DYNAMIC ANALYSIS OF STRIKE-SLIP FAULTING

by

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SYNOPSIS

Strike-slip generated by wave motions and wave motions generated by strike-slip, are analyzed for vertical surface faults on which motion is opposed by friction. In this analysis the fault is a plane of discontinuity in a homogeneous, isotropic, linearly elastic half-space. The half-space is subjected to gravity stresses and spatially uniform anti-plane shear stresses. The principal results of this paper are the rate of advance of the tip of the zone of sliding in the initial stages of the sliding process, and an expression for the maximum depth of penetration of the zone of sliding. Curves for motions on the surface of the fault are also presented.

INTRODUCTION

A theoretical analysis of the quasi-static mechanics of strike-slip on straight faults can be carried out in a relatively simple manner if it is assumed that the length of the zone of sliding is much greater than its depth, and if the crustal rock in which the fault is located is assumed to behave as a homogeneous, isotropic, linearly elastic solid. The analytical model then is an elastic half-space with a plane of discontinuity of infinite length and finite depth. In a typical analysis it is assumed that prior to sliding the stress distribution in the half-space includes body stresses with components τ_x, τ_y and τ_z, and tectonic stresses with components τ_{xz}, see Fig. 1. The body stresses are due to the weight of the material, and they may be assumed to increase linearly with depth. The tectonic stresses are due to a horizontal shear τ_∞, which is uniform with depth, and applied at far distances.

A fault normal to the surface was examined by Berg [1], who included an analysis of the effect of resistance to slip caused by the static and kinetic soil mechanical strength of breccia lining the fault. Strike-slip for faults of arbitrary dip on which motion is opposed by a frictional shear stress, was investigated by Walsh [2]. The aforementioned work is <u>quasi-static</u> in nature in that the horizontal shear stress τ_∞ is gradually applied, and the removal of the stresses on the

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fault surface also takes place gradually, so that static equilibrium exists at all times. In this paper we will consider two corresponding transient dynamic problems. The two problems can be treated by a single mathematical analysis. In the first problem, it is assumed that instead of a stress τ_∞ being gradually applied at $x = \pm \infty$, a stress τ_∞ is so rapidly applied at $x = -\infty$ that it gives rise to a horizontally polarized shock wave. The wave propagates through the body and strikes the fault plane. If the amplitude of the stress wave is large enough, sliding in conjunction with a process of reflection and diffraction of the incident wave is initiated as the wave reaches the fault. This type of problem is relevant to the generation of earthquakes by wave motions that are caused by explosions or by sliding at some other location. The second problem is concerned with the case where a critical condition for sliding has been reached quasi-statically, but where the actual unloading over the zone of sliding is treated as a transient dynamic problem involving the generation of elastic waves. A problem of the latter kind was also considered by Burridge and Halliday [3].

The uniform shear loading τ_∞ shown in Fig. 1 produces a displacement component $w(x,y,t)$ which is normal to the x-y plane, and which depends only upon the coordinates x and y and on the time t. The stress field caused by τ_∞ has only two components, namely, τ_{xz} and τ_{yz}. In the two-dimensional geometry of Fig. 1 the anti-plane deformation defined by $w(x,y,t)$ is not coupled to the in-plane deformations that are associated with the body stresses. Wave motions which involve displacements $w(x,y,t)$ only are called horizontally polarized shear waves.

We consider a fault plane which extends vertically into the earth. It is assumed that the faces of the fault are lined with pulverized fault breccia. For relative local displacement of the fault faces the resistance to sliding of the pulverized material must be overcome. The resistance to sliding depends strongly on the pressure on the plane of sliding. Here it is assumed that the dependence can be approximated by a Coulomb-Rankine law. If the pressure increases linearly with depth the resistance to sliding may then be expressed in the form

$$\tau_s = \tau_{os} + k_s y \ . \qquad (1)$$

Equation (1) governs the initiation of sliding. The coefficient k_s, which is the rate at which the resistance to sliding increases with depth, is a static coefficient, and τ_{os} is the shear stress required to start sliding with zero pressure. Once sliding has been initiated the resistance to further relative motion drops to

$$\tau_d = \tau_{od} + k_d y \ , \qquad (2)$$

where

$$\tau_{od} < \tau_{os} \ , \text{ and } \ k_d < k_s \ . \qquad (3a,b)$$

It is assumed that τ_{os}, k_s, τ_{od} and k_d are uniform over the faces of the fault.

MATHEMATICAL FORMULATION

We consider an elastic half-space oriented with regard to a cartesian coordinate system as shown in Fig. 1. For a plane two-dimensional geometry the propagation of horizontally polarized shear waves in a homogeneous, isotropic, linearly elastic medium is governed by the two-dimensional wave equation

$$\partial^2 w/\partial x^2 + \partial^2 w/\partial y^2 = \partial^2 w/\partial s^2 , \qquad (4)$$

where $s = c_T t$, and $c_T = (\mu/\rho)^{\frac{1}{2}}$. Here t is the time, and μ and ρ are the shear modulus and the mass density, respectively. The nonvanishing stresses are the antiplane shear stresses

$$\tau_{xz} = \mu\ \partial w/\partial x , \quad \text{and} \quad \tau_{yz} = \mu\ \partial w/\partial y . \qquad (5a,b)$$

Let us investigate the initiation of sliding when a horizontally polarized plane step-stress wave of magnitude τ_∞ strikes the plane of the fault. The incident displacement wave is of the form

$$w_i(x,y,s) = -(\tau_\infty/\mu)(s - x)\ H(s - x) , \qquad (6)$$

where H( ) is the Heaviside step function. The wave strikes the fault at time $t = 0$. If τ_∞ is smaller than the resistance to sliding, the incident wave will pass the fault as if the material were homogeneous. If, on the other hand, τ_∞ exceeds τ_{os}, the incident stress wave exceeds the resistance to sliding over a depth defined by

$$0 \le y < a = (\tau_\infty - \tau_{os})/k_s . \qquad (7)$$

As the incident wave strikes the fault, the initiation of sliding generates a plane reflected wave and a plane transmitted wave, while a cylindrical diffracted wave is generated at the tip of the region of slide initiation. In view of the inequalities (3a,b), the resistance to further sliding decreases after sliding has started. In addition, a stress singularity which is located at the tip of the region of slide initiation, and which is associated with the cylindrical diffracted wave, is relieved by a further extension of the area of sliding in the direction of increasing y. Taking these effects into account the position of the tip of the region of sliding is defined by

$$y = Y(s) , \qquad (8)$$

where $Y(s)$ is to be determined. We generally have $b = Y(0) > a$.

The pattern of wavefronts of the transmitted, reflected and diffracted waves can easily be sketched on the basis of elementary considerations. For $c_T t < b$ the pattern of wavefronts is shown in Fig. 2.

As t increases the cylindrical diffracted wave reaches the surface of the half-space, and is subsequently reflected. When the reflected cylindrical wave reaches the edge of the zone of sliding, defined by $y = Y(s)$, it may again be diffracted depending on the magnitudes of the associated stresses.

The problem of reflection, transmission and diffraction described above for the half-space $y \geq 0$ can conveniently be analyzed by considering an equivalent full-space problem which is obtained on the basis of symmetry considerations with respect to $y = 0$. Thus we consider a full-space, with a fault plane defined by $x = 0$, while the static and dynamic resistances to sliding are given by $\tau_s = \tau_{os} + k_s|y|$ and $\tau_d = \tau_{od} + k_d|y|$, respectively. The incident wave is given by Eq. (6). Indeed, in view of the symmetry with respect to $y = 0$, the shear stress τ_{yz} will vanish at $y = 0$, so that the boundary condition for the half-space is satisfied.

If there were no fault at $x = 0$, the incident wave would give rise to the shear stress τ_∞ in the plane $x = 0$. If sliding is initiated the fault can transmit the stress τ_d. The difference cannot be transmitted by the fault, and this difference gives rise to diffraction of the incident wave. The solution to the diffraction problem is obtained by superimposing on the incident wave the wave motion that is generated in an initially undisturbed medium by shear stresses that are equal and opposite to $\tau_\infty - \tau_d$, and that are applied on both sides of a slit defined by $x = 0$, $|y| < Y(s)$. Through the superposition the stresses on the sliding part of the fault are just equal to the resistance to sliding. Since the wave motion induced by equal shear tractions acting on the sides of a slit is antisymmetric, the displacement vanishes for $|y| \geq Y(s)$. Considering the half-plane $x \geq 0$ for $s > 0$, the wave motion that is superimposed, and whose displacement is denoted by $w_s(x,y,s)$, then must satisfy the governing equation (4) with the following conditions at $x = 0$:

$$|y| < Y(s) \; : \; \mu \frac{\partial w_s}{\partial x} = \tau_s = -\tau_\infty + \left[\tau_{od} + k_d|y|\right] \tag{9}$$

$$|y| \geq Y(s) \; : \; w_s = 0 \; . \tag{10}$$

In addition we have for $s < 0$:

$$w_s(x,y,s) = \frac{\partial w_s(x,y,s)}{\partial s} \equiv 0 \; . \tag{11}$$

The conditions (9) - (11) are pertinent to the initial stages of sliding, before the depth of the zone of sliding begins to decrease.

The displacement solution for problem 1, that is, the problem of sliding generated by an incident wave, follows as $w_1(x,y,t) = w_i(x,t) + w_s(x,y,t)$, where $w_i(x,t)$ is defined by Eq. (6). Problem 2 is concerned with sudden sliding in a stress field defined by a quasi-statically applied τ_∞ which exceeds the resistance to sliding at $y = 0$. It is evident that the dynamic part of the solution to problem 2 is $w_2(x,y,t) = w_s(x,y,t)$.

METHOD OF SOLUTION

Following the works of Kostrov [4] and Achenbach [5] the wave propagation problem defined by Eqs. (9) - (11) can be analyzed by employing a Green's function technique. Thus we employ the function $G(x, y - \bar{y}, s - \bar{s})$, which represents the solution to the two-dimensional wave equation (4) for the half-plane $x \geq 0$ with the boundary condition at $x = 0$:

$$\tau_{xz} = \mu \frac{\partial w}{\partial x} = \delta(y - \bar{y})\, \delta(s - \bar{s}) \ , \tag{12}$$

where $\delta(\ )$ is the Dirac delta function. The boundary condition (12) represents an impulsive anti-plane shear load applied at time $s = \bar{s}$ along the line defined by $y = \bar{y}$. For a half-space which is initially at rest, the region disturbed by this surface disturbance is defined by

$$(s - \bar{s}) - [x^2 + (y - \bar{y})^2]^{\frac{1}{2}} \geq 0 \ , \qquad 0 \leq \bar{s} \leq s \ . \tag{13}$$

Within this region the displacement wave is

$$G(x,\ y - \bar{y},\ s - \bar{s}) = -1/\pi\mu R \ , \tag{14}$$

where R is defined as

$$R = [(s - \bar{s})^2 - x^2 - (y - \bar{y})^2]^{\frac{1}{2}} \ . \tag{15}$$

If the surface shear tractions at $x = 0$ were known, say $\tau_{xz}(0,y,s) = \tau(y,s)$, linear superposition could be employed to write the displacement $w_s(x,y,s)$ in the half-plane $x \geq 0$ in the form

$$w_s(x,y,s) = -\frac{1}{\pi\mu} \iint_S \frac{\tau(\bar{y},\bar{s})}{R}\, d\bar{y}\, d\bar{s} \ , \tag{16}$$

where S is that part of the s-y plane which falls inside the cone defined by Eq. (13).

For $x = 0$ the region of integration S reduces to a triangular region in the s-y plane, as shown in Fig. 3. The conditions (9) and (10), together with the integral representation (16) now provide an integral equation for the shear stress $\tau_{xz}(0,y,s)$ in the region $|y| > Y(s)$. As shown by Achenbach and Abo-Zena [6] this equation can be reduced to an integral equation of the Abel type, which can easily be solved.

RESULTS

From the requirement that the shear stress should be bounded at the tip of the zone of sliding the position of the tip can be computed. For details we refer to Ref. [6]. Along AB, where $s \leq y$, See Fig. 3, we find

$$Y(s) = s/3 + (\tau_\infty - \tau_{od})/\ k_d \tag{17}$$

The instantaneous depth of the area of sliding, $Y(0) = b$, follows immediately by setting $s = 0$:

$$b = Y(0) = (\tau_\infty - \tau_{od})/\ k_d \tag{18}$$

Note that $b > a$, where a is defined in Eq. (7), except when $\tau_{od} = \tau_{os}$ and $k_d = k_s$, i.e., when the static and dynamic resistances to sliding are identical. In the latter case we have $a = b$. It is noted that for a step-stress wave the zone of sliding initially extends linearly with time. Along BC ($y \leq s \leq y + b$) we find

$$16[Y(s)]^3 = s[3b + 3Y(s) - s]^2. \tag{19}$$

The rate of advance of the zone of sliding follows from Eq. (19) as

$$\frac{dY}{ds} = \frac{Y(s)\,[b + Y(s) - s]}{s[3b + Y(s) - s]}\ . \tag{20}$$

Solving from Eq. (19) for $s = y + b$ and substituting the result into Eq. (20) we obtain $dY/ds \equiv 0$ at point C. It should be noted that $s = Y + b$ corresponds to the arrival time at $y = Y(s)$ of the cylindrical diffracted wave from the point $y = -b$. For the half-space $y \geq 0$ this is actually the reflection from the free surface of the cylindrical wave centered at $y = +b$. It is easily verified that for the boundary conditions (9) and (10) the stresses are opposite in sign in the regions $|y| < b$ and $|y| > b$, respectively. As a consequence the cylindrical diffracted wave arriving at $y = Y(s)$ actually relieves the stress at $y = Y(s)$. The foregoing observations strongly suggest that a maximum value of the region of sliding has been reached, and that $Y(s)$ will not increase further. The value of $Y(s)$ at $s = y + b$ is denoted by d, where

$$d = b\ /\ (2^{2/3} - 1) \tag{21}$$

As s increases beyond $s = y + b$, we must distinguish the region over which actual relative motion is still taking place from the region over which relative motion, which has now subsided, has taken place. The now receding edge of the region of motion is denoted by $y = F(s)$. In Fig. 3 the "closing line", which is defined by $y = F(s)$, is indicated by CD. To determine CD Eq. (16) is employed to compute the displacements $w_s(o,y,s)$ at specific locations y, as functions of s. The analytical expressions for $w_s(o,y,s)$ are rather lengthy, and they are, therefore, not reproduced here. Curves of $w_s(o,y,s)$ versus s are shown in Fig. 4. The points at which $\partial w_s/\partial s$ vanishes, which are the end points of the curves, define the points of the closing line CD.

The maximum depth of the zone of sliding in this paper has been determined on the basis of dynamic considerations. It may be compared with the depth which was computed by Walsh [2] and Berg [1] on the basis of static considerations. The cases corresponding to the ones considered by Walsh and Berg are included in the analysis of this paper by setting $\tau_{od} = \tau_{os}$, $k_d = k_s = k$, and $\tau_{od} = 0$, respectively. We find

$$\frac{d}{d_W} = \frac{d}{d_B} = \frac{2}{\pi} \frac{1}{2^{2/3} - 1} \simeq 1.08 , \tag{22}$$

where d is defined by (21). Thus, the present dynamic analysis shows an overshoot of the static depth of the fault. It is to be expected that the overshoot decreases if the shear load τ_∞ is applied more gradually.

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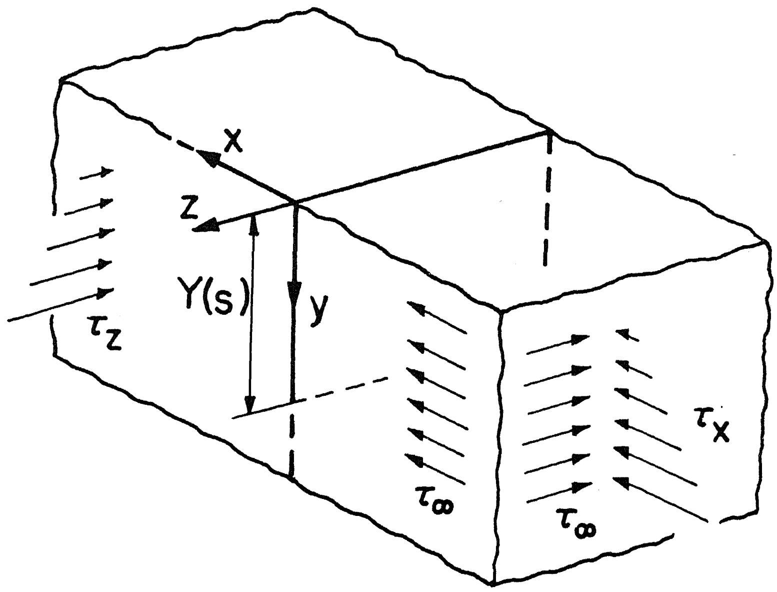

Fig. 1. A strike slip fault of time dependent depth Y(s)

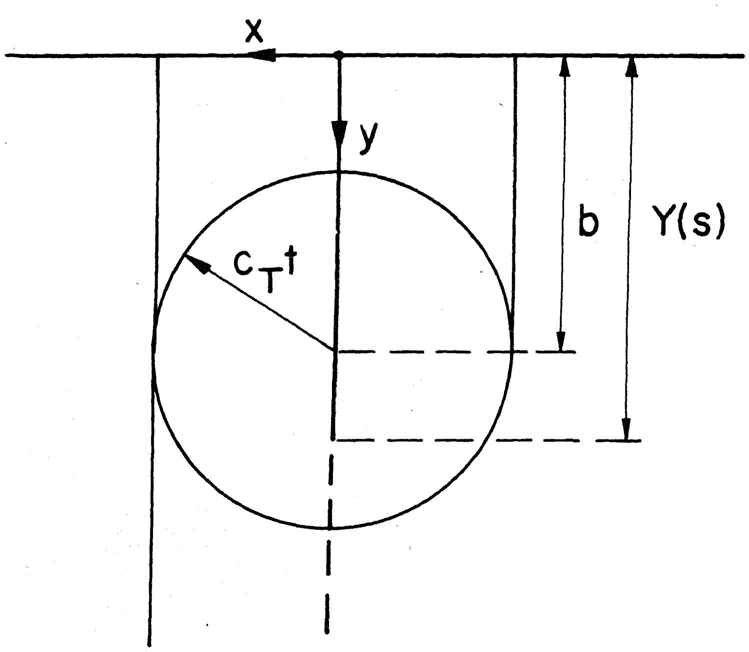

Fig. 2. Pattern of wavefronts for the motions transmitted, reflected and diffracted by the fault in the time interval $0 < c_T t < b$

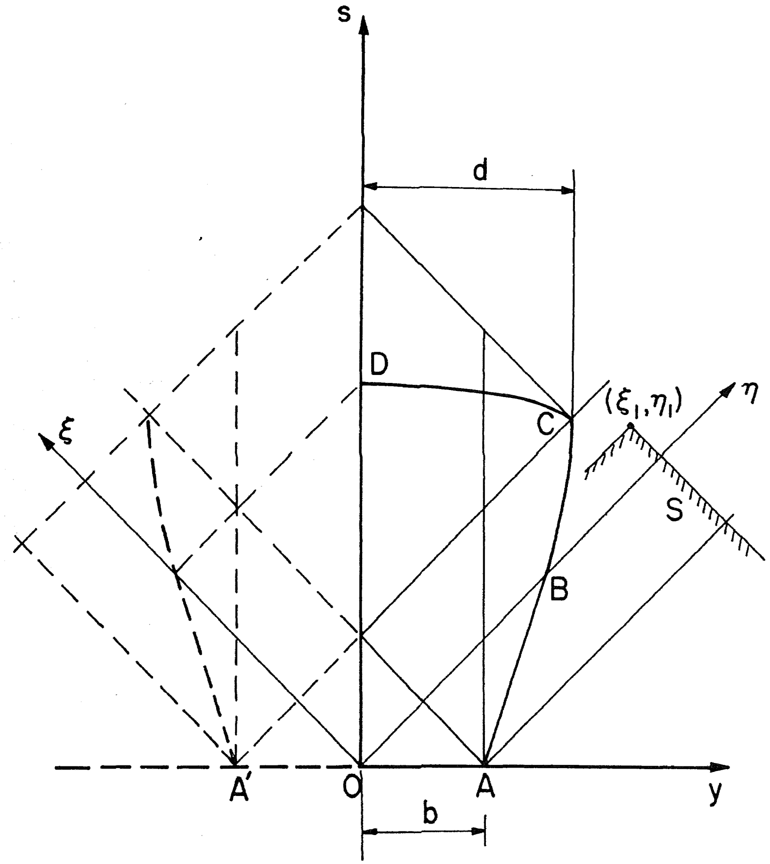

Fig. 3 Wavefronts and the trajectory of the tip of the zone of sliding in the s-y plane

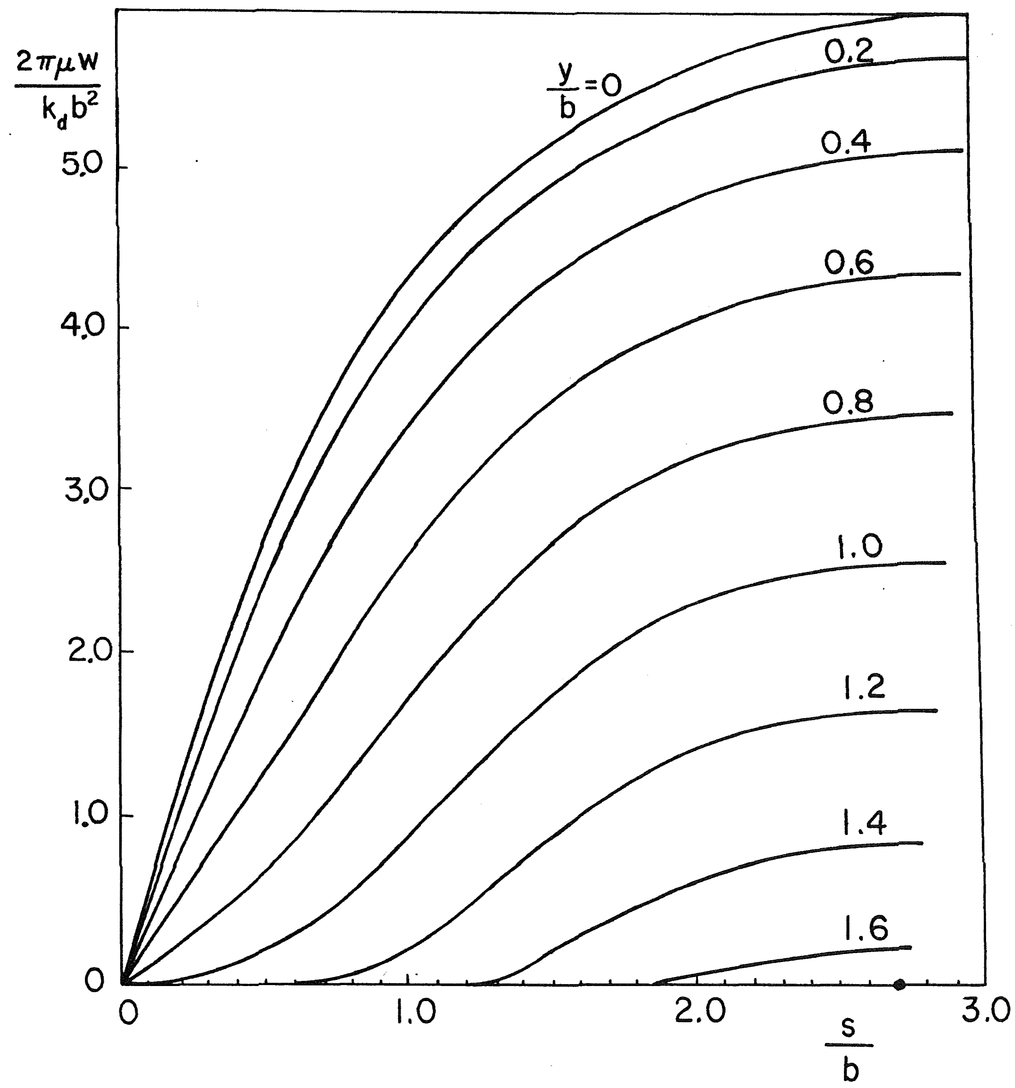

Fig. 4. Displacement of the fault surface