DESIGN DECISIONS FOR SEISMIC STRUCTURES

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SYNOPSIS

The structural design problem of a seismic structure is formulated on the basis of decision analysis. For the purpose of illustration, the design of columns for a one-story building structure is considered. The decision involves the choice between two alternative materials, namely steel and reinforced concrete.

INTRODUCTION

Decision analysis [1] provides a rational basis for making decisions in the face of uncertainty. The practical advantage of decision analysis results from the decomposition of a complex problem into its simpler parts. The treatment of uncertainty is based upon the mathematical laws of probability, the application of which is essential in earthquake engineering. The objective of this study is to apply decision theory to design problems involving the choice of materials for structures which are being designed to resist seismic loads.

DECISION THEORY

When all the facts are known, it is possible to choose the most desirable action from a given set of alternatives. However, in most problems, the relevant facts are frequently uncertain. In the face of this uncertainty, decision theory provides a rational solution.

In the present analysis it is assumed that the problem can be simplified to a two-step decision. The following four domains are involved, two of them being probabilistic and the other two being deterministic:

Experiment E: alternative ways to gather more information about uncertainties with element e,

Outcome Ω: results from experiment with element ω, following probability law,

Action A: decisions with element a, either following an experiment or without experiment,

State of nature Θ: the way the things really are, with element θ.

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It is further assumed that the decision-maker can assign a relative loss function $\ell(\theta,a)$ for the given set of state of the nature, θ, and action, a. Because uncertainties are involved in the decision making process, the most logical procedure is to minimize the expected loss. The notation E is used for expectation, the optimum decision rule a* will be

$$E[\ell(\theta,a^*)] = \min_{A} E[\ell(\theta,a)] \tag{1}$$

Following the definition of expectation,

$$E_{\Theta}[\ell(\theta,a)] = \int_{\Theta}\int_{A} \ell(\theta,a)f(\theta,a)d\theta da \tag{2}$$

in which $f(\theta,a)$ denotes the density function of θ accompanying action a. By applying the multiplication rule, it follows that

$$E_{\Theta,\Omega}[\ell(\theta,a)] = \int_{\Omega}\int_{\Theta}\int_{A}\ell(\theta,a)\ P_d(a(\omega)/\omega,\theta)f(\omega/\theta)\tau(\theta)da d\theta d\omega \tag{3}$$

in which $\tau(\theta)$ is the prior (prior to experiment ω) density function of θ, $f(\omega/\theta)$ is the conditional density function of ω given θ, and $P_d(a(\omega)/\omega,\theta)$ is a decision rule. A single action is taken, i.e., $P_d(a_1(\omega)/\omega,\theta) = \delta(a-a_1(\omega))$ where $\delta(\ )$ represents the Dirac delta function. Rearrangement of equation (3) yields

$$E_{\Theta,\Omega}[\ell(\theta,a)] = \int_{\Omega}\int_{A} P_d(a(\omega)/\omega)[\int_{\Theta}\ell(\theta,a)f(\omega/\theta)\tau(\theta)d\theta]da d\omega \tag{4}$$

With the substitution of equation (4) into equation (1), the optimum decision rule is then given by

$$\left.\begin{aligned} &\int_{\Theta}\ell(\theta,a^*)f(\omega/\theta)\tau(\theta)d\theta = \min_{A} \int_{\Theta}\ell(\theta,a)f(\omega/\theta)\tau(\theta)d\theta \\ &\text{and} \\ &\int_{A} P_d(a^*(\omega)/\omega)da = 1 \end{aligned}\right\} \tag{5}$$

The expected loss for this specific experiment, e, is then obtained by substituting equation (5) into (4). Thus

$$E_{\Theta,\Omega}[\ell(\theta,a^*)] = \int_{\Omega}\int_{\Theta} \ell(\theta,a^*)f(\omega/\theta)\tau(\theta)d\theta d\omega \tag{6}$$

The final decision for the choice of experiment e* is made such that

$$E_{\Theta,\Omega}[\ell(e^*,\omega,a^*,\theta] = \min_{\substack{a\epsilon A \\ e\epsilon E}} E_{\Theta,\Omega}[\ell(e,\omega,a,\theta)] \tag{7}$$

PROBABILISTIC DESCRIPTION OF EARTHQUAKE

Recently, it has been recognized that probabilistic analysis is essential in earthquake engineering because of the uncertainties in the character of ground motion. In this study, the model for the ground acceleration is assumed to be a white noise with two variables, the duration and the measure of magnitude of the motion. Furthermore, the duration for each magnitude is assumed to be uniformly distributed between zero and the maximum possible duration which has been estimated from the existing records. The spectral density of the white noise is determined by the magnitude of the earthquake. Because an earthquake represents the discharge of energy from the earth, it is reasonable to assume that the rate of occurrence is a function of time [2]. A Weibull distribution is used to predict the probabilities of occurrences of earthquakes.

AN EXAMPLE

Specifically, the failure of a linear one-story building structure as shown in Figure 1 resulting from dynamic response to earthquake is considered. From the known characteristics of the structure, the statistics of dynamic response can be obtained, and the probability p_y of first yielding can then be estimated from these statistics. As shown in Figure 2, the choice of materials between steel and reinforced concrete for the columns of the structure can be made on the basis of minimizing the total cost.

As numerical examples, columns are to be designed according to present specifications and for the Alaska region. For the purpose of these illustrative examples, the probabilities of occurrences of strong-motion earthquakes within a service life of 40 years as estimated previously for the Alaska area [2] are used. The parameters γ and μ are shape and scale factors, respectively, in a Weibull distribution, and r_o is the number of occurrences in the next 40 years.

Probability of first yielding as a function of earthquake duration is shown in Figure 3. Total cost, for the purpose of the illustrative examples, has been defined as the sum of the initial cost and the product of failure (yielding) probability and failure cost. The decision between two materials can be made according to Equation (7) after the initial cost and failure cost have been determined.

CONCLUSION

The application of decision analysis to the design of seismic structures is formulated herein. Limited results indicate that it is possible for structural engineers to make a rational choice among alternative designs if and when the initial cost and failure cost can be estimated in a realistic manner. For the specific example, it was found that the probability of first yielding is higher for concrete columns than that for steel columns if the columns, in each case, are designed according to present specifications.

REFERENCES

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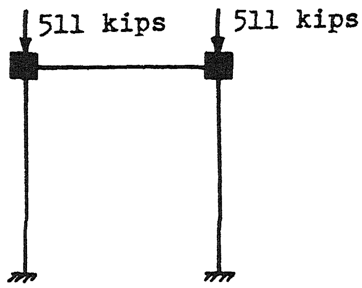
Figure 1. Structural Model



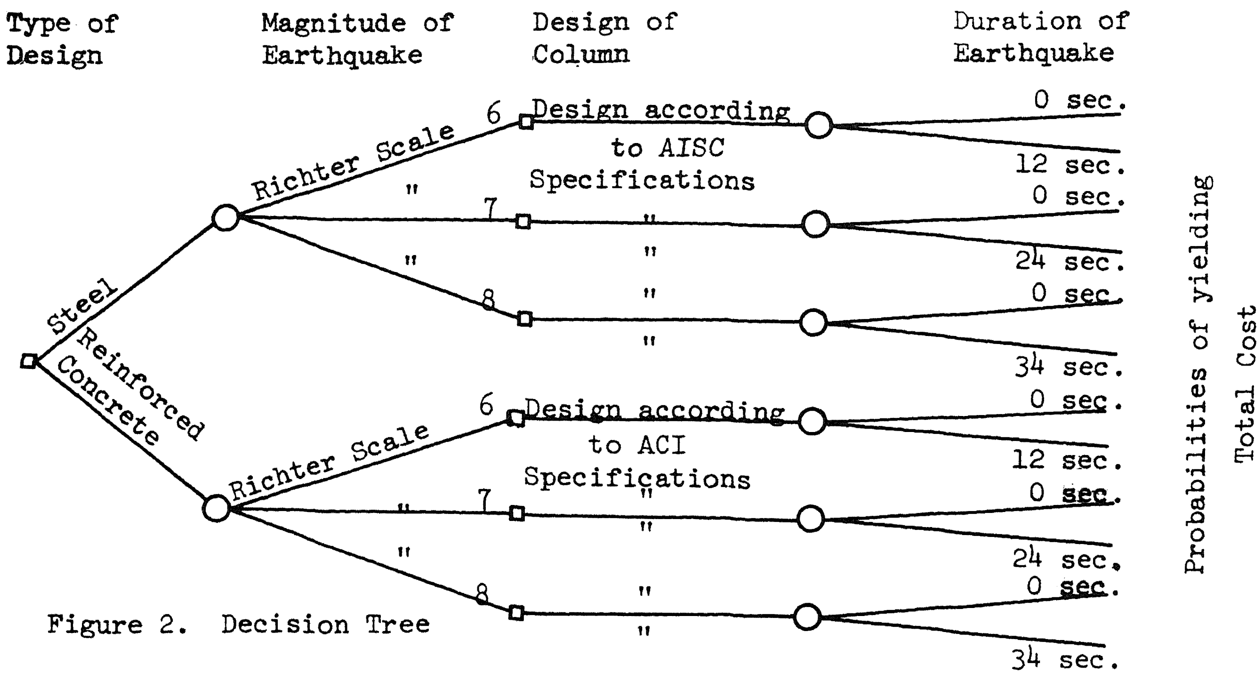


Figure 2. Decision Tree

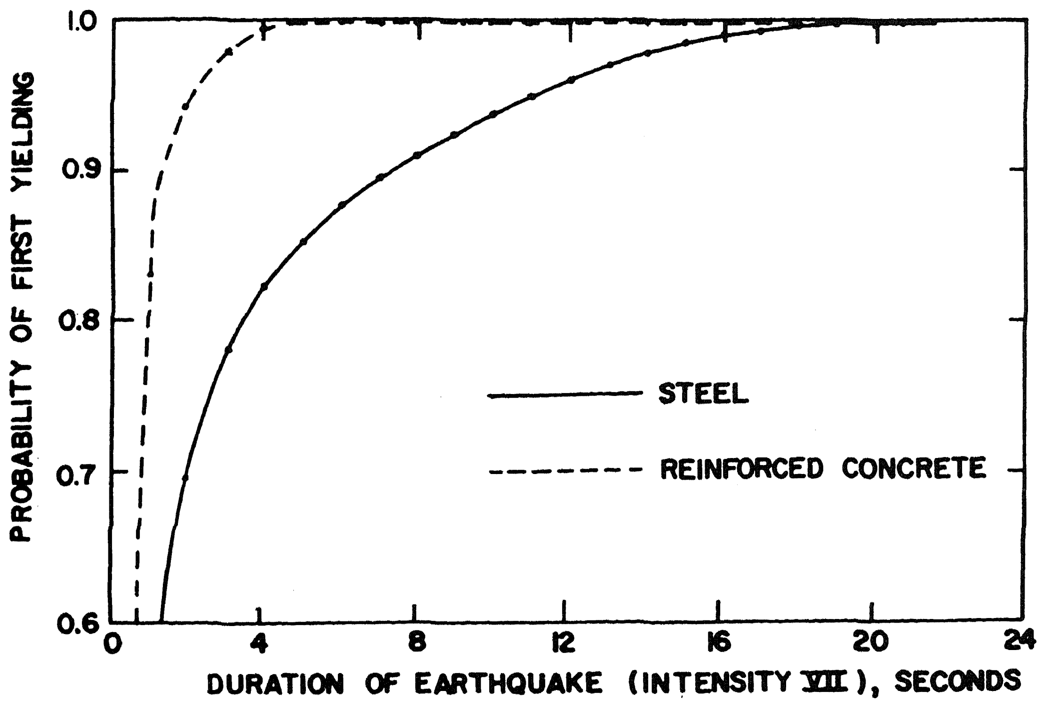


FIGURE 3 PROBABILITY OF FIRST YIELDING AS A FUNCTION OF EARTHQUAKE DURATION