METHODOLOGY FOR OPTIMUM SEISMIC DESIGN

by

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SYNOPSIS

The paper introduces a method based on Markov decision theory to describe the effects and evaluate the performance of building systems subjected to strong-motion earthquakes. The effect of a single earthquake is described in terms of (i) a matrix of probabilities of transition from one state of system serviceability to another, and (ii) a matrix of associated losses and benefits. Average discounted future costs are computed for various multi-state models of a building. Throughout the paper reference is made to a major study in which this methodology is applied to the evaluation of seismic design criteria for tall buildings in U.S. eastern metropolitan areas.

INTRODUCTION

In seismic design of buildings a fundamental trade-off between costly higher protection levels and higher risks of various levels of social and economic losses must be made. A rational formulation of seismic design decisions requires combining the uncertainties and the values and losses involved. The uncertainties entering into the computation of expected future losses due to earthquakes are of two types: first, the uncertainty in the occurrence characteristics of earthquakes of various intensities, secondly, the uncertainty in the effect each earthquake has on the building or the class of buildings being studied.

Methods of evaluation of seismic risk which aim at obtaining intensity-versus-return period curves have been developed[1-3]. They basically rest on the assumption that the times between successive exceedances of relatively high intensity levels at a given site are independent and exponentially distributed. The effect on structures of earthquakes with various intensities is more difficult to model. A number of recent papers [2,3,4] consider damage models in which the effects of successive earthquakes are stochastically independent. It is well-known, however, that previous damage may significantly influence future behavior. The model studied here allows the incremental damage at any stage to be stochastically dependent on the damage to date. It provides an excellent

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tool for analyzing the performance of structures for which several levels of unserviceability (e.g., minor nonstructural damage, minor structural and major nonstructural damage, collapse) can be identified. Intermediate damage states may contribute to the total losses either directly, in terms of repair costs, or indirectly, by causing a change in the risk of collapse during subsequent strong-motion shaking. For example, the natural period of a building may significantly increase as a result of a moderate earthquake. This may have the beneficial effect of moving the structural period out of the range of predominant frequencies in the ground motion at the site, but it may also have the opposite effect.

One of the most useful properties of a Markov model is that it provides a relatively simple framework for quantifying a building's performance in terms of economic loss. This aspect of the theory of discrete state Markov processes has received wide attention in the field of control theory and operations research[5,6,7]. Basically, the building (or class of structures) is visualized to accumulate a series of benefits and costs as it proceeds through various states of damage. The method allows all future benefits and losses to be discounted. The total expected cost which forms the basis for design decisions is the algebraic sum of the discounted losses and benefits and the cost of construction. Alternative designs or design strategies can then be evaluated by comparing their total expected costs, and the optimum design or design strategy is one which minimizes total expected cost. The use of Markov models to represent earthquake damage was first proposed by Vanmarcke[8]. An application to structures in the San Francisco Bay Area has recently been described by Shah and Vagliente[9].

EARTHQUAKE OCCURRENCE CHARACTERISTICS

Methods of seismic risk analysis[1-3] allow one to make reasonable estimates for the probability of exceeding a given site intensity say, MMI scale in any one year, by appropriate analysis of local historical records and of geological information. It is generally assumed that within a given seismically active area, strong earthquakes occur independently according to a Poisson process, and that earthquake occurrences and sizes in non-overlapping seismically active areas are independent. Under further mild assumptions relative to the laws governing intensity attenuation it can be shown that earthquakes causing a site intensity in excess of a high level y also follow a Poisson process with average annual occurrence rate λ_y. This mean rate λ_y, viewed as a function of the threshold intensity y, defines the risk-versus-intensity curve which characterizes the site seismicity.

It is often appropriate to discretize the intensities. If y_0 denotes the intensity below which associated building damage is negligibly small, one may choose to evaluate λ_y for a discrete set of y values, i.e., $y = y_0, y_1, \ldots, y_n$. Of particular interest in subsequent analysis are the related quantities $q_{y_k} = (\lambda_{y_{k+1}} - \lambda_{y_k})/\lambda_{y_0}$, $k = 0,1,\ldots n-1$ in which q_{y_k} = the probability that the site intensity equals y_k given an event in which it is at least y_0. Also, $q_n = \lambda_{y_n}/\lambda_{y_0}$ = the probability that the site intensity is at least y_n given that it is at least y_0. Note that $\sum_{k=0}^{n} q_k = 1$ and $\lambda_{y_k} = \lambda_{y_0} \sum_{j=k}^{n} q_j$. It will be notationally convenient to

use λ to denote λ_{y_0} in what follows.

STATES AND TRANSITIONS

To model the uncertain effect of various levels of earthquake ground motion, a structure or a building system is idealized by a finite number of states. Between earthquake occurrences, the system always occupies one and only one state. Transitions from one state to another may take place during or immediately following an earthquake. Any particular history of seismic effects can be represented by a list of successive states starting with state 0, e.g., 0→0→0→2→2.... This particular sequence indicates that 4 seismic events occurred: the first two left the building in its original state, the third caused a "jump" from state 0 to state 2, the building remained in state 2 through the fourth earthquake, and so on. Many different multi-state models of building behavior are possible. The choice will depend on the building system under study, on the repair and replacement policy considered, and on the level of accuracy sought. The only theoretical restriction is that all transitions must be so defined that every possible sample history can be represented and interpreted in an unambiguous way in terms of a sequence of successive states.

Figure 1 shows a few of the possible three-state representations of the behavior of a constructed facility during earthquakes[10]. In these so-called transition diagrams, the states are labeled 0, 1 and 2, and the arrows indicate the direction in which transitions can take place. One always starts in state 0 and makes a transition when an earthquake occurs. Each transition diagram corresponds to a different repair and replacement policy, as explained below.

Model A: Structure Deteriorates. No Replacement Upon Failure
This model represents a simple deterioration process: the structure can be serviceable (state 0), damaged, left unrepaired, but still serviceable (state 1), or completely unserviceable (state 2). When state 2 is reached, the structure is not replaced (or, at least, replacement is not considered in the analysis).

Model B: Immediate Repair or Replacement
This transition diagram is symmetrical with respect to the three states. It indicates that whichever state is occupied at a given instant, there are exactly two paths, labeled d (for damage) and f (for failure), out of that state, and a loop labeled s (for survival) which signifies return to that state. The sample history 0→2→1→0 indicates that the system failed three times during successive earthquakes and was replaced (by a nominally identical one) each time.

Model C: Two modes of Failure. No Replacement Upon Failure
This model is appropriate when there are two ways in which a building may become unserviceable. For example, an earthquake may cause (i) enough structural damage that the building is declared unsafe, or (ii) actual collapse of the building.

Model D: Immediate Damage Repair. No Replacement Upon Failure
This transition diagram combines elements of both Models A and B.

Note that it is symmetrical with respect to states 0 and 1. Repairable damage is sustained when the transition $0 \to 1$ or $1 \to 0$ occurs and transitions $0 \to 0$ and $1 \to 1$ imply no damage. But the transitions $0 \to 2$ and $1 \to 2$ signify that the system got trapped in the failure state.

It is important and rather easy to construct multi-state extensions of the models just presented. For Models A and B, for example, while preserving the respective "no repair" and "immediate repair" policies, additional intermediate damage states can be considered. In other situations, it will be appropriate to deal in a single model with both types of damage, one which is repaired and another which cannot be or is not repaired.

In an M.I.T. study aimed at evaluating tall building seismic design strategies[11], the model adopted is a direct extension of Model B: building performance is evaluated in terms of nine damage states ranging from "undamaged" to "collapse", and repair or replacement are assumed to be instantaneous.

CONDITIONAL AND MARGINAL TRANSITION PROBABILITIES

For any m-state model of a building system the uncertain effect of a single earthquake with known intensity y_k can be summarized in terms of an $m \times m$ matrix $P_{y_k} = [p_{ij|y_k}]$ of <u>conditional</u> one-step transition probabilities. The element $p_{ij|y_k}$ can be interpreted as the fraction of (nominally identical) buildings expected to move from state i to state j during an earthquake causing an intensity y_k at the site. The <u>marginal</u> one-step transition probability p_{ij} is the average of $p_{ij;y_k}$ with respect to y_k, i.e.,

$$p_{ij} = \sum_{k=0}^{n} p_{ij|y_k} \, q_{y_k} \tag{1}$$

It equals the probability of transition from state i to state j under an earthquake with site intensity $\geq y_0$. The $m \times m$ matrix of marginal transition probabilities is denoted by $P = [p_{ij}]$.

All quantities of interest, e.g., the probability that the system will be in a given state at some specified future time, can be shown to depend directly on the marginal transition probabilities p_{ij}[8]. In other words, the conditional probabilities $p_{ij|y_k}$ are only needed to construct the matrix $P = [p_{ij}]$ for a given site, building system and design strategy.

Figure 1 shows the P matrices for Models A through D. Note that only three elements (p_{01}, p_{02} and p_{12}) can be chosen independently in Models A and D. Also, the probability of failure will differ depending on the state the building occupies just prior to the earthquake; in particular, the ratio p_{12}/p_{02} must exceed one. In Models B and C, only the probabilities of transition out of state 0 (p_{01}, p_{02} and $p_{00} = 1 - p_{01} - p_{02}$) are needed. The same conclusion holds for the multi-state extensions of Models B and C: only the probabilities p_{0i}, $i = 0,1,\ldots,m-1$, are needed. The corresponding conditional probabilities are $p_{0i|y_k}$, and they

constitute the elements of the damage probability matrices whose evaluation forms the topic of another paper at this conference[12].

LOSSES AND REWARDS

For any m-state model of a building an m×m cost matrix $C = [c_{ij}]$ can be constructed. The element c_{ij}, $i \neq j$, represents the loss sustained (or benefit received) if the building system makes a transition from state i to state j. The quantity c_{ii} is the cost per unit time when the system occupies state i. (Note that c_{ii} and c_{ij}, $i \neq j$, do not have the same dimensions.

Figure 1 lists the C matrices for Models A through D. For example, for Model A, the elements $c_{01} = c_d$ and $c_{02} = c_f$ are the costs associated with damage and failure, respectively. c_{11} covers operating costs (minus benefits) per unit time; it could also represent the premium for insurance against earthquakes. c_{22} is the annual cost of operating a damaged structure. Diagonal elements can be put equal to zero if one is only interested in estimating future losses resulting from actual earthquake damage.

It is realized that modifications to this format are needed when losses associated with transitions cannot be expressed in monetary value. Multi-attributed losses must then be considered, and a different "cost" matrix may be needed for each attribute.

EXPECTED FUTURE LOSSES: BASIC RELATIONS

We define the following quantities:

$C_i(t)$ = the expected discounted total loss due to earthquakes during the time interval 0 to t if the system starts in state i.

δ = the discount rate: a unit quantity of money received after a very short time interval Δt is now worth $1 - \delta\Delta t$.

$q_i = c_{ii} + \sum_{k \neq i} \lambda p_{ik} c_{ik}$ = the mean loss rate of the system when it occupies state i.

It is very important in decision making involving constructed facilities to discount future losses. The choice of an appropriate discount factor raises complex issues[13], however, and this question will not be further pursued here.

The expected future losses $C_i(t)$ are governed by a set of ordinary differential equations[6,7]:

$$\frac{dC_i(t)}{dt} = q_i + \lambda \sum_{j=0}^{m-1} p_{ij} C_j(t) - (\delta+\lambda) C_i(t) \qquad (2)$$

$$i = 0,1,\ldots,m-1$$

Laplace transformations can be used to obtain closed-form solutions. At large values of t, however, $C_i(t)$ asymptotically approaches an upper bound $C_i = \lim_{t\to\infty} C_i(t)$. These long-range expected values are of considerable practical interest in earthquake engineering since the operational lifetimes of constructed facilities are often long and seldom predetermined. It is clear that when $t\to\infty$, the time derivative $dC_i(t)/dt \to 0$, and therefore the costs C_i can be determined from a simple system of m linear equations:

$$(\delta+\lambda)C_i = q_i + \lambda \sum_{j=0}^{m-1} p_{ij}\, C_j \qquad i = 0,1,\ldots,m-1 \qquad (3)$$

Generally only the expected total future loss C_0 will be of interest (since the building system always starts in state 0), except when the decision at hand involves questions of maintenance or condemnation of buildings.

To find C_0 for the three state models discussed earlier requires solving a set of three simultaneous linear equations. The resulting expressions are given in Figure 1. (For details, see Reference 10.) From these it is relatively easy to construct the expressions for C_0 corresponding to several important multi-state models. For example, in the multi-state extension of Model B (instantaneous repair and replacement), there are m-1 damage states in addition to the "no damage" state 0, m transition probabilities p_{0i}, and m transition costs $c_{0i} = c_i$ (where $c_{00} = c_0 = 0$). The resulting value for the total expected future loss C_0 is:

$$C_0 = \frac{\lambda}{\delta} \sum_{i=0}^{m-1} p_{0i}\, c_i \qquad (4)$$

It is further worth noting that if the damage costs c_i are uncertain, it is theoretically correct to replace them by their respective average values. Inserting Eq. 1 into Eq. 4 yields

$$C_0 = \frac{\lambda}{\delta} \sum_{i=0}^{m-1} \left(\sum_{k=0}^{n} p_{0i|y_k}\, q_{y_k} \right) c_i = \frac{\lambda}{\delta} \sum_{k=0}^{n} \bar{c}_{y_k}\, q_{y_k} = \sum_{k=0}^{n} \bar{c}_{y_k}\, \lambda_{y_k}$$

in which $\bar{c}_{y_k} = \sum_{i=0}^{m-1} p_{0i|y_k}\, c_i$ = the mean damage ratio[12] times the building replacement cost), given the intensity is y_k. In addition to direct repair costs, important human and social costs must be considered in evaluating the effects of strong earthquakes. These associated costs can be analyzed by the same method, and might be measured, for example, in number of lives lost. The conversion to equivalent monetary losses is possible only by placing a monetary value on life[12].

OPTIMUM SEISMIC DESIGN

The approach just outlined combines information about earthquake risks and consequences to obtain the average values, C_0, of the discounted future loss due to earthquakes for a building or for a class of buildings. The other major component in optimum seismic design is represented by A_0,

the increase in the initial cost due to special provisions for earthquake protection. If all losses can be expressed in monetary terms, then a reasonable design objective is to minimize the total expected cost, $A_0 + C_0$. In regions of low or moderate seismicity, other design considerations, e.g., functional requirements or resistance against wind, may enable one to design buildings for sufficiently low seismic risk levels without adding to the initial cost.

The methodology is quite general and can be applied to many types of buildings and other engineered facilities. Figure 2 outlines, by means of a flow chart, the methodology as it is used in a study at M.I.T. which focuses upon seismic design criteria for a class of tall buildings to be constructed in Boston, Massachusetts. The alternative design strategies considered correspond to the four seismic zones (0, 1, 2 and 3) of the Uniform Building Code. The repair policy adopted is one whereby all damaged buildings are repaired and all unserviceable buildings are replaced immediately following each earthquake. Hence, Eq. 5 can be used to evaluate average future losses. Reference 12 gives the categorization of levels of damage. These levels of damage are described both by words and by the average damage costs $c_i = d_i A$, in which A = replacement cost, and d_i = ratio, to replacement cost, of physical damage to the building and its contents. Damage probability matrices are estimated for each particular building and its contents. Damage probability matrices are estimated for each particular building system and each design strategy. Two approaches are followed in assembling these matrices: (i) actual observed damage (and non-damage) during past earthquakes is correlated with ground motion intensity[12], and (ii) theoretical predictions of dynamic response are used to interpret and extrapolate from the empirical information concerning damage and non-damage. The initial cost, in this study, is a function of the design strategy. It is expressed as the extra cost to design for, say, Zone 2 requirements as compared to making no provision for earthquake resistance.

CONCLUSION

A methodology based on Markov decision theory has been presented which evaluates the future performance of buildings under earthquakes and determines optimum seismic design levels and repair policies. To apply the analysis to a particular location, a suitable set of damage states, intensity categories and decision alternatives must be specified, and the corresponding probabilities and cost factors must be estimated.

ACKNOWLEDGEMENT

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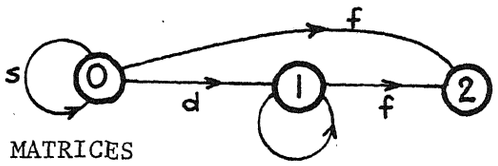
Model A



MATRICES

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ 0 & p_{11} & p_{12} \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} c_{00} & c_d & c_f \\ 0 & c_{11} & c_f \\ 0 & 0 & 0 \end{bmatrix}$$

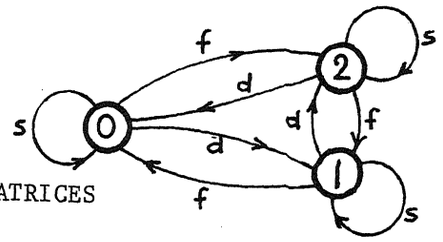
SOLUTION

$$C_0 = \frac{c_{00}+\lambda(p_{01}c_d+p_{02}c_f+p_{01}C_2)}{\delta+\lambda(p_{01}+p_{02})}$$

$$C_1 = (c_{00}+\lambda p_{12}c_f)/(\delta+\lambda p_{12})$$

$$C_2 = 0$$

Model B



MATRICES

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{02} & p_{00} & p_{01} \\ p_{01} & p_{02} & p_{00} \end{bmatrix}$$

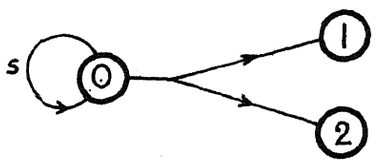
$$C = \begin{bmatrix} c_{00} & c_d & c_f \\ c_f & c_{00} & c_d \\ c_d & c_f & c_{00} \end{bmatrix}$$

SOLUTION

$$C_0 = C_1 = C_2 =$$

$$\frac{1}{\delta}\,[c_{00}+\,(p_{01}c_d+p_{02}c_f)]$$

Model C



MATRICES

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

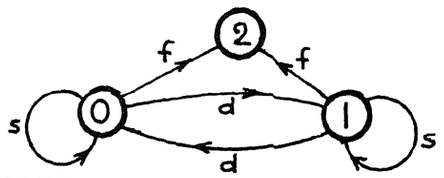
$$C = \begin{bmatrix} c_{00} & c_f & c_f' \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

SOLUTION

$$C_0 = \frac{c_{00}+\lambda(p_{01}c_f+p_{02}c_f')}{\delta+\lambda(p_{01}+p_{02})}$$

$$C_1 = C_2 = 0$$

Model D



MATRICES

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{01} & p_{00} & p_{02} \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} c_{00} & c_d & c_f \\ c_d & c_{00} & c_f \\ 0 & 0 & 0 \end{bmatrix}$$

SOLUTION

$$C_0 = C_1 = \frac{c_{00}+\lambda(p_{01}c_d+p_{02}c_f)}{\delta+\lambda p_{02}}$$

$$C_2 = 0$$

Figure 1: Some Three State Models of Seismic Performance of Buildings

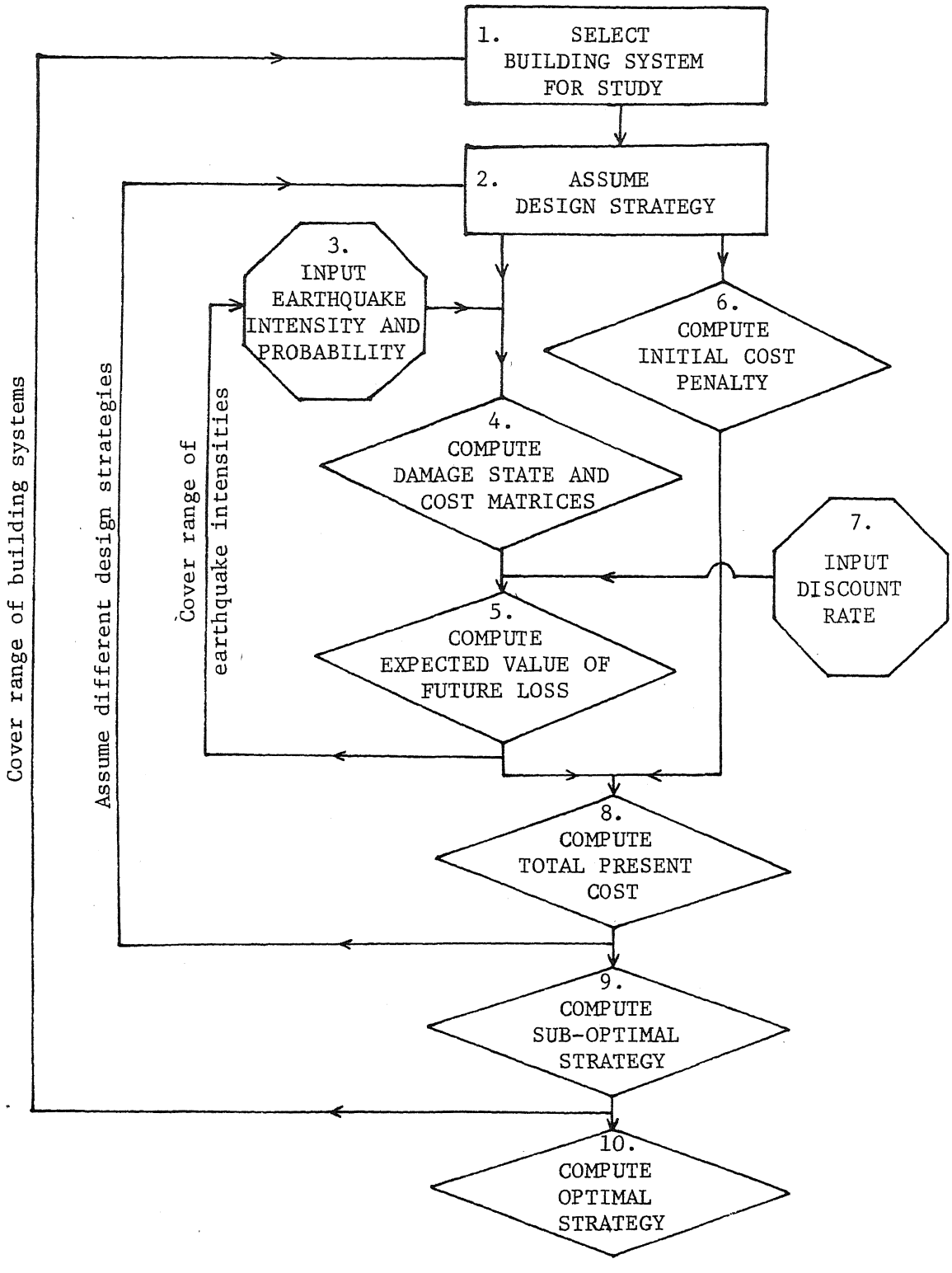


Figure 2: Flow Diagram for General Methodology