \right\} \right]^{1/2} \tag{1}$$

where t_d is the duration of the earthquake and σ_r^2 and $\sigma_{\dot{r}}^2$ are the variances of r(t) and $\dot{r}(t)$.

Response Variances -- To calculate the probability-based response of Eq. 1, the variances σ_r^2 and $\sigma_{\dot{r}}^2$ are required. Figure 2 shows a shear-beam secondary system connected to a primary system. An exact approach is to treat both the primary and the secondary systems as a single coupled system.

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However, in practical problems, this coupled approach generally leads to a large number of degrees of freedom; moreover, when there are large differences between the masses and stiffnesses of the secondary and primary systems, there may be a loss of calculational accuracy. In a decoupled approach the dynamic interaction between the primary and the secondary system is neglected. The response of the primary system to a specified ground motion serves as the input to the secondary system. Except for resonant systems, the error arising from decoupling is small for light secondary systems (2). With this decoupled approach, the expressions for the spring distortions for the system of Fig. 2 is

$$z_{sk} = \sum_{m=1}^{n_s} \{\gamma_{sm}[\psi_{sm}(k)-\psi_{sm}(k-1)]\int_0^t h_{sm}(t-\tau)\ \ddot{x}_1(\tau)d\tau\} \tag{2}$$

where γ_m is the participation factor in mode m; $\psi_m(k)$ is the kth element of the mth modal vector; $h_m(t)$ is the impulse response function in mode m; $\ddot{x}_1$ is the absolute acceleration of the supporting primary mass, and n is the number of modes considered. The subscript s refers to the secondary system. Based on the above equation the required variances are (3):

$$\sigma^2_{z_{sk}} = \sum_{m=1}^{n_s} \sum_{n=1}^{n_s} \sum_{j=1}^{n_p} \sum_{\ell=1}^{n_p} [A_{mj}(k)\ A_{n\ell}(k)\ \sigma^2_{mnj\ell}] \tag{3}$$

$\sigma_{\dot{z}_{sk}}$ is obtained by replacing $\sigma^2_{mnj\ell}$ by $\dot{\sigma}^2_{mnj\ell}$ in Eq. 3;

where $$A_{mj}(k) = \gamma_{sm}[\psi_{sm}(k)-\psi_{sm}(k-1)]\ \gamma_{pj}\ \psi_{pj}(1)\ \omega^2_{pj} \tag{4}$$

$$\sigma^2_{mnj\ell} = \int_{-\infty}^{\infty} H_{sm}(\omega)\ H^*_{sn}(\omega)\ H_{pj}(\omega)\ H^*_{p\ell}(\omega)\ G(\omega)d\omega \tag{5}$$

and $\dot{\sigma}^2_{mnj\ell}$ is obtained by replacing $\omega^2G(\omega)$ for $G(\omega)$ in Eq. 5;

in which $H_j(\omega)$ is the complex frequency response function in mode j, and $G(\omega)$ is the spectral density of the ground acceleration. The subscripts s and p refer to the secondary and primary systems, respectively. The above integrals are evaluated by the method of residue.

COMPARISON WITH TIME-HISTORY SOLUTIONS

For the structure of Figs. 2 and 3, the responses obtained by the proposed method at 50% and 10% probability of exceedance levels are compared with the average and second highest of eight time-history solutions. The two horizontal components of the following four earthquakes were used in the time-history analysis: El Centro (12/30/1934) NE,EW; Taft (7/21/1952) N21E, S69W; Olympia (4/13/1949) S80W,S10E; and El Centro (5/18/1940) NS, EW. The records were adjusted for base line correction and normalized by equating the area under the undamped velocity spectra. In the random vibration approach, the following expressions for the spectral density for ground acceleration (1) is used

$$G(\omega) = s_o \frac{1 + 4\beta_f(\omega/\omega_f)^2}{(1-\omega^2/\omega_f^2)^2 + 4\beta_f^2(\omega/\omega_f)^2} \cdot \frac{a+4\beta_g^2(\omega/\omega_g)^2 + c(\omega/\omega_g)^4}{(1-\omega^2/\omega_g^2)^2 + 4\beta_g^2(\omega/\omega_g)^2} \quad (6)$$

where $\beta_f = 0.64$, $\omega_f = 15.5/\text{sec}$, $a = 0.46$, $\beta_g = 0.81$, $c = 1.25$, $\omega_g = 12.6/\text{sec}$, and $s_o = 0.0024$. The constants a, β_g, and c were found by minimizing the squared error in the average velocity spectra obtained by Eq. 1 relative to those obtained from the above eight earthquakes. The calculated results are presented in columns 4 and 5 of Table 1. It is observed that except for resonant conditions, the random vibration results are in good agreement with time-history solutions (columns 2 and 3 of Table 1) and are far better than the corresponding response spectrum values (column 6). Even for more complex systems, the results of the proposed random vibration method are equally good (3); however, those of the response spectrum method can be far worse than those presented in Table 1 (3).

CONCLUSIONS

A random vibration method is presented for predicting the maximum response of light secondary systems to earthquake excitations. Except for resonant conditions, the method provides a convenient and reliable means for estimating the maximum response (at specified exceedance probability) of secondary systems to an earthquake with specified power spectral density.

APPENDIX I--REFERENCES

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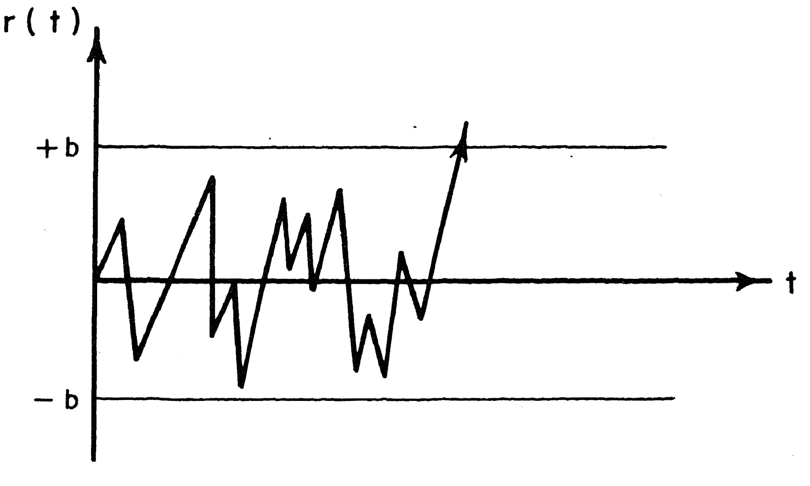

FIG. 1

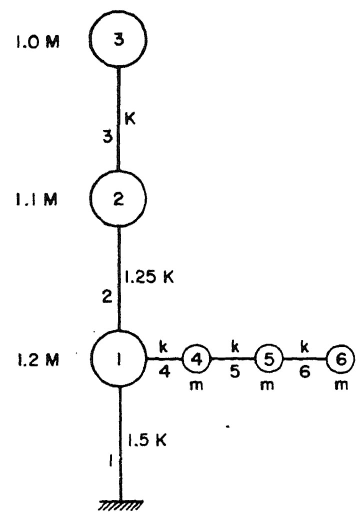

FIG. 2

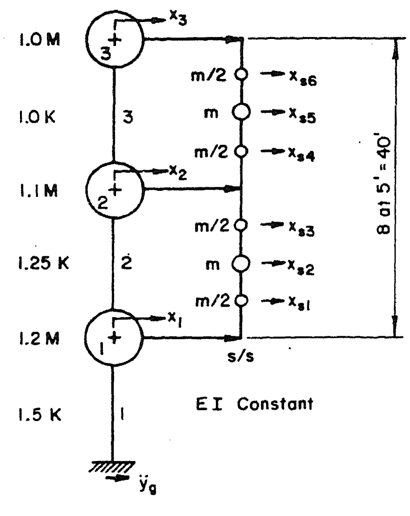

FIG. 3

TABLE 1 COMPARISON OF TIME HISTORY, RANDOM VIBRATION, AND RESPONSE SPECTRUM SOLUTIONS

$\frac{T_s(1)}{T_p(1)}$ (1)	TIME HISTORY SOLUTION AVERAGE (2)	TIME HISTORY SOLUTION 2ND HIGH (3)	$\|b_{.50}\|_{RV}$ AVERAGE (4)	$\|b_{.10}\|_{RV}$ 2ND HIGH (5)	$\|b\|_{RS}$ AVERAGE (6)
*STRUCTURE OF FIG. 2, MASS RATIO = 0.01, β_p = 0.05 AND β_s = 0.01					
ACCELERATION OF MASS 4					
0.50	5.65	6.39	1.10	1.13	0.98
0.75	7.41	9.66	0.99	0.88	0.86
1.00	11.52	12.07	1.36	1.52	0.93
1.25	8.07	9.59	0.98	0.96	0.85
1.50	4.89	5.85	0.98	0.95	0.90
2.00	2.67	3.00	0.96	1.01	0.91
3.00	4.67	5.84	1.24	1.18	0.85
DISTORTION IN SPRING 5					
0.50	0.030	0.035	1.10	1.08	0.90
0.75	0.027	0.029	1.13	1.23	1.33
1.00	0.165	0.198	1.61	1.60	0.96
1.25	0.125	0.162	0.98	0.91	0.70
1.50	0.103	0.130	1.05	1.01	0.68
2.00	0.119	0.151	0.95	0.91	0.66
3.00	0.186	0.223	0.95	0.97	0.61
*STRUCTURE OF FIG. 3, MASS RATIO = 0.01, β_p = 0.02 AND β_s - 0.02					
RELATIVE DISPLACEMENT OF MASS 5.					
0.50	0.052	0.059	0.94	0.96	1.35
0.75	0.062	0.074	0.99	0.97	1.85
1.00	0.251	0.299	1.59	1.59	1.08
1.25	0.335	0.427	0.93	0.87	0.95
2.00	0.370	0.413	1.01	1.09	0.86
3.00	0.297	0.347	0.96	1.00	0.73
RELATIVE DISPLACEMENT OF MASS 6					
0.50	0.044	0.056	1.03	0.95	0.97
0.75	0.051	0.059	0.95	0.97	1.26
1.00	0.174	0.209	1.61	1.59	0.98
1.25	0.193	0.252	0.92	0.84	0.77
2.00	0.163	0.188	0.99	1.02	0.91
3.00	0.124	0.150	0.99	1.00	0.82

* Primary structure frequencies -- 6.28 16.0 23.3 rad/sec.
Secondary structure frequencies: System of Fig. 2 -- ω, 2.79ω, 4.04ω
System of Fig. 3 -- ω, 1.54ω, 4.88ω, 5.94ω, 8.96ω, 10.2ω