DETERMINATION OF SUBSURFACE GROUND STRESSES FROM A GIVEN SURFICIAL DESIGN RESPONSE SPECTRA

by

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SYNOPSIS

Current seismic design techniques for nuclear plants make use of ground level design response spectra as the standard for predicting the dynamic response of structures and the subsurface dynamic soil stresses. In many instances, it is impractical to perform soil exploration to bedrock or to define the ground motion at bedrock, particularly when bedrock is deeper than 500 feet below the ground surface. This paper presents a method by which the subsurface shear stresses are determined solely as a function of the surficial design response spectra and soil properties to the depth of interest, thus eliminating the need to determine soil strata properties to bedrock. Also, the method can be used to determine a site dependent bedrock time history input which will reproduce a specified surficial time history or design response spectra.

NOMENCLATURE

X_i - Relative displacement of mass point i.

Y_i - Absolute displacement of mass point i.

Z - Spatial direction measured from the surface.

t - Time.

G_i - Dynamic shear modulus of soil layer i.

γ_i - Unit weight of soil layer i.

$2h_i$ - Depth of soil layer i.

k_i - Stiffness of layer i.

m_i - Mass of layer i.

c_i - Damping associated with layer i.

τ_i - Shear stress in layer i.

ρ_i - Mass density of layer i.

μ_i - Viscosity coefficient of layer i.

U_g - Bedrock displacement.

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METHOD OF ANALYSIS

Current seismic design techniques for nuclear plants make use of ground level design response spectra for determining the dynamic responses of structures. In order to predict the magnitude of dynamic soil stresses from the design spectrum, it is necessary to proceed through several steps of mathematical analyses. First, a time history for the ground surface which reproduces the specified surficial design response spectra for the site must be established using any of several methods [1],[2],[3]. Then, assuming the subsurface soil stratification can be idealized as a series of semi-infinite linear visco-elastic layers, the equilibrium equation within each layer is the one-dimensional wave equation for a linear visco-elastic shear beam:

$$(\partial^2 X/\partial t^2) - (\mu/\rho)\,(\partial^3 X/\partial Z^2 \partial t) - (G/\rho)\,(\partial^2 X/\partial Z^2) = -\ddot{U}_g\,(t) \qquad (1)$$

Damping is proportional to the rate of strain and is, therefore, internal as opposed to velocity dependent damping which is external. Utilizing a lumped parameter soil model, as suggested in reference (4), the equations of motion become:

$$M_{ij}\ddot{X}_j + C_{ij}\dot{X}_j + K_{ij}\,X_j = -M_{ij}\ddot{U}_{gj} \qquad (2)$$

in which:

$$\begin{aligned} K_{ij} &= k_{i-1} + k_i \quad \text{for } i = j \\ K_{ij} &= -k_i \;;\; -k_j \quad \text{for } j = i+1 \text{ and } j = i-1 \text{ respectively} \\ K_{ij} &= 0 \qquad \text{for all other } i,j\text{'s} \;; \end{aligned} \qquad (3)$$

the k_i's are the stiffness parameters of each of the soil layers, given by

$$k_i = G_i/2h_i \;; \qquad (4)$$

the mass matrix, M_{ij} is diagonal and defined by

$$M_{ii} = m_i = \{(\gamma_{i-1}h_{i-1}) + \gamma_i h_i\}/g \qquad (5)$$

where terms with a negative or zero subscript are defined equal to zero. The damping matrix C_{ij} is proportional to the stiffness matrix for internal visco-elastic damping. The value is taken as the percent of critical for each layer, as defined by Hardin[5], which varies inversely as the square root of the soil overburden stress at each soil strata.

Equation (2) is written in terms of the relative displacements X_i. Defining the absolute displacement of each mass, Y_i, as;

$$Y_i = X_i + U_g \qquad (6)$$

equation (2) may be written;

$$(c_i\dot{Y}_{i+1}) + (k_i Y_{i+1}) = (m_i\ddot{Y}_i) + (c_i\dot{Y}_i) + (k_i Y_i) + \{c_{i-1}(\dot{Y}_i - \dot{Y}_{i-1})\} + \{k_{i-1}(Y_i - Y_{i-1})\} \quad i = 1,N \qquad (7)$$

Equation (7) is the differential equation in terms of the displacement of the i + 1 layer and may be solved after the displacement functions Y_i and Y_{i-1} of the two layers above are known. Since the absolute responses of the ground Y_1, $\dot{Y}_1$ and $\ddot{Y}_1$ are known from the specified surficial time history acceleragram, and at time zero $Y_i(0) = 0$, the equations of motion at each layer can be solved in a stepwise manner. (i = 1, N corresponds to increasing depth of soil strata.)

As the solution of equation (7) requires repeated differentiation with respect to time, it is necessary to utilize the Finite Fourier Transform in order to solve equation (7) at each layer. The use of the Fourier Transform allows the differential operator to be replaced by an algebraic operator, thus eliminating the numerical problems associated with higher order differentiation.

Once the displacement at each layer is determined, the shear stress can be found from

$$\tau_i = k_i\,(Y_i - Y_{i+1}) \qquad (8)$$

A computer program was developed to perform the above computations and was checked by inputting a saw tooth acceleration at the base of a 10 mass soil model to determine the surface accelerations; then the above method was used to regenerate the bedrock acceleration. A comparison of the input base acceleration time history to the regenerated base time history is shown on Figure I. The 10 percent difference between the two time histories can be attributed to the number of Fourier terms used, the number of time points at which responses were calculated and the assumption that the time history is linear between time points.

For actual soils, the shear modulus and damping are highly dependent on the shear strain [(6),(7)]. To account for the strain dependent characteristics of soils, the above procedure can be applied in an iterative manner, revising the shear modulus and layer damping consistent with the strain, until compatible shear strains and soil properties have been determined.

CONCLUSION

For sites admitting to the semi-infinite layered soil model, the above procedure has the advantage of being able to generate the dynamic shear stresses at succeeding layers to any desired depth requiring knowledge of only the soil properties to the desired depth. Also, the basis of the dynamic stress calculations is the ground level design response spectra, which is also used in the structural design. In addition, the method can be used to generate a bedrock time history which matches a defined surficial design response spectrum to be used in more sophisticated finite element soil-structure interaction models.

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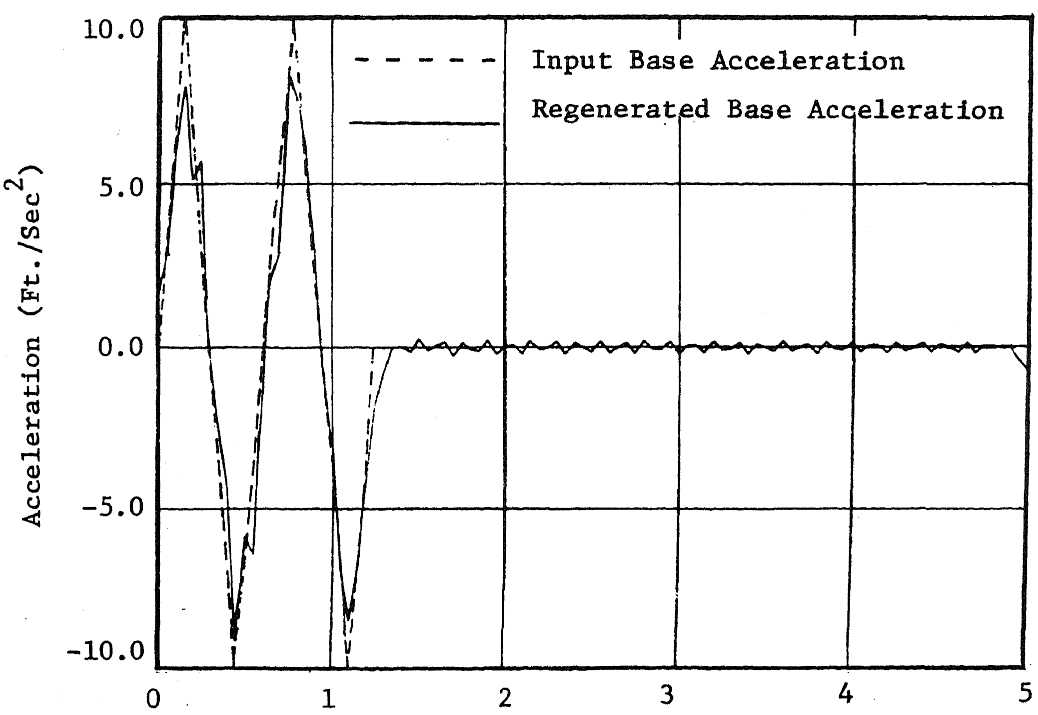

FIGURE I