INTERPRETATION OF APPARENT UPTHROW OF OBJECTS IN EARTHQUAKES

by

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SYNOPSIS

There is credible evidence in a number of instances of objects having moved upward relative to the ground in an earthquake. Competent observers have sometimes cited this evidence as an indication of vertical earthquake accelerations greater than the acceleration of gravity. Although it is necessary that the actual acceleration of a near-surface object be greater than the acceleration of gravity for it to move upward relative to the ground, this is not necessarily an indication that the general vertical ground acceleration is indeed greater than that of gravity.

INTRODUCTION

This brief study of upthrow of near-surface objects in earthquakes considers the behavior of a small compact mass m supported on the ground near or at the ground surface. This mass rests on the ground but can be considered as resting on a series of springs connecting it to the ground surface. The upward coordinate of motion of the mass is designated by x and that of the ground by y, as in Fig. 1. The relative displacement u of the mass and the ground is designated by:

$$u = x - y \tag{1}$$

In this discussion, which is presented merely to illustrate the general nature of the problem, it is convenient to consider that the spring support is elastic with a spring constant k. Then the equation of motion of the mass is given by:

$$m\ddot{x} + ku = o \tag{2}$$

It can be seen directly that the acceleration of the mass is at all times, except when there is a separation of the mass from the ground, given by:

$$\ddot{x} = -\omega^2 u \tag{3}$$

where ω is the circular natural frequency of vibration.

It is convenient to rewrite Eq. (2) in the form:

$$m\ddot{u} + ku = -m\ddot{y} \tag{4}$$

If the mass remained attached to the ground, with the springs being able to carry tensile force, then the problem is merely thatof the usual analysis of response of a simple oscillator to the earthquake motion, and the maximum values of relative displacement can be plotted in the form of a response spectrum, as a function of frequency of vibration of the mass spring system. Although this is not a tenable assumption, it is

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obvious from the nature of the problem that the response acceleration can indeed be greater than 1g, where g is the acceleration of gravity, even though the maximum ground acceleration is considerably less than 1g.

In the range of frequencies of the order of about 2 to 10 or more hertz, the response acceleration is often greater than 1g even for maximum ground accelerations of intensity equal to or slightly greater than about 0.3g, and is usually greater than 1g when the maximum ground acceleration is greater than about twice this value. Since vertical accelerations of the order of 0.3 to 0.6g are associated with moderately intense to very intense earthquakes, one can expect occasional indications of uplift or upthrow of compact objects near the surface of the ground in strong earthquakes even though the vertical earthquake intensity, and even the horizontal earthquake intensity, may correspond to maximum ground accelerations less than the acceleration of gravity.

BASIS OF ANALYSIS

We shall proceed to make somewhat more reasonable assumptions as to the nature of the problem and the parameters. We begin by assuming that the inclusion rests on the ground and has a static deflection, due to its own weight, of u_o. The latter is defined by:

$$u_o = mg/k = g/\omega^2 \tag{5}$$

If we now consider a pattern of ground accelerations as in Fig. 2, consisting of a series of pulses roughly sinusoidal in shape, the relative response of the inclusion can readily be determined. However, the problem is nonlinear in the sense that, if the upward relative displacement u from the initial position exceeds u_o, the object will separate from the ground surface. This inequality holds for the algebraic value of the displacement, but not the absolute value, since a downward motion or a compression in the spring support can, of course, be greater than the static deflection u_o without causing separation.

Under the condition where the upward relative displacement exceeds the static displacement, then:

$$\ddot{x} = -\omega^2 u_o = -g \tag{6}$$

In other words, where there is separation, the downward acceleration of the mass will be equal to the acceleration of gravity. It is also clear from Eq. (6) that, at the limiting point where $u = u_o$, the downward acceleration is indeed equal to the acceleration of gravity. Separation does not occur unless the response acceleration of the mass is greater than the acceleration of gravity.

The possibility of separation of an inclusion from the ground surface can be inferred from the normal type of response spectrum drawn for the linear case where the spring is a linear spring, or drawn for a nonlinear situation where one might wish to take account of nonlinear behavior of the support under the inclusion. However, after separation takes place, the problem is changed and a separate calculation is involved.

BEHAVIOR AFTER SEPARATION

Let us consider the situation which develops during the motion of the inclusion, when a value of relative displacement u greater than u_o is reached. At this time, the ground surface moves in accordance with its prescribed acceleration-time history, as in Fig. 2, but the inclusion leaves the ground surface with a velocity $\dot{u}_o$ relative to the ground, traveling in the upward direction. It then rises to a maximum height z above the position of the ground surface at the time of separation, where:

$$z = \dot{u}_o^2/2g \qquad (7)$$

The time it takes to reach this maximum value of separation is Δt, where:

$$\Delta t = \dot{u}_o/g \qquad (8)$$

The situation at the time of "maximum" separation is shown in Fig. 3. The dashed line marked s_o in the figure is the position of the ground surface at the time of separation, and the point r_o is the point at the bottom of the center of the mass m at the time of separation. The mass is shown as rising to a height z above this line, and moving transversely to a point $r_{a'}$, because the ground has imparted to it, during its motion while it was attached, a horizontal component of velocity and acceleration. During the rise to the new position, the ground continues to move, and at the time of maximum the ground has reached a new position s_a, where the center point of the contact area is designated by r_a.

Actually the true maximum separation of the mass in the ground may be greater or less than z because the ground has continued to move during the time Δt.

Following the maximum rise of the mass, it begins to fall, still with an acceleration downward of g, until it makes contact with the ground again. It intersects the ground with a velocity of approximately $\dot{u}_o$, but this may be greater or less than $\dot{u}_o$, depending on the height it has to fall to reach the new ground position. The total time, from the departure of the mass from the ground surface until its intersection with it again, is approximately $2\Delta t$.

Because of the fact that the ground and the mass both have horizontal as well as vertical components of motion, the relative positions of the mass and the ground during the motion are quite complicated. In Fig. 4 there is shown a sketch of possible positions during the motion history. This indicates a lateral motion of the mass having taken place after the earthquake, possibly being overturned or changed in position in such a way as to leave the conclusion that it had in fact been moved by a ground acceleration greater than the acceleration of gravity. Figure 4(a) shows the original position of a mass such as a tombstone or other compact mass somewhat taller vertically than it is wide. Figure 4(b) shows the position of the mass at its highest relative displacement from its original position, with the ground having moved during the separation. In Fig. 4(c) the mass has intersected a new position of the ground, hitting on a corner and indenting the ground. In Fig. 4(d) the situation is shown after the ground and the mass have come to rest.

It is believed that this kind of situation can be developed even under conditions where the maximum ground acceleration may in fact have been only 0.5 to 0.6g. Erroneous conclusions can, therefore, be drawn from observations if one does not take into account the nature of the amplification of response of an object resting on the ground.

ESTIMATES OF RELATIVE MOTION

It is possible to make reasonable estimates of situations which might show separation of near-surface masses in an earthquake from the general shape of response spectra that might be applicable, and from possible surface properties of sediments.

For example, at a natural frequency of 10 hertz, with about 5 percent damping, one can expect that the response acceleration will often be amplified from the ground acceleration by about a factor of 2 or more, and even at a natural frequency of 20 hertz the amplification will be nearly 2 on occasions. At a natural frequency of 5 hertz the amplification will often be a factor of 3. These frequencies correspond to static deflections under dead load of the following:

freq. = 5 hertz, u_o = 0.391 in.

freq. = 10 hertz, u_o = 0.098 in.

freq. = 20 hertz, u_o = 0.024 in.

A near-surface subgrade modulus of 10 to 40 psi/in is quite possible for sediments. Hence, rocks of about 2 to 3 ft in height might be possible candidates for upthrow under vertical earthquake ground accelerations of the order of about 0.5g.

It is apparent, however, from the relations stated above, that the height of rise of the inclusion or object is relatively small, generally of about the order of u_o, unless the actual ground acceleration does in fact exceed 1g. Consequently, the only objects that might normally be expected to turn over, or to be displaced in a manner similar to that shown in Fig. 4, in moderately intense earthquakes, would be tall relatively slender objects such as tombstones, rather than low, broad or flat objects.

Nevertheless, broad, flat objects may show large accelerations resulting from their impact with the ground upon striking it after the separation ends. Hence, one should be careful in placing recording instruments to avoid the instrument recording a peak value determined by the response of the block on which it rests rather than by the general ground movement. This precaution applies whether the actual ground acceleration is less than or more than the acceleration of gravity.

USEFUL APPROXIMATIONS

It is convenient to have a means of estimating the fact of separation and the relative velocity $\dot{u}_o$ at the time of separation. One can do this with fair accuracy from the results of the usual response analysis, or from the maximum value of the response acceleration $\ddot{x}$ of the mass m. Let this value be designated by Ag, where A is dimensionless and g the acceleration of gravity.

The values of A can be obtained from the usual response spectrum for the particular frequency f of the inclusion. The frequency is given by the result obtained using Eq. (6):

$$f = \omega/2\pi = \sqrt{g/u_o} \tag{9}$$

Usually the upward and downward values of maximum response acceleration are nearly equal. In fact, spectra are usually drawn without distinguishing between them.

Then the energy available in the spring-mass system can be approximated by assuming that it is equal to the energy stored in the spring at its point of maximum compression. This leads to the result:

$$\text{Available Energy} = (m\ Ag)^2/2k \tag{10}$$

where k is the spring constant.

At the time of separation, some energy will have been lost in raising the mass to the relative displacement u_o. This energy is mg $u_o/2$. The difference is available as kinetic energy. Equating the kinetic energy at liftoff, m $\dot{u}_o^2/2$, to the net available energy, we obtain the result:

$$\frac{m\ \dot{u}_o^2}{2} = \frac{(m\ Ag)^2}{2k} - \frac{mg\ u_o}{2} \tag{11}$$

Upon simplification, noting that $\omega^2 = k/m$, and using Eq. (5) to replace u_o in Eq. (11), we find:

$$\dot{u}_o^2 = g^2(A^2 - 1)/\omega^2 \tag{12}$$

This relation can be used for an estimate of z, by use of Eq. (7):

$$z = \frac{g(A^2 - 1)}{2\omega^2} = u_o(A^2 - 1)/2 \tag{13}$$

It should be noted that z is actually the height of rise over the position of the ground surface assuming that the ground surface moves during separation at the same velocity that it had at the time of liftoff. Hence, the actual height of separation may differ materially from z. However, Eq. (12) is applicable directly.

The errors in Eq. (12) may be considerable, because the actual maximum response acceleration before liftoff governs the liftoff. Hence, if the maximum response spectrum acceleration occurs late in the history of motion, it may be considerably larger than the value which governs the first period of separation. After separation takes place, the energy losses on subsequent impact make further analysis somewhat doubtful. Nevertheless, these relations can be used to indicate the general nature of the problem of separation of an inclusion from the ground in an earthquake.

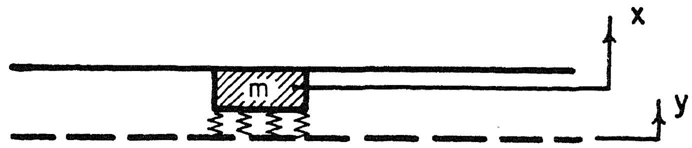

FIG. 1 INCLUSION AT GROUND SURFACE

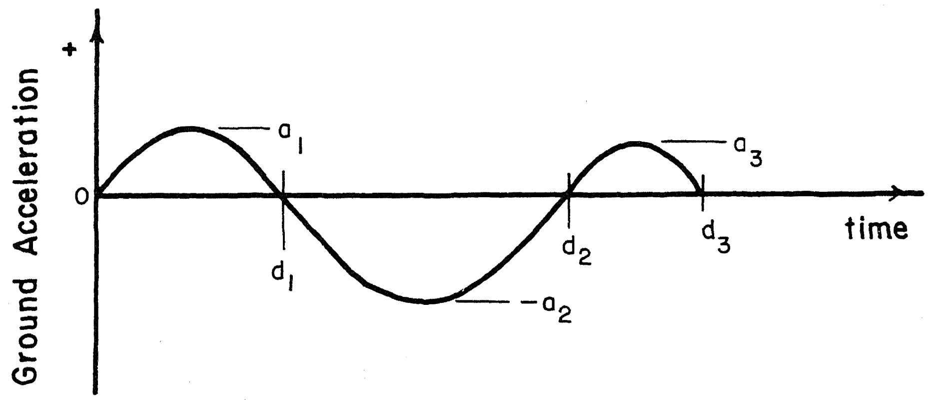

FIG. 2 GROUND ACCELERATION CONSIDERED

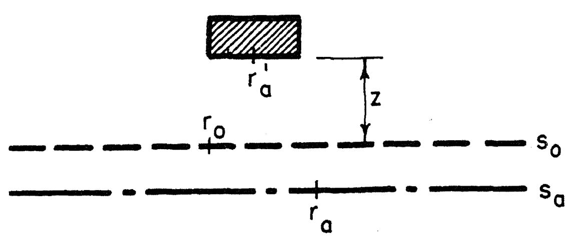

FIG. 3 POSITION OF INCLUSION AT TIME OF "MAXIMUM" SEPARATION

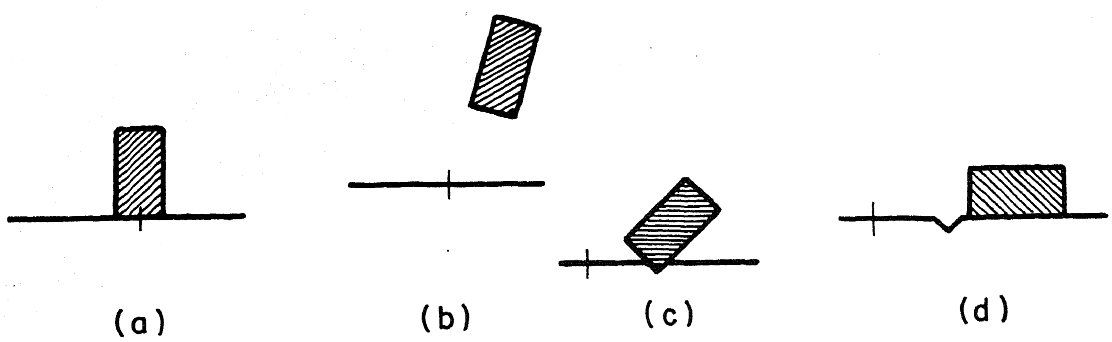

FIG. 4 SUCCESSIVE POSITIONS OF UNSTABLE MASS