FORCE DEFLECTION-RATIO, ITS CONTROL AND DESIGN

Rodrigo Flores A.[I], Ramón Arias F.[II]

SYNOPSIS.

The possibility is explored by means of model tests to materialize structural elements with capacity to resist horizontal forces and with a prescribed non-linear force-deflection ratio. The behaviour of bilinear one-degree-of-freedom systems subjected to El Centro earthquake 1940 is studied, finding an adequate dynamic response when the systems are assumed to be of reinforced concrete. For such structural elements the degradation associated with hysteretic processes can be avoided and an increased ductility for failure conditions can be obtained.

I. INTRODUCTION.

It's a well known fact that structures designed for conventional elastic conditions actually behave in the plastic range when subjected to a destructive earthquake. The degradation of mechanical properties due to yielding and cracking associated with the hysteretic process is accepted as an unavoidable evil.

The force-deflection history is mainly determined by the structural properties of the system and by the earthquake characteristics. The structural properties are dependent on the kind of structural element concerned, on the nature of their mutual connections and on the adopted material. For a particular problem a large choice of possible designs appears feasible. This is not actually the case; in the great majority of situations the structural design is subordinated to functional and economic requirements. Therefore, the designer can only perform a restricted control of the force-deflection ratio and has to renounce to adopt the best solution from the seismic point of view.

From a different approach, in this paper the effort is made to demonstrate that by using conventional materials-reinforced concrete-it is possible to design structural elements with a controllable non-linear force-deflection ratio; the degradation or "softening" of the material normally found in non-elastic behaviour being avoidable. The simplified, particular problem of a bilinear one-degree-of-freedom system is analyzed and its relationship with a special structural element-composite wall-is discussed. The composite wall can be utilized both as a structural element and as a system to minimize earthquake effects.

II. COMPOSITE WALL.

2.1. Description of System.

In Fig. 1 a composite wall is shown formed by several units-columns-held together by friction induced by a transversal compression q, normal

I Prof. of Structural Engineering, University of Chile.

II Research and Assistant Professor, University of Chile.

to the contact faces applied by steel cables or bars-in the manner of transversal prestressing. Column dimensions are chosen to meet two conditions : a) individual columns deform mainly in flexure, and b) a shear wall having the composite wall dimension deforms mainly in shear.

2.2. Force-Deflection Ratio.

It is easy to anticipate an approximately bilinear relationship between a variable lateral force H acting at the top of the composite wall and the horizontal deflection δ induced thereby. In Fig. 2 the force-deflection history is shown. When H is a small force the corresponding shear stresses are not capable to overcome the Coulomb friction induced by the compression q and no relative sliding between contiguous faces take place. Therefore a shear deformation of the composite wall will prevail. If the force H is increased, the friction no longer will prevent the sliding, and the columns will deform in bending decreasing the wall rigidity. If the force H is now decreased, a stage will be reached where the shear stresses-now in a reserved sense-will no longer be able to overcome the friction and the deformation will be commanded by shear. If the force H is increased up to the point to produce the composite wall failure, this will be reached when the flexural capacity of the columns is exhausted and therefore a ductile failure will occur.

In order to prove practically the bilinear behaviour, an experimental programm with plastic models, of a qualitative character was undertaken. The test arrangement is schematically shown in Figs. 3 and 4.

Each model column is composed of two plexiglass pieces each 18 cm. heigh, 3 cm. width and 6 mm. thick joined together by transversal elements located at the top and bottom of the combined piece and forming a hollow section to permit the inside placement of 4 springs to provide the lateral compression q.

Two series of tests differing only in the end support conditions for the columns of the composite wall, were performed. In series N°1, shown in Fig. 3, the base end is hinged and the top end has a sliding support. In series N°2, shown in Fig. 4, the base end is transformed into a fixed support. For both series, the influence of the lateral compression q on the top horizontal deflection of the composite wall, was investigated.

By inspection of figures 3 and 4 an acceptable agreement between experimental results and theoretical hypotheses is observed. In fact, for small deformations the model behaves as a shear wall; when the friction between contiguous faces is exceeded a relative column displacement is produced with a lowering of the total rigidity. In the first experimental series a behaviour similar to the elastoplastic system with a well defined yield deformation, is observed. In this case the columns cannot develop bending resistance. In the second series, the behaviour is bilinear-like, because the built-in columns can resist in bending when the friction is surpassed. The yield deformation is found to be vaguely defined, what can be explained by the different shear stress level at the various interface contact planes which causes a non simultaneous starting point of relative sliding of contiguous columns. For both experimental series the possibility to control the bilinear behaviour by varying the lateral compression q, is observed. This control can also be operated by modifying the moment of inertia and the number and the height of columns. Actually a large range of bilinear behaviours can be obtained, limited only by the properties of the structural materials.

III. DYNAMIC RESPONSE OF BILINEAR SYSTEMS.

3.1. General.

Accepting that it is feasible to design and construct a system with a bilinear-or-similar behaviour, a study was undertaken to investigate its dynamic response characteristics from a seismic point of view. The behaviour of the mechanical model of Fig. 5 was studied. This model is composed of a conventional elastic system of mass M connected to the ground by a spring with rigidity k and a damper defined by its damping coeficient c. A massless element acted by the normal force N can develop friction along its contact surface with the ground. This element-called base element-is connected to the mass M with a spring of rigidity km. Both springs k and k_m connected in series produce the bilinear behaviour in the force-displacement diagram.

The following notation has been adopted

x = absolute mass displacement
x_b = absolute base displacement
x_m = relative displacement between mass and base element
μ = Coulomb friction coefficient
C = μN = maximum friction
Δ_y = yielding displacement
Δ = friction equilibrium position for the elastic system with dry friction.
ω^2 = bilinear frequency
Ω = elastic frequency

Accordingly, we can write

$$\Delta_y = \frac{C}{k_m} ; \quad \Delta = \frac{C}{k} ; \quad x = x_b + x_m ; \quad |x_m| \leq \Delta_y$$

3.2. Free Vibration.

It is assumed in this case that c, damping coefficient is equal to zero. From Fig. 2 it can be seen, that the study of the bilinear system can be interpreted in terms of the elastic system when the adequate axis translations and considerations of initial values of displacements and/or velocities are taken into account.

To simplify the analytical expressions, let us assume as initial conditions : $|X_o| > \Delta_y$ and $\dot{X}_o = 0$.

The differential equation of movement is :

$$M\ddot{x} + (k + k_m)\ \{x + \Delta_m \operatorname{sgn}[\dot{x}(t_o + \tau)]\} = 0$$

where Δ_m is defined by

$$x_{mo} = -\frac{k + k_m}{k}\ \Delta_m \operatorname{sgn}\ [\dot{x}(t_o + \tau)]$$

The function sgn $[\dot{x}]$ takes the value + 1 or - 1 according to the positive or negative sign of the argument $\dot{x}$, respectively.

The former equation is valid for $\tau < \tau_o$, where the interval τ_o is defined by the condition that for the instant $t_1 = t_o + \tau_o$ the base begins to slide;also at that instant $x_{m1} = x_{mo}$ and therefore

$$\Omega\ \tau_o = \text{arc cos}\ \frac{k\ \Delta_m - k_m\Delta_y}{k\ \Delta_m + k_m\Delta_y} < 2\pi,$$

and

$$x_1 = 2\Omega \sqrt{\frac{k}{k_m}\ \Delta_y\ \Delta_m}\ \ \operatorname{sgn}\ [\dot{x}(t_1 + \tau)]$$

Finally for $t \geq t_1$, the equation is

$$M\ddot{x} + k\ \{x + \Delta \operatorname{sgn}[\dot{x}(t_1 + \tau)]\} = 0$$

valid for $t_1 \leqslant t \leqslant t_2$ where the instant $t_2 = t_1 + \tau$, is defined by the condition $\dot{x}_2 = 0$ and therefore

$$\text{tg}\ \omega\ \tau_1 = \frac{\dot{x}_1}{\omega\ \{x_1 + \Delta\ \ \operatorname{sgn}[\dot{x}(t_1 + \tau)]\}}$$

and

$$x_2 = \operatorname{sgn}\ [\dot{x}(t_1 + \tau)]\ \{- \Delta + \sqrt{\{x_1 + \Delta \operatorname{sgn}\ [\dot{x}(t_1+\tau)]\}^2 + \frac{\dot{x}_1^{\,2}}{\omega^2}}\ \}$$

To simplify the previous expressions, variables are defined as follows

$$\theta = -\frac{x}{\Delta_y}\ \operatorname{sgn}\ [\dot{x}] \quad \text{and} \quad \phi = \frac{F}{F_y}$$

The parameters are defined as follows

$$\xi = \frac{\Delta}{\Delta_y} = \frac{k_m}{k}\ \ ;\ \text{and} \quad \kappa = \frac{C}{F_y} = \frac{k_m}{k + k_m} = \frac{\xi}{1 + \xi}$$

In order to generalize the previous analysis, instead of instants t_o, $t_1$and t_2, let us consider instants t_{2i}, t_{2i+1}, and t_{2i+2} respectively, according to the following definition t_{2i} is the instant where $\theta_{2i} = 0$; t_{2i+1} is the instant marking the beginning of sliding and t_{2i+2} is the instant where again $\dot{\Theta}_{2i+1} = 0$ as it is shown in Fig. 6. Then, by comparison with the expression derived for x_2, we can write

$$\Theta_{2i+2} = - \xi + \sqrt{(\Theta_{2i} - \xi)^2 + 4 \xi}$$

In Fig. 7, Θ_{2i+2} as function of Θ_{2i} for different values of ξ is represented. It is found that $\xi \leq 1$ corresponds to $\Theta_{2i+2} > 1$. That is when $\Delta \leq \Delta_y$ or $k_m \leq k$, the mass has the tendency to move as an elastic system with a frequency Ω and amplitude $2\Delta_y$ without remanent displacement. If $\Delta > \Delta_y$ there is an instant t_{2i+j} where $\Theta_{2i+j} \leq 1$. When $t > t_{2i+j}$ the mass movement will be a free vibration of an elastic system with a frequency Ω, amplitude less than $2\Delta_y$ and centered around a ficticious equilibrium position, distant from the true equilibrium position, a distance Δ_f, defined by :

$$\Delta_f = (1 - \Theta_{2i+j}) \Delta_y$$

3.3. Forced Vibrations.

The differential equation of movement of the mass M subjected to the ground acceleration $\ddot{x}_g$ is

$$\ddot{\Theta} = - \frac{1}{\Delta_y} \ddot{x}_g - \frac{1}{\Delta_y} \frac{F}{M} - 2 \Omega h \dot{\Theta}$$

where $h = \frac{c}{c_c}$ is the fraction of critical damping.

We can also write

$$\frac{F}{M} = \frac{\Omega^2}{k+k_m} F$$

$$\frac{F_y}{F} = \Omega^2 \Delta_y \text{ ; and } \frac{C}{M} = \kappa \Omega^2 \Delta_y$$

In the general case, the ratio F/M must satisfy one of the four following expressions, depending on the velocity and the position of the mass M.

1.- $\frac{F}{M} = \Omega^2 \Delta_y \left[\Theta + \kappa (1 - \Theta_{2i})\right]$ If $\Theta_{2i} - 2 < \Theta < \Theta_{2i}$

2. $\frac{F}{M} = \Omega^2 \Delta_y \left[\Theta - \kappa (1 + \Theta_{2i})\right]$ If $\Theta_{2i} < \Theta < \Theta_{2i} + 2$

3. $\frac{F}{M} = \Omega^2 \Delta_y \left[\Theta (1 - \kappa) + \kappa\right]$ If $\dot{\Theta} > 0$

4. $\frac{F}{M} = \Omega^2 \Delta_y \left[\Theta (1 - \kappa) - \kappa\right]$ If $\dot{\Theta} < 0$

The ground acceleration was defined by the first 10 seconds of the N-S component of the El Centro Earthquake 1940. A numerical procedure of "step by step" integration was computer-programmed in order to avoid the analytical complication of the differential equation of movement. Following basic parameters were investigated :

a) The natural elastic period $T = 2\pi/\Omega$ in the range of 0,1 to 2,5 sec.
b) The stiffness ratio $\kappa = k_m/k+k_m$ in the range 0 to 1.
c) The yielding displacement Δ_y in the range 0,75 to 3 cm.
d) The **fraction** of critical damping $h = c/cc$ in the range 0 to 0,15.

An IBM 360 computer was used to obtain the results given directly in tabular form.

The ratio -called "elastic acceleration"- between the maximum displacement and the elastic displacement of the system subjected to a lateral force equal to the mass multiplied by the gravity acceleration is related to the elastic period for different values of the remaining basic parameters. It's interesting to notice that for the elastic system, the "elastic acceleration" is equivalent to the maximum acceleration of the system. In Fig. 8, 9 and 10 the most relevant table values are presented in graphical form.

The study of the computer results suggests the following comments

a) There is a limit vibration period, below the value of which, the bilinear system has an elastic behaviour and for higher values of the vibration period the yielding displacement is surpassed. When the yield displacement or the damping are increased, the limit vibration period is also increased because in both cases the yield displacement is more difficult to surpass.

b) For elastic vibration periods only slightly larger than the limit vibration period, the bilinear systems have a maximum displacement which is smaller than that corresponding to the elastic system. Fig. 11 shows the maximum displacement versus the restoring force for different stiffness ratios with a period of vibration equal to 0.4 sec. and no damping: it can be seen that the smaller maximum displacement is obtained for a stiffness ratio equal to 0.625. It is amazing to notice the remarkable uniformity of this family of curves, in contrast with those shown in Figs. 8 to 10.

c) For vibration periods much larger than the limit period, the maximum displacement of the bilinear system is larger than that corresponding to elastic system and is nearly proportional to the stiffness ratio.

d) A larger damping influences the response in such a way as to bring closer the bilinear and the elastic behaviour.

e) A large resonant response appears under certain conditions, as can be shown in Fig. 8. The resonance period is variable and increases as the stiffness ratio decreases. This resonance zone is strongly reduced when a larger yielding displacement or a larger damping is considered. The real meaning of the resonance characteristics is to be found in the properties of the given ground motion and is beyond the scope of this paper.

The previous remarks apparently can be explained by the interaction of three factors, the preponderance of which is dependent on the elastic period and the stiffness ratio.

a) Flexibility. For bilinear systems a larger stiffness ratio means a larger flexibility and the systems tend to behave as elastic systems with a longer vibration period. This is significant for short elastic vibration periods.

b) Energy absorption. When the stiffness ratio increases, the energy absorption increases. This is also significant for short elastic vibration periods.

c) Progressive cumulative displacement. When the stiffness ratio and/or the period increases -condition for very flexible systems- the existing probability that the restoring force be not capable to overcome the friction is larger and therefore the production of a one-sense movement with an increased progressive displacement, is facilitated.

In relation with composite walls, several aspects of practical implication can be derived.

a) The free vibration analysis has shown that under certain conditions it is possible to have bilinear systems without remanent displacement. Nevertheless if such condition is not satisfied and a remanent displacement exists at the end of the earthquake, it is possible by relaxing the system and annulling transitorily the normal force N, to have it return to its equilibrium position. This can be achieved in the composite wall by nullifying transitorily the lateral compression q.

b) The relative displacement between the structure and the ground is not very different from what can be expected of an elastic system. For short vibration periods the bilinear system may have a maximum displacement even smaller than that corresponding to an elastic system. Although this is not true for systems with a long vibration period, this doesn't imply their seismic inadequacy; in fact, considering that the bilinear are more flexible than the corresponding elastic systems, the seismic forces are actually reduced.

c) Although the existance of the resonance zone is disturbing, it can be controlled by increasing the yielding displacement. If the damping inherent to any structure is considered, the resonance is further reduced.

d) It is possible to avoid the degradation by controlling the yield displacement, in such a way as to have a column-relative-sliding prior to the shear failure of the wall.

e) A destructive earthquake would induce in the composite wall a ductile failure commanded by the flexural behaviour of the columns.

f) The previously commented aspects permit us to conclude that the proposed bilinear system is benefitial to minimize the seismic effects.

IV. SCOPE AND LIMITATION OF THE RESEARCH.

The range of variation of basic parameter values adopted in the study of the dynamic response of bilinear systems, has been chosen having in mind composite walls built with reinforced concrete. Considering that composite walls may find application for multistory buildings, it is clear that the study made for a one-degree-of-freedom system is insufficient for practical purposes. On the other hand, in general, the study of the nonlinear structural behaviour can reach valid general conclusions only if the stochastic nature of the movement is considered. Nevertheless, the aim of this paper has been only to demonstrate the possibility of designing the force-deflection ratio, and for this purpose, a sinple case-composite walls-subjected to one single earthquake has been studied.

The authors hope to arouse interest in relation to the general problem of designing the force-deflection ratio to be applied to earthquake resistant structures.

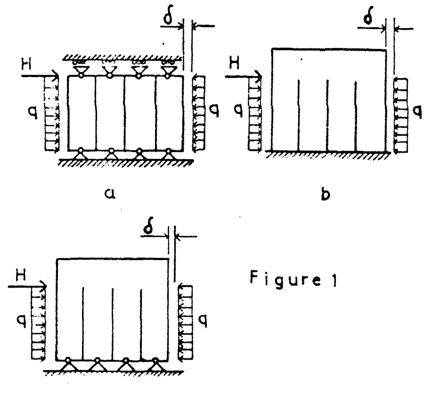

Figure 1

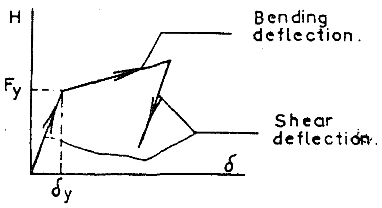

Figure 2

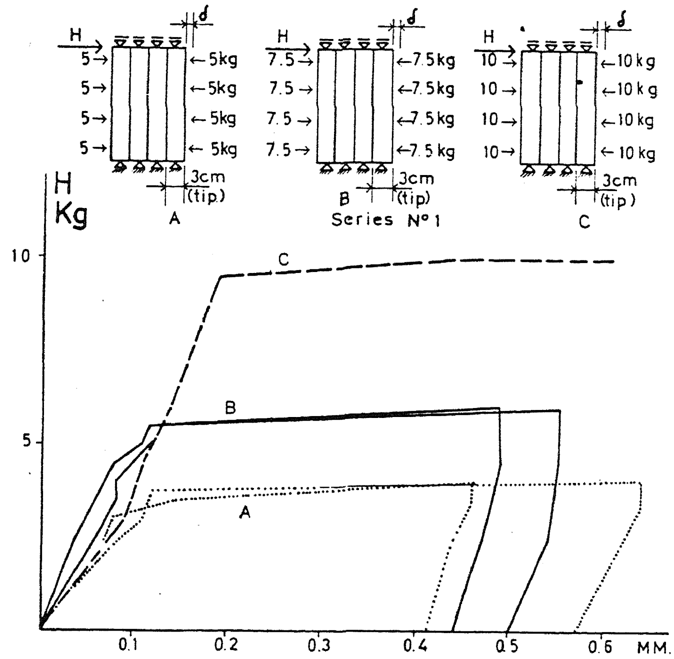

Figure 3

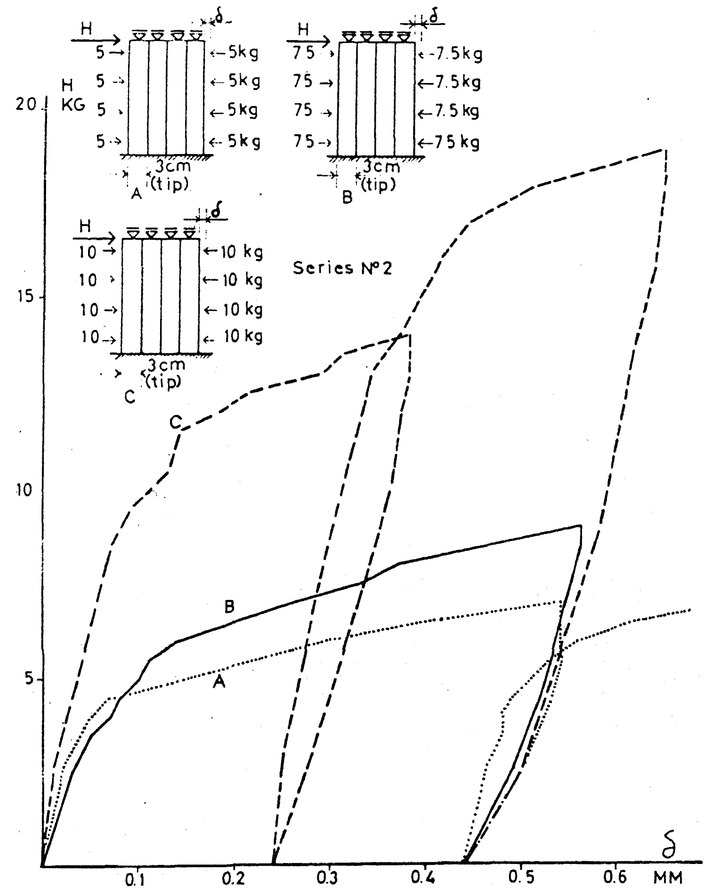

Figure 4

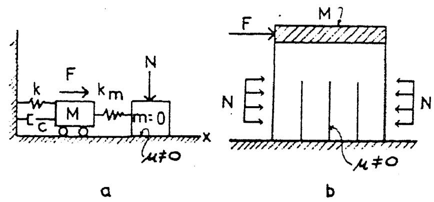

Figure 5

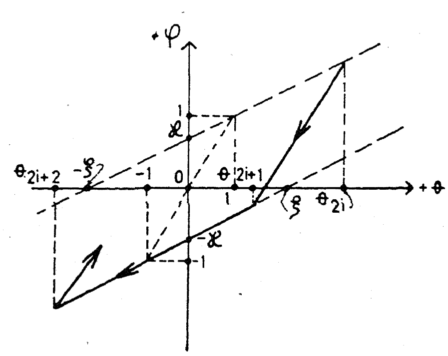

Figure 6

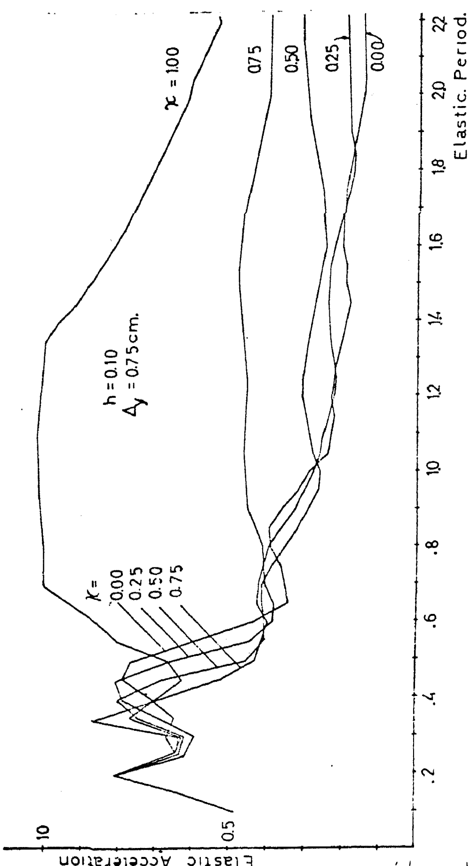

Figure 9.

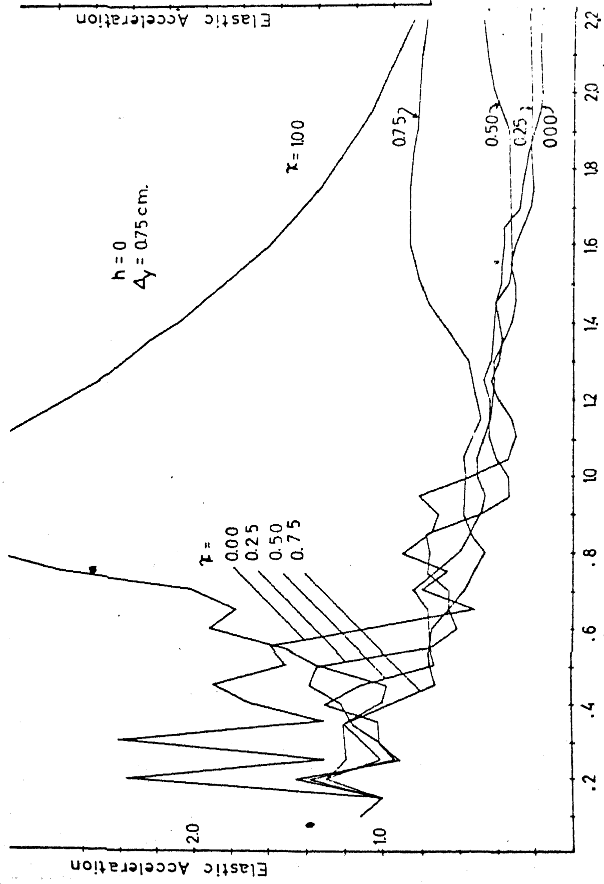

Figure 8.

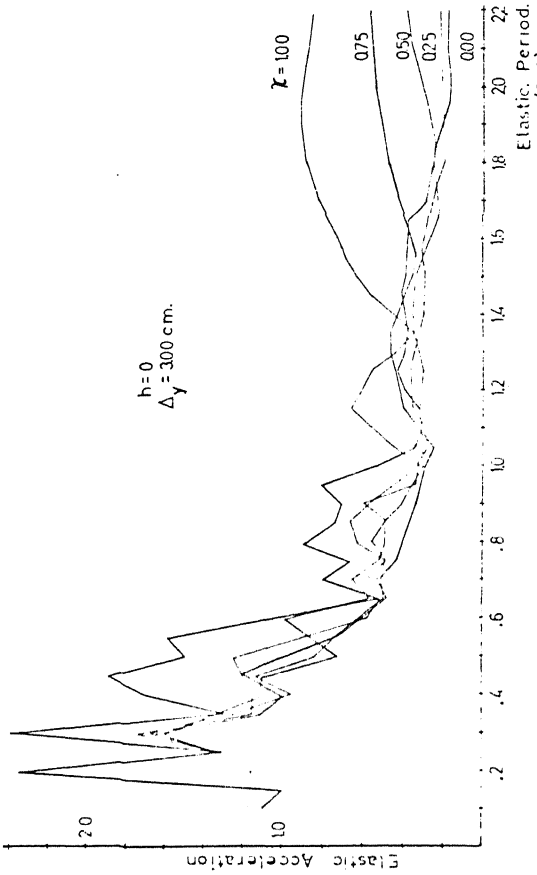

Figure 10.

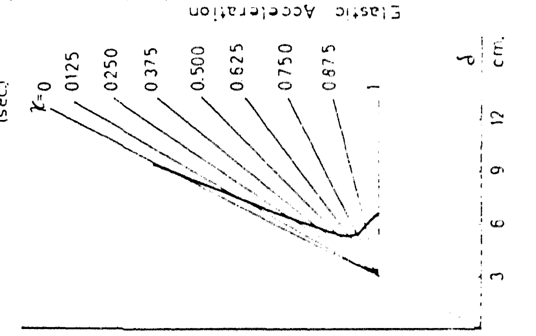

Figure 11.

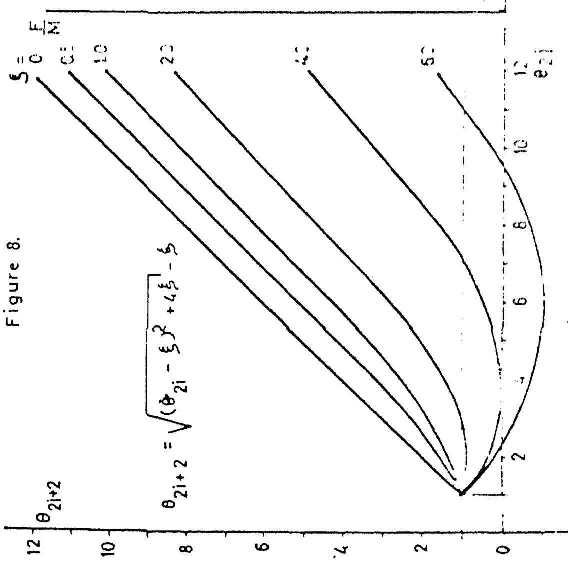

Figure 7.