STATISTICAL ANALYSIS OF NONLINEAR PARAMETRIC SYSTEMS BY STRONG-MOTION EARTHQUAKE EXCITATION

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Synopsis

The paper discusses problems of working out the method of statistical analysis of nonlinear parametric systems under random dynamic effects such as seismic ones. The results of estimating the basic characteristics of non-linear parametric systems required to determine conditions of dynamic stability and to evaluate safety are presented. The analysis is based on the method of building up and solving the appropriate Kolmogorov equations.

Introduction

The analysis of the results of a number of earthquakes that were observed here and abroad has shown that there is characteristic instability of latticed structures of radio masts, electric power transmission lines and bridge trusses, as well as of thin-wall tanks. Members and structures may lose their stability due to the horizontal components of the earthquake, vertical component and due to the combined effect of both of them / 1-5 /.

The peculiar feature of thin-wall structures is that in their design the finiteness of displacements and others non-linear factors should be taken into account / 1,2 /.

I. Statistical analysis of a non-linear parametric system

Let's consider a problem of forced vibrations of a sructure in the system if the x, y coordinates which moves progressively as regards the inertial system X, Y (Fig. Ib). The progressive motion of the moveable system of coordinates is determined by the functions $\ddot{x}_0(t)$ and $\ddot{y}_0(t)$,

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regarded as stationary independent random functions of time with known statistical parameters (the distribution law and correlation functions are known). The problem of a rod structure vibrating at the horizontal and vertical seismic motion of the ground (if to accept the hypothesis of the seismic effect being stationary) under the effect of tracking force resolves itself to the foregoing model. In particular, this may be a column of the framework of a single-storey building (Fig.Ia).

If to assume the hypothesis of non-linear viscous resistance for damping forced, to take account of the forces of inertia of the concentrated mass M_c due to the effect of seismic forces, non-linear inertia of the mass M_c and power non-linear elasticity, as well as to take account of only the first mode of bending vibrations, then the elastic axis of the rod can be determined by the equality:

$$v(x,t) = \varphi(x) f(t) = (1 - \cos\frac{\pi x}{2l}) f(t) \qquad (\mathrm{I})$$

Making use then of the Bubnov-Galiorkin method, the equation will be attained for $f(t)$, determining the law of oscillation of the rod, the model of which was presented by a singledegree-of-freedom system:

$$(1+s)\ddot{f} + 2\beta_0\dot{f} + [\Omega^2(1-2\mu x(t)) + Q(t)[\frac{\pi}{2l}r - R]]f + 2\varkappa(\ddot{f}f + \dot{f}^2)f + \gamma_0 f^3 + \gamma_{10} f^5 + \varepsilon_0 \dot{f} f^2 = N(t) r + M(t)\tau - \vartheta\eta(t) , \qquad (2)$$

where

$$s = M_c b/m\cdot c; \quad P_э = EJ\pi^2/4l^2; \quad R = a/mc; \quad r = b/mc; \quad \beta_0 = \beta/m;$$

$$\Omega^2 = \omega^2(1 - P_0/P_э); \quad \omega^2 = EJ\pi^2 a/4l^2 mc; \quad \tau = d/mc; \quad \ddot{x}_0(t) = A_1 x(t);$$

$$\mu = M_1 A_1/2(P_э - P_0); \quad d = \int_0^l \varphi(x)dx; \quad a = \int_0^l \cos(\frac{\pi x}{2l})\varphi(x)dx; \quad c = \int_0^l B\varphi(x)dx;$$

$$b = \int_0^l (l-x)\varphi(x)dx; \quad B = [\pi^2(2lx - x^2) - 8l^2\sin\frac{\pi x}{2l} + 4\pi l^2 - 4\pi l(l-x)\cos\frac{\pi x}{2l}]/2\pi^2;$$

$$\varkappa = K_1 M_c a/mc; \quad K_1 = \frac{1}{2}\int_0^l [\varphi'(x)]^2 dx; \quad \gamma_0 = \gamma K_2/mc; \quad \gamma_{10} = \gamma_0 K/mc; \quad \varepsilon_0 = \varepsilon K_2/mc;$$

$$K = \int_0^l [\varphi(x)]^6 dx; \quad K_2 = \int_0^l [\varphi(x)]^4 dx; \quad \vartheta = B_1 s; \quad \ddot{y}_0(t) = B_1\eta(t).$$

The function $\varphi(x)$ approximates the shape of bending , $\varphi(t)$ - is the angle of rotation of the end section, $Q(t)$, $N(t)$, $M(t)$ - are the tangent, normal of the force and moment which are considered conditionally as known the static design for the time moment t . When deriving (2), it was supposed that deflections were small: $Q_x(t) \simeq Q(t)$; $Q_y(t) \simeq Q(t)\varphi(t)$; $N_x(t) \simeq N(t)\varphi(t)$; $N_y(t) \simeq N(t)$; $\varphi = v'(l)$.

It is assumed that intensity of disturbances $x(t)$ and $\eta(t)$ does not result in great changes in the amplitude and the phase of the system being out of the period; the value β_0 is small and the system is of a narrow band.

Under the above suppositions the output of the system is close to quasi-harmonic vibrations with the amplitude and phase changing slowly in time. According to the asymptotic method it can be taken in the first approximation

$$f(t) = A(t)\cos[\Omega t + \psi(t)]; \quad \dot{f}(t) = -\Omega A(t)\sin[\Omega t + \psi(t)]. \qquad (3)$$

For the quasi-harmonic output of system (3) the following eguations will be obtained for the amplitude and phase:

$$\dot{A}(t) = \frac{1}{A(t)\Omega^2}\Big[\frac{-s\Omega^2 A^2(t)\sin 2\Phi}{2} - \beta_0\Omega^2 A^2(1-\cos 2\Phi) - \frac{\varkappa A^4\Omega^3}{2}\sin 4\Phi + \frac{\gamma_0 A^4\Omega}{4}(\sin 2\Phi + \frac{1}{2}\sin 4\Phi) + \frac{\gamma_{10} A^6\Omega}{16}(\frac{1}{2}\sin 6\Phi + 2\sin 4\Phi + \frac{5}{2}\sin 2\Phi) - \frac{\varepsilon_0\Omega^2 A^4}{4}(1-\cos 4\Phi) - \mu A\Omega x(t)\sin 2\Phi + \frac{\pi}{4l}\Omega^{-1}Q(t)rA\sin 2\Phi - \frac{Q(t)RA}{2\Omega}\sin\Phi - \frac{N(t)r}{\Omega}\sin\Phi - M(t)r\Omega^{-1}\sin\Phi + \vartheta\Omega^{-1}\eta(t)\sin\Phi; \qquad (4)$$

and

$$\dot{\psi}(t) = -\frac{1}{A^2\Omega}\Big[\beta_0\Omega A^2\sin 2\Phi + \varkappa A^4\Omega^2(\frac{1}{2} + \cos 2\Phi + \frac{1}{2}\cos 4\Phi) - \frac{\gamma_0 A^4}{8}(3 + 4\cos 2\Phi + \cos 4\Phi) - \frac{\gamma_{10} A^6}{16}(\frac{1}{2}\cos 6\Phi + 3\cos 4\Phi + \frac{15}{2}\cos 2\Phi + 15) + \varepsilon_0 A^4\Omega(\frac{1}{8}\sin 4\Phi + \frac{1}{4}\sin 2\Phi) + \frac{s\Omega^2 A^2}{2}(1+\cos 2\Phi)\Big] - \mu x(t)\Omega(1-\cos 2\Phi) + \frac{\pi}{4l}Q(t)r\Omega^{-1}(1+\cos 2\Phi) - \frac{Q(t)}{2\Omega}R(1+\cos 2\Phi) - N(t)r\cos\Phi(\Omega A)^{-1} - M(t)r\cos\Phi(\Omega A)^{-1} + \vartheta\eta(t)\cos\Phi(\Omega A)^{-1}. \qquad (5)$$

Here $\Phi = \Omega t + \psi(t)$.

Let's exclude vibrational functions out of these equations, passing from simple averaging over a period to the second approximation. It will be supposed here, that the correlation time of the function $\psi(t)$ is substantially less than the period of natural vibrations.

Let's write down Equations (4), (5) in the following form:

$$\dot{A} = \beta_0 G^* + G; \quad \dot{\psi} = \beta_0 H^* + H, \qquad (6)$$

where G^* and H^* are regular terms, and G, H are fluctuating terms. Excluding fluctuating terms from these equations and expanding the functions G^* and H^* into series, with regard to the small parameter β_0, we'll get:

$$\dot{A} = \beta_0 G_1 + \beta_0^2 G_2 + \ldots; \quad \dot{\psi} = \beta_0 H_1 + \beta_0^2 H_2 + \ldots \qquad (7)$$

The functions G_1 and H_1 are determined by simple averaging of the functions G^* and H^* as regards Φ;

the functions G_2 and H_2 determine the non-vibrational terms respectively from expressions

$$\left(\frac{\partial G^*}{\partial A} u_1 + \frac{\partial G^*}{\partial \psi} v_1\right); \qquad \left(\frac{\partial H^*}{\partial A} u_1 + \frac{\partial H^*}{\partial \psi} v_1\right)$$

and functions u_1 and v_1 are determined from equations

$$\frac{\partial u_1}{\partial \varphi} = \Omega^{-1}\left[G^* - \langle G^* \rangle_\varphi\right]; \quad \frac{\partial v_1}{\partial \varphi} = \Omega^{-1}\left[H^* - \langle H^* \rangle_\varphi\right] \tag{8}$$

Let the fluctuating terms of Equations (4) and (5) be presented as a sum of mean m_1, m_2 and centered random components $\xi_1(t)$, $\xi_2(t)$ with δ-like correlation functions:

$$\begin{aligned} G &= \nu\Omega^{-1}\eta(t)\sin\Phi - \mu A\Omega x(t)\sin\Phi + \pi Q(t) r A \sin 2\Phi/4l\Omega - \\ &- RAQ(t)\sin 2\Phi/2\Omega - rN(t)\sin\Phi/\Omega - rM(t)\sin\Phi/\Omega = \\ &= m_1 + \xi_1(t); \end{aligned} \tag{9}$$

$$\begin{aligned} H &= \nu A\eta(t)\cos\Phi/\Omega - \mu\Omega x(t)(1+\cos 2\Phi) + \pi r Q(t)(1 + \\ &+ \cos 2\Phi)/4l\Omega - RQ(t)(1+\cos 2\Phi)/2\Omega + rN(t)\cos\Phi/\Omega A - \\ &- rM(t)\cos\Phi/\Omega A = m_2 + \xi_2(t). \end{aligned} \tag{10}$$

The coefficientes of the intensitiens of processes $\xi_1(t)$ and $\xi_2(t)$ are equal to

$$\begin{aligned} K_1 &= \int_{-\infty}^{\infty} \langle \xi_1(t)\xi_1(t+\tau)\rangle d\tau = \int_{-\infty}^{\infty} R[G(t), G(t+\tau)]d\tau + \int_{-\infty}^{0} R[G(t), H(t+\tau)]d\tau + \int_{-\infty}^{0} R[G(t+\tau), H(t)]d\tau = \\ &= \nu^2 S_\eta(\Omega)/2\Omega^2 + \mu^2\Omega^2A^2 S_x(2\Omega)/2 + \frac{1}{2}[\pi^2r^2A^2/16l^2\Omega^2 - \pi RrA^2/4l\Omega^2 + R^2A^2/4\Omega^2]S_Q(2\Omega) + \\ &+ r^2 S_N(\Omega)/2\Omega^2 + r^2 S_M(\Omega)/2\Omega^2; \end{aligned} \tag{11}$$

$$K_2 = \int_{-\infty}^{\infty} \langle \xi_2(t)\xi_2(t+\tau)\rangle d\tau = \int_{-\infty}^{\infty} R[H(t), H(t+\tau)]d\tau + \int_{-\infty}^{0} R[H(t), H(t+\tau)]d\tau + \int_{-\infty}^{0} R[H(t+\tau), G(t)]d\tau =$$

$$= \nu^2 S_\eta(\Omega)/2\Omega^2 A^2 + \mu^2\Omega^2[S_\chi(0) + \frac{1}{2}S_\chi(2\Omega)] + \pi^2 r^2 [S_Q(0) + \frac{1}{2}S_Q(2\Omega)]/16\ell^2\Omega^2 +$$
$$+ R^2(S_Q(0) + \frac{1}{2}S_Q(2\Omega))/4\Omega^2 + r^2 S_N(\Omega)/2\Omega^2 A^2 + \tau^2 S_M(\Omega)/2\Omega^2 A^2 \quad , \tag{12}$$

where $R[\ldots]$ is the correlation function;

$S(\ldots)$ are the spectral densities of the processes at respective frequencies.

The value m_1 is calculated by the general formula

$$m_1 = \int_{-\infty}^{0} R[\frac{\partial G(t)}{\partial A}, G(t+\tau)]d\tau + \int_{-\infty}^{0} R[\frac{\partial G(t)}{\partial \psi}, H(t+\tau)]d\tau = \nu^2 S_\eta(\Omega)/4\Omega^2 A + \frac{3}{4}\mu^2\Omega^2 A S_\chi(2\Omega) -$$
$$- \frac{3}{16}\pi r R A S_Q(2\Omega)/\ell\Omega^2 + 3\pi^2 r^2 A S_Q(2\Omega)/64\ell^2\Omega^2 + 3R^2 A S_Q(2\Omega)/16\Omega^2 + r^2 S_N(\Omega)/\Omega^2 A + \tag{13}$$
$$+ \tau^2 S_M(\Omega)/\Omega^2 A .$$

Similarly m_2 is calculated. The expressions

$$\int_{-\infty}^{0} R[G(t+\tau), H(t)]d\tau \; , \qquad \int_{-\infty}^{0} R[G(t), H(t+\tau)]\,d\tau$$

give only vibrational terms. In the first approximation the random processes $\xi_1(t)$ and $\xi_2(t)$ can therefore be considered as independent.

If $S_\chi(0) = S_\chi(2\Omega) = S_Q(0) = S_Q(2\Omega) = S_N(\Omega) = S_M(\Omega) = S_\eta(\Omega) = 1$,

then a particular case of the effect of "white noise" will be obtained.

So the shortened evolutionary equation for the amplitude with excluded vibrational functions in the second approximation has the form of

$$\dot{A} = -a_1 A - J_1 A^3 + J_2 A^5 + J_3 A^7 + m_1 + \xi_1(t) , \tag{14}$$

where $a_1 = \beta_0$; $J_1 = \frac{\varepsilon_0}{8} - \frac{\beta_0 \varkappa}{2}$; $J_2 = \frac{\gamma_0 \varepsilon_0}{32\Omega^2}$; $J_3 = \frac{5}{256}\frac{\gamma_{10}\varepsilon_0}{\Omega^2}$.

The convenience of the above method is that for any type of non-linearities Equation (14) does not depend upon the relavant equation for $\psi(t)$, which permits the statistical characteristics of the amplitude to be studied separately by working out a one-dimensional FPK equation for $w(A,t)$ of the form:

$$\frac{\partial w(A,t)}{\partial t} = -\frac{\partial}{\partial A}\left\{\left[(0{,}5a_7+a_{10}+a_{11})A^{-1}-(a_1-\frac{3}{2}a_8-a_9)A-J_1A^3+J_2A^5+\right.\right.$$
$$\left.\left.+J_3A^7\right]w(A,t)\right\}+\frac{1}{2}\frac{\partial^2}{\partial A^2}\left\{[(a_7+a_{10}+a_{11})+(a_8+a_9)A^2]w(A,t)\right\} \quad , \tag{I5}$$

where

$$a_{11}=\frac{r^2}{\Omega^2}S_M(\Omega);\quad a_{10}=\frac{r^2}{\Omega^2}S_N(\Omega);\quad a_9=\left[\frac{3}{64}\frac{\pi^2r^2}{l^2\Omega^2}-\frac{3}{16}\frac{\pi r R}{l\Omega^2}+\frac{3}{16}\frac{R^2}{\Omega^2}\right]S_R(2\Omega);$$

$$a_8=\mu^2\Omega^2S_\chi(2\Omega)/2;\quad a_7=\vartheta^2S_\eta(\Omega)/2\Omega^2$$

Stationary distribution

$$w_{st}(A)=\frac{C}{(a_7+a_{10}+a_{11})+(a_8+a_{10})A^2}\exp\left[2\int\frac{(0{,}5a_7+a_{10}+a_{11})A^{-1}-(a_1-\frac{3}{2}a_8-a_9)A-J_1A^3+J_2A^5+J_3A^7}{(a_7+a_{10}+a_{11})+(a_8+a_{10})A^2}dA\right] \tag{I6}$$

The constant C is determined in terms of normalizing $\int w_{st}(A)\,dA=1$

To determine the non-stationary distribution $w(A,t)$ the Bubnov-Galiorkin method may be used.
Let's take the function $w_m(A)$ in the form

$$w_m(A)=\frac{A}{\sigma_1^2}\exp\left\{-\frac{A^2}{2\sigma_1^2}\right\}L_m\left(\frac{A^2}{2\sigma_1^2}\right);\quad \sigma_1^2=(0{,}5a_7+a_{10}+a_{11})/(a_1-\frac{3}{2}a_8-a_9) \tag{I7}$$

$L_m(\ldots)$ are Lagerr polynoms. Function (I7) correspond to the solution of the linear problem of forced vibrations. The solution is to be sought in the form

$$w(A,t)=w_{St}(A)+\sum_{m=1}^{n}T_m(t)w_m(A) \tag{I8}$$

For the function $T_m(t)$ the following system of equations will be obtained:

$$\sum_{m=1}^{n}\left[\alpha_{mj}\dot{T}_m(t)+\beta_{mj}T_m(t)\right]=0 \ , \tag{I9}$$

where $\alpha_{mj}=\frac{\sqrt{2}}{\sigma_1}\int_0^\infty x^{1/2}e^{-2x}L_m(x)L_j(x)dx;\quad \beta_{mj}=-(0{,}5a_7+a_{10}+a_{11})N_{1mj}/\sigma_1^3-$
$-2\sqrt{2}(a_8+a_9)N_{5mj}/\sigma_1-(a_1-\frac{3}{2}a_8-a_9)N_{2mj}/\sigma_1+J_1\sigma_1N_{3mj}-J_2\sigma_1^3N_{4mj}$

The values of coefficients are given in Table I. Equation (I9) is solved in the form $T_m(t) = C_m \exp\{\lambda t\}$, and the values $\lambda_1, \ldots, \lambda_n$ will be found from the equation

$$\begin{vmatrix} \alpha_{11}\lambda + \beta_{11}, \ldots, & \alpha_{n1}\lambda + \beta_{n1} \\ \cdots\cdots\cdots & \cdots\cdots\cdots \\ \cdots\cdots\cdots & \cdots\cdots\cdots \\ \alpha_{1n}\lambda + \beta_{1n}, \ldots, & \alpha_{nn}\lambda + \beta_{nn} \end{vmatrix} \qquad (20)$$

The values $C_1^{(K)}, \ldots\ldots, C_m^{(K)}$ corresponding to λ_K will be found from the system of equations

$$\sum_{m=1}^{n} (\alpha_{mj}\lambda_K + \beta_{mj}) C_m^{(K)} = 0 \quad (j = 1, \ldots, n) \qquad (2I)$$

The coefficients $C_m^{(K)}$ are determined to the accuracy of an arbitrary multiplier $\mathcal{D}_K$, i.e. $C_i^{(K)} = \mathcal{D}_K K_i^{(K)}$,

where $K_i^{(K)}$ are minors of the members of the first line of the determinator (20). Then the complete solution of System (I9) will be written in the form

$$T_m(t) = \sum_{m=1}^{n} \mathcal{D}_K K_m^{(K)} e^{\lambda_K t} \qquad (22)$$

The coefficients $\mathcal{D}_K$ are found from the initial conditions: at $t = 0$, $w(A,0) = w_0(A)$. It is not difficult to show /8/ that

$$\sum_{K=1}^{n} \mathcal{D}_K K_m^{(K)} = \int_0^\infty w_0(A) \mathcal{L}_m\left(\frac{A^2}{2\sigma_1^2}\right) dA - \int_0^\infty w_{st}(A) \mathcal{L}_m\left(\frac{A^2}{2\sigma_1^2}\right) dA \quad (m = 1, 2, \ldots, n) \qquad (23)$$

So, the non-stationary distribution of the amplitude is determined by the relation:

$$w(A,t) = w_{st}(A) + \sum_{m=1}^{n} \sum_{K=1}^{n} \mathcal{D}_K K_m^{(K)} e^{\lambda_K t} w_m(A) \qquad (24)$$

The value $w(A,t)$ from (24) can be obtained to any degree of accuracy depending on the number n, and the given scheme of solution is readily programmed on a digital computer.

Here the criterion of stability is taken according to the first approximation. In this case all the non-linea - rities should be neglected and $N(t)=M(t)=\ddot{y}_0(t)=\eta(t)=0$ should be assumed. Then, taking into account (I3) and (I4) we'll get

$$\dot{A}=-a_1A+\frac{3}{4}\mu^2A\Omega^2S_\chi(2\Omega)-\frac{3}{16}\frac{\pi r RA}{\ell\Omega^2}S_Q(2\Omega)+\frac{3}{64}\frac{\pi^2r^2A}{\ell^2\Omega^2}S_Q(2\Omega)+ \\ +\frac{3}{16}\frac{R^2A}{\Omega^2}S_Q(2\Omega)+\xi_1(t) \quad (25)$$

and $[\langle A\rangle]>0;\ A\neq 0$ corresponds to the instable condition. Let's average (25). Since $\langle\xi_1(t)\rangle=0$, we obtain the condition under which parametric vibrations of the system occur in the form

$$\beta_0<\frac{3}{4}\mu^2\Omega^2S_\chi(2\Omega)+\left[\frac{3}{64}\frac{\pi^2r^2}{\ell^2\Omega^2}+\frac{3}{16}\frac{R^2}{\Omega^2}-\frac{3}{16}\frac{\pi rR}{\ell\Omega^2}\right]S_Q(2\Omega) \quad (26)$$

In this case vibrations increase unrestrictedly and dynamic instability is observed. The value corresponds to the cri - tical value of the coefficient

$$\beta_{0_{cr}}=\frac{3}{4}\mu^2\Omega^2S_\chi(2\Omega)+\left[\frac{3}{64}\frac{\pi^2r^2}{\ell^2\Omega^2}+\frac{3}{16}\frac{R^2}{\Omega^2}-\frac{3}{16}\frac{\pi rR}{\ell\Omega^2}\right]S_Q(2\Omega) \quad (27)$$

and this value determines the magnitude of the excitation factor $\mu=\mu_1$, above which lies the area of dynamic instability of the system. For the sake of illustration, we'll consider a seismic effect that is regarded as a stationary random peocess with a normalized correlation function

$$R_{\ddot{x}_0}(\tau)=e^{-\alpha_1|\tau|}\cos\beta_1\tau;\quad \alpha_1=7\ 1/\text{sec};\quad \beta_1=18\ 1/\text{sec} \quad (28)$$

Untroduce the design acceleration of the seismic ef - fect $A_1=\bar{K}\sqrt{B(0)}=gK_s$; $\sqrt{B(0)}$ is the standard of acceleration $\ddot{x}_0(t)$; $\bar{K}$ is the number of standards; K_s is the seismicity factor; is the gravity acceleration. The diagram in Fig. 2 will taken as a design one Transform (27) as follows

$$y_1-y_2=0 \quad (29)$$

where $y_1=\xi\dfrac{(4\bar{\varepsilon}+7{,}6\xi^2)(1-\bar{\varepsilon})^2}{16\bar{\varepsilon}^2-44{,}8\bar{\varepsilon}\xi^2+57{,}6\xi^4}$; $y_2=\dfrac{\delta_0}{2}\bar{\varepsilon}$; $\xi=\dfrac{\alpha_1}{\omega}$; $\gamma_0=\dfrac{\beta_1}{\alpha_1}=2{,}57$;
$\bar{\varepsilon}=1-P_0/P_э$; $\delta=8\gamma/3K_s^2$.

The parameter $\gamma=\delta/\pi$ characterizes the dissipative forces, where δ is the logarithmic decrement of attenuation of vibration. For metal structures γ =0,05; for reinforced concrete structures γ =0,I / 2 /. The graphical solution of Equation (29) is presented in Fig.2 ; the dotted lines show the criteria for the stability of the system as regards the average logarithm of the amplitude velocity / 2 /. In this case $\delta_0 = 4\gamma / K_s^2$ The points of intersection y_1 and y_2 define the magnitudes ε and , consequently , the critical value of the mass M at the given intensity of the seismic effect. If rod mem - bers of structures (frames, trusses, etc.) are loaded with forces close to P_3 , the instability can occur at a very low intensity of acceleration of the ground motion / 9,10 /.

References

I. Goldenblat J.J., Nikolaenko N.A. Raschet konstrukzij na deistvie seismicheskikh i impulsivnikh sil. Gosstroiisdat, I96I.

2. Nikolaenko N.A. Verojatnostnie metodi dinamicheskogo rescheta mashinostroitelnikh konstrukzij. Mashinostroenie, I967.

3. Nokolaenko N.A. Shtol A.T. Dinamicheskaja ustojchivost i statisticheskij analis kolebanij nelinejnoj parametricheskoj sistemi. "Stroitelnaja mechanika i raschet sooruzhenij", I970, N I.

4. Nikolaenko N.A., Uljanov S.V. K statisticheskomu analisu parametricheskikh sistem pri dinamicheskikh vosdeistvijakh. "Stroitelnaja mechanika i reschet sooruzhenij", I972, N 2.

5. Nikolaenko N.A., Uljanov S.V., Karimova R.Ch. Statisticheskij amalis, optomizazija struktury i ozenka nadezhnosti parametricheskikh sistem pri sluchajnikh vosdeisvijakh. In.: "Problema nadezhnosti v stroitelnoi mekhanike", Vilnus, I97I.

6. Nitropolskij Ju.A. Metod usrednenija v nelineinoj mekhanike. Naukova Dumka, Kiev, I97I.

7. Stratonovich R.L. Isbrannie voprosi teorii fluktuazii v radiotekhnike. Sovetskoe Radio, I96I.

8. Khasminskij R.Ch. Ustoichivost sistem differenzialnikh uravnenij pri sluchainikh vozmushenijakh ikh parametrov.

9. Goldenblat J.J. , Karzivadze G.N., Napetvaridze Sh.G., Nikolaenko N.A. Proektirovanie seismostoikikh gidro - tekhnicheskikh , transportnikh i spezialnikh sooru - zhenij. V. 4, Stroiisdat, I97I.

IO. Nikolaenko N.A., Uljanov S.V., Shtol A.T. Statisti - cheskij analis i ozenka nadezhnosti parametricheskikh sistem pri seismicheskikh vosdeisvijakh. In: "Materi- ali k Usesojusnomu soveshaniju po proektirovaniju i stroitelstvu seismostoikikh zdanij i sooruzhenij " , Frunze, I97I.

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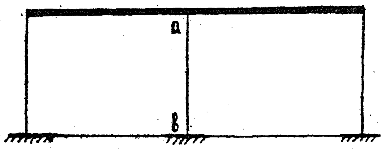

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