INFLUENCE OF THE NONELASTIC DEFORMATIONS ON THE STRUCTURES ON THE CHANGING OF DYNAMIC CHARACTERISTICS AND RESPONSE

by

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SUMMARY

Designing of the structures for seismic forces requires determining of their dynamic characteristics both in elastic, elasto-plastic and plastic stages.

Theoretical and experimental investigations on reinforced concrete frame elements and buildings for determination of a variation of dynamic characteristics, redistribution of the stresses and response to a strong motion in function of non-elastic deformations were carried out.

The investigated elements are models of reinforced concrete frame buildings with or without filling masonry subjected to a horizontal reversive load. The strength, the deformative and dynamic characteristics are used for investigation of the redistribution of the stresses in an elastic and elasto-plastic stage.

The response of a six-story building for the San Fernando 1971 earthquake S16°E component was investigated taking into account nonelastic deformations. The equations of motion are solved using a step-by-step integration procedure with changing of the stiffness matrix and some other characteristics.

I. INTRODUCTION

The dynamic response of structures to strong motion earthquakes is the problem which drew the attention of many investigators in recent years. Due to the introduction of computer technics during the last years, numerous investigations on the seismic response of structures in elastic, elastic-plastic and plastic stages were carried out [1, 2, 3, 4, 5]. The investigations in this field began with idealized models of structures formed at a large number of assumptions and initial conditions. With the data accumulation and investigation improvement the structures' models get complicated and become close to the characterization of the actual behaviour of the structures during seismic actions.

Although the greater part of the investigations on the structure response is carried out on frame buildings and shear type buildings, recently the investigations on complex structures, (frame buildings, diaphragms, partition walls, stairwell towers) are extended.

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The investigation of structure response in a nonelastic stage requires: data about the strength and deformative characteristics of the different structure elements over the boundary of elasticity at reversive loading; data about the simultaneous work and interaction of the structural elements, their ability to absorb the energy, to redistribute the stresses and the possibility of big deformations without any destructions.

The investigations mentioned above are in a close connection with the concrete structural system which is mostly applied in a given country and which will be studied.

Our first investigations [6, 7] considered the stresses and deformations in frame buildings with infilling walls in an elastic stage. The next investigations [8] considered the dynamic characteristics of complex structures consisting of a frame, infilling walls and stairwell towers.

The purpose of the present investigations is to study the response of a frame reinforced concrete building with infilling masonry to strong earthquakes by investigating the structure at the appearance of nonelastic deformations in different elements and their influence on the change of the dynamic characteristics of the structure and redistribution of the stresses.

II. MODELS OF ANALYSES

Real six-story reinforced concrete frame structures consisting of three types of vertical diaphragms (Fig. 1) are investigated. The vertical diaphragm A (Fig. 1) is constructed from a frame with infilling masonry; diaphragm B - from frame only, and diaphragm C - from frame and infilling masonry with openings.

The reinforced concrete frame is formed of columns and beams with uniform cross section in all storeys. The strength of the concrete is 200 kg/cm^2, modul of elasticity 2,65.10^5 kg/cm^2. The masonry is built of normal clay bricks (25/12/6,5 cm) with a strength - R_1 = 100 kg/cm^2 and mortar - R_2 = 25 kg/cm^2. The strength and deformative characteristics of the masonry are determined according to the norms taking into account some specific properties.

The compressed strength [9] is

$$R = AR_1\left(1-\frac{a}{b+\frac{R_2}{2R_1}}\right)\gamma = 18\ \mathrm{kg/cm^2} \tag{1}$$

where

$$A = \frac{100+R_1}{100m+nR_1}$$

a = o,2; b = 0,3; m = 1,25; n = 3; γ = 1 - coefficients R_1, R_2 -

compressed strength of the bricks and mortar. The shear strength [6] is

$$H_{sh} = k_1 \frac{3ld(\beta - 0{,}3\beta^2)}{F} R_o \tag{2}$$

where

$$k_1 = 1 - 0{,}4\frac{a_1}{l} - 0{,}6\frac{b_1}{h}$$

$$F = 1 + \beta^2 - \sqrt{\beta^4 - 0{,}8\beta^2 + 1}$$

$$\beta = l/h$$

l, h, d - length, height and thickness of the masonry;

a_1, b_1 - length and height of the opening in the masonry;

R_o, R_{sh} - tension and shear strengths of masonry;

$$R_o = \frac{2{,}5}{1 + \frac{25}{R_2}}; \qquad R_{sh} = 2R_o = \frac{5}{1 + \frac{25}{R_2}} \tag{3}$$

The modul of elasticity is

$$E_m = \alpha R = 750.18 = 13500 \ \text{kg/cm}^2 \tag{4}$$

The modul of shearing

$$G = 0,4E_m = 5400 \ \text{kg/cm}^2 \tag{5}$$

The stiffness of the masonry

$$K_m = k_1 \frac{0{,}83}{h} l d G \tag{6}$$

The field of the deformations as a function of stresses, relative deformation (ε) and speed of the loading are taken into account according to Fig. 2.

The characteristics mentioned are used for investigation of the building in two stages:

a) investigation of the influence of different assumptions and non-elastic deformation on the redistribution of the stresses and variation of the dynamic characteristics;

b) investigation of the response of the building model of the San Fernando earthquake, 1971, in elastic and non-elastic deformations in some elements.

III. INFLUENCE OF THE NONELASTIC DEFORMATIONS IN THE STRUCTURES AND ASSUMPTIONS ON THE REDISTRIBUTION OF THE STRESSES AND CHANGE OF THE DYNAMIC CHARACTERISTICS.

The size of the seismic forces for the building on Fig. 1 is determined according to the norms [10] and are accepted as being distributed in height by the triangle law. The seismic force in a given point of the structure is

$$S_i = \psi \beta \eta K_s Q \tag{7}$$

where

$\psi = 1$ - damping coefficient of the structure

$\beta = \frac{0,9}{T}$ - dynamic coefficient

T - natural period of the structure

η - coefficient of the form of vibration.

The total horizontal force is distributed between the vertical diaphragms and frames proportional to the rigidities by equalizing the horizontal displacement on every floor.

$$\delta_{iA} = \delta_{iB} = \delta_{iC} \tag{8}$$

This assumption is possible if it is accepted that there is a nondeformability of the floor structures in their planes.

The masonry with and without openings in the vertical diaphragms A and C is replaced by equivalent diagonals (Fig. 1b) hinged with the frame and taking up only compressed stresses.

The strength and deformative characteristics of the replacing compressed diagonals are determined depending on the properties of infilling masonry

$$F_o = \frac{K m \ell}{E \cos^3 \alpha} \tag{9}$$

The investigation of the stress redistribution in the vertical diaphragms for the corresponding loading is carried out under the following states:

(1) Columns, beams and diagonals are deformable;

(2) Columns and diagonals are deformable, beams are flexible but they are neither shortened nor lengthened.

3 Diagonals are deformable, columns are flexible but neither lengthened nor shortened, beams are rigid (not deformable).

4 The same as 2 but in plastic stage.

Influence of the nonelastic deformations on the redistributions of the stresses is carried out by means of the successive approximations as follows:

a) The stresses in the elements of the diaphragms in the elasticity stage are determined;

b) Using the obtained stresses and the force-deformation diagram (Fig. 3), the new elastic-plastic characteristic of masonry is obtained by (10)

$$K_i = \alpha_i K_m = K_m\left(1 - \frac{\sigma_i}{1{,}5R_o}\right) \tag{10}$$

Relation (10) and the work diagram of masonry (Fig. 3) are determined on the basis of the experimental investigations on elements subject to the reversive loading (Fig. 4).

The stresses in the masonry on the story are different; that is why the rigidities determined by (10) vary for each story.

c) By using the new rigidities the natural periods and mode of vibration, the size of seismic forces and their redistribution among the diaphragms are determined.

d) The stresses in the diaphragms for the new rigidities are determined according to (10) and so on.

The investigations are carried out by Fortran program with a ICL computer.

On the basis of the investigations the following conclusions are drawn:

1. The horizontal deformations in the diaphragms determined under different assumptions (Fig. 5) differ from each other. The horizontal deformations in the diaphragms determined under assumptions 3 ,page 6 are about 20 - 35% smaller than the deformations determined under assumptions 2 and about 25% smaller than the deformations determined under assumptions 1

2. Natural periods of the building determined under the different assumptions 1 - 3 in elastic stage differ from each other about 25%.

3. The natural periods of the building determined in elastoplastic stage 4 differ up to 80% compared with the elastic stage. In this case the size of the seismic force decreases according to the norms twice.

4. The distribution of the stresses between the frame and the filling masonry depends also on the assumptions.

Distributions of the stresses under assumption (1) and concentration of the horizontal forces on the point of the floor lead to different stresses in the masonry of a given floor, Fig. 6 - (1) .

The acceptance of nonlengthened beams (2) leads to a decrease of the difference in the stresses of the masonry for a given story but changes the distribution of the stresses. In this case the maximum stress is on the second story instead of on the first one. (Fig. 6)

The acceptance of rigid beams and nonlengthened columns leads to equalization of the stresses in the masonry for a given story. The sum of the horizontal forces on the given floor in this state (3) is equal to the sum from the forces in the previous states (1) or (2) .

5. The redistribution of the stresses in elasto-plastic state leads to decreasing of the forces in lower stories (Fig. 6). (4)

IV. RESPONSE OF THE STRUCTURE TO A STRONG MOTION EARTHQUAKE.

After investigating the redistribution of the stresses in the diaphragms as a consequence of an appearance of elastic-plastic deformations more exact data were obtained concerning the variation of structure rigidity as a whole on the different floors and diaphragms in certain stages of loading.

These data served as a basis for composing a more precise model of investigating the structure response under strong motion excitation.

The models originally accepted created a series of difficulties when solving the system of differential equations because of which it was necessary to investigate first the structure response with a more simple model.

The response of a six-story structure (Fig. 1) for the San Fernando 1971 earthquake, S16^{o}E component [11] was investigated by the model formed under the following assumptions:

1. Floor slabs are infinitely rigid.
2. The rigid floors remain parallel during vibrations.
3. The mass of the structure is concentrated on each floor level.
4. The modes of vibrations are independent. Coupling between the various modes of vibration do not exist.
5. The fundamental mode of vibration is triangular.
6. The strength and the stiffness of the structure in the elastic stage is equal for all storeys.
7. In elasto-plastic stage the structure stiffness is a function of the stresses (Fig. 3).
8. The damping is a viscous one.

The system of differential equations of motion of the structure is:

$$
\begin{aligned}
&m_1 \ddot{x}_1 + \alpha_1 K_1 x_1 - \alpha_2 K_2 (x_2 - x_1) + c_1 \dot{x}_1 - c_2 (\dot{x}_2 - \dot{x}_1) = -m_1 \ddot{x}_g \\
&m_2 \ddot{x}_2 + \alpha_2 K_2 (x_2 - x_1) - \alpha_3 K_3 (x_3 - x_2) + c_2 (\dot{x}_2 - \dot{x}_1) - c_3 (\dot{x}_3 - \dot{x}_2) = \\
&\qquad - m_2 \ddot{x}_g \\
&m_3 \ddot{x}_3 + \alpha_3 K_3 (x_3 - x_2) - \alpha_4 K_4 (x_4 - x_3) + c_3 (\dot{x}_3 - \dot{x}_2) - c_4 (\dot{x}_4 - \dot{x}_3) = \\
&\qquad - m_3 \ddot{x}_g \\
&m_4 \ddot{x}_4 + \alpha_4 K_4 (x_4 - x_3) - \alpha_5 K_5 (x_5 - x_4) + c_4 (\dot{x}_4 - \dot{x}_3) - c_5 (\dot{x}_5 - \dot{x}_4) = \\
&\qquad - m_4 \ddot{x}_g \\
&m_5 \ddot{x}_5 + \alpha_5 K_5 (x_5 - x_4) - \alpha_6 K_6 (x_6 - x_5) + c_5 (\dot{x}_5 - \dot{x}_4) - c_6 (\dot{x}_6 - \dot{x}_5) = \\
&\qquad - m_5 \ddot{x}_g \\
&m_6 \ddot{x}_6 + \alpha_6 K_6 (x_6 - x_5) \qquad + c_6 (\dot{x}_6 - \dot{x}_5) \qquad = m_g \ddot{x}_g
\end{aligned}
\tag{11}
$$

where

x_i - displacement of mass m_i relative to a fixed reference position;

x_g - ground displacement;

K_i - story elastic spring constant;

α_i - function of the nonelastic deformations

$$\alpha_i = \left(1 - \frac{\sigma_i}{1{,}5 R_0}\right) \tag{12}$$

C_i - viscous damping coefficient for story i.

The six simultaneous equations of motion (11) have been solved using a step-by-step integration procedure.

The system of equations (11) is solved in an elastic stage up to the moment when the stresses in some of the elements pass the limit of elasticity and plastic deformations are obtained in these elements. The stiffness matrix is changed by means of (12) and the solution continues up to the next stage of appearance of plastic deformations when a new correction in the stiffness matrix is made.

The response of the structure for the San Fernando 1971 earthquake, S16°E component at t = 2,1890 sec; t = 2,853 sec and t = 7,7550 sec is given on Table 1.

It is evident that the horizontal forces for t = 7,7550 sec are extremely big to be used for practical purposes. The horizontal forces determined in elastic-plastic stage are smaller but rather bigger than those determined according to the norms [10].

V. CONCLUSION

The conclusions made on the basis of the nonelastic deformations influence and assumptions about the changing of dynamic characteristics of the structure and their response shows that the structure response depends on the structural peculiarities, strength and deformative characteristics of the different structural elements and their interaction during exitation.

The plastic deformations in the masonry redistribute the stresses and decrease the size of seismic forces. Decreasing of the forces depends on the allowable deformations in each story which is a function of the structural system (its ability to absorb energy, to redistribute the stresses), the deformative characteristics of the materials, etc. For reinforced concrete frame buildings with infilling masonry walls 2.6 cm, horizontal story relative displacement can be accepted for response analyses.

Response analyses of the structures to a strong motion earthquake have to be provided taking into account redistribution of the stresses in a whole space structure and participation of the non-structural elements.

REFERENCES

1. Penzien J., Elasto-plastic response of idealized multi-storey structures subjected to a strong motion earthquake. Proc. Second World Conference on Earthquake Engineering, Tokyo, 1960.
2. Veletsos, A.S., Newmark, N.M., Effect of inelastic behaviour on the response of simple systems to earthquake motions. Proc. Second World Conference on EE, Tokyo, 1960.
3. Housner, G.W., The plastic failure of frame during earthquake, Second World Conference on EE, Tokyo, 1960.
4. Jennings, P.C., Force-deflection relations from dynamic tests, ASCE vol. 93, No. EM2, April 1967.
5. Vitiello, E., Seismic elastic plastic behaviour of multi-story buildings with "shear-bending type" resistant structures. Proc. of Third European Symposium on EE, Sofia, 1970.
6. Sachansky, S., Analysis of the earthquake resistance of frame buildings taking into account the carrying capacity of the filling masonry. Proc. Second World Conference on EE, Tokyo, 1960.
7. Sachansky, S., Karamanski, T., Method nouvelle pour determiner les tension dans les poutres murales constituees des materiaux differents. Symposium on the use of computers in civil engineering. Lisbon,

Portugal, 1962, paper No. 42.

8. Sachanski, S., Sotirov, P., Investigation of package lift slabs building in plastic stage by method of extrapolation (Russian). Proc. Third European Symposium on EE, Sofia 1970.

9. Sementsov, S. A., Kameiko, B. A., Handbook for designer of masonry constructions, Moscow, 1968.

10. Earthquake Resistant Regulations - A World list, International Assn. for Earthquake Engineering, Tokyo, 1970.

11. Hudson, D. E., Strong motion instrumental data on the San Fernando earthquake, February 9, 1971, California Institute of Technology.

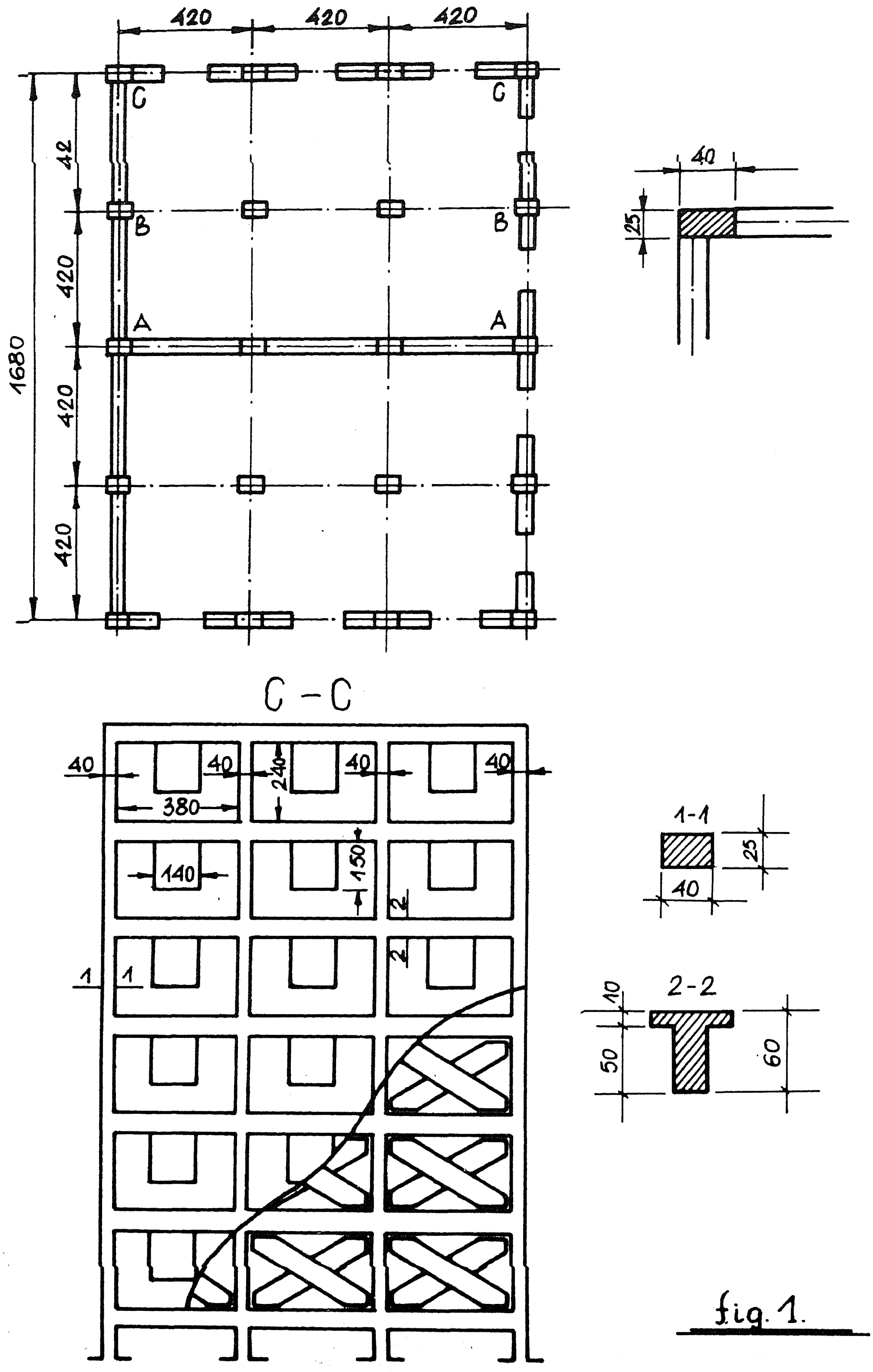
Table 1 - RESPONSE OF THE STRUCTURE FOR SAN FERNANDO 1971 EARTHQUAKE S16°E COMPONENT [11]

a. Response of the whole structure

Floor	Horizontal forces in tons for ... sec			
	2,1890	2,8530	7,7550	7,7550 plastic
6	56	153	453	204
5	33	118	437	192
4	25	74	400	168
3	28	25	334	130
2	26	-26	249	84.5
1	23	-37	145	43.5

b. Response of the diaphragms

Floor	Forces in diaphragms for ... sec								
	2,1890			2,8530			7,7550		
	Frame	Diaphragms		Frame	Diaphragm		Frame	Diaphragm	
	B - B	C - C	A - A	B - B	C - C	A - A	B - B	C-C	A-A
6	3,55	15,41	18	9,4	42	50	31	128	135
5	0,72	8,50	14	3,3	30	51	12	112	188
4	0,64	6,30	12	2,1	18	33	12	100	176
3	0,72	6,70	13	0,25	6	13	9	82	153
2	0,37	6,11	13	-2,2	-7	-5,6	3	58	127
1	2,11	3,95	11	-0,2	-12	-11	17	17	76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420
420
1680
42
420
420
420
C
C
B
B
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A
40
25
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40
40
40
40
380
240
140
150
1
1
2
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1-1
25
40
2-2
10
50
60

fig. 1.

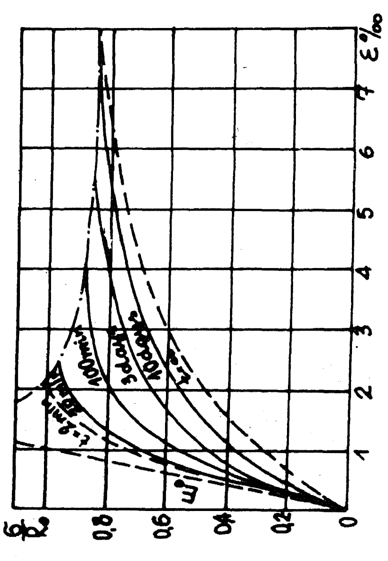

fig 2

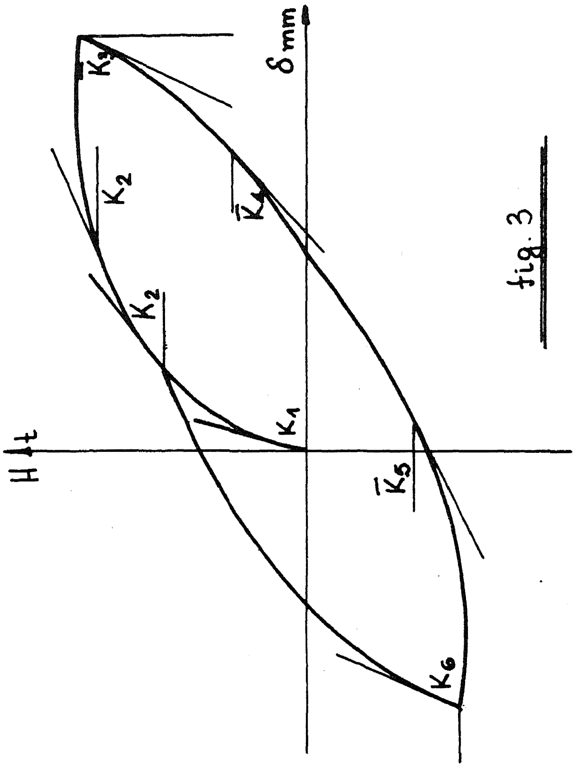

fig 3

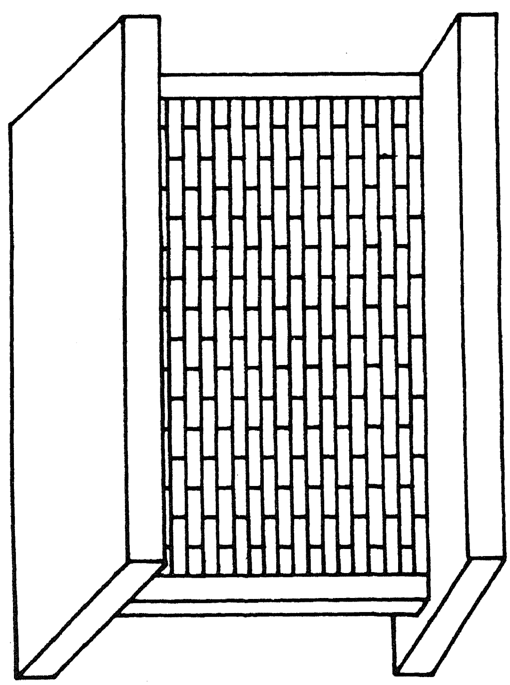

fig 4

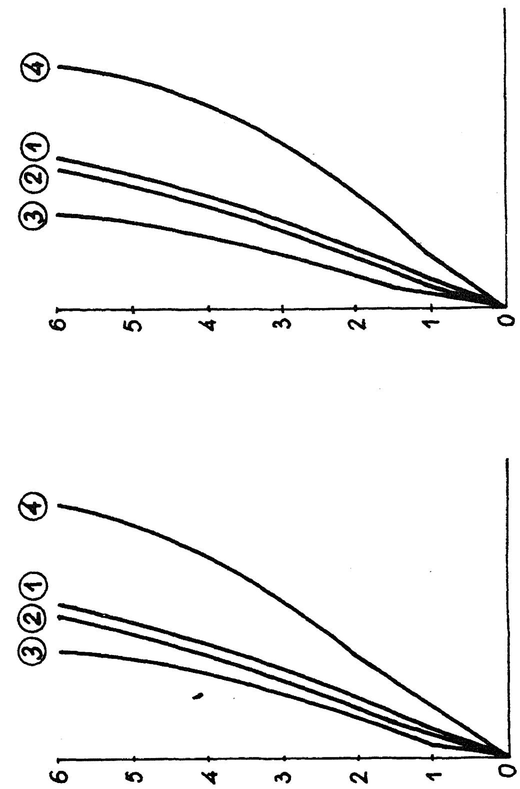

fig 5

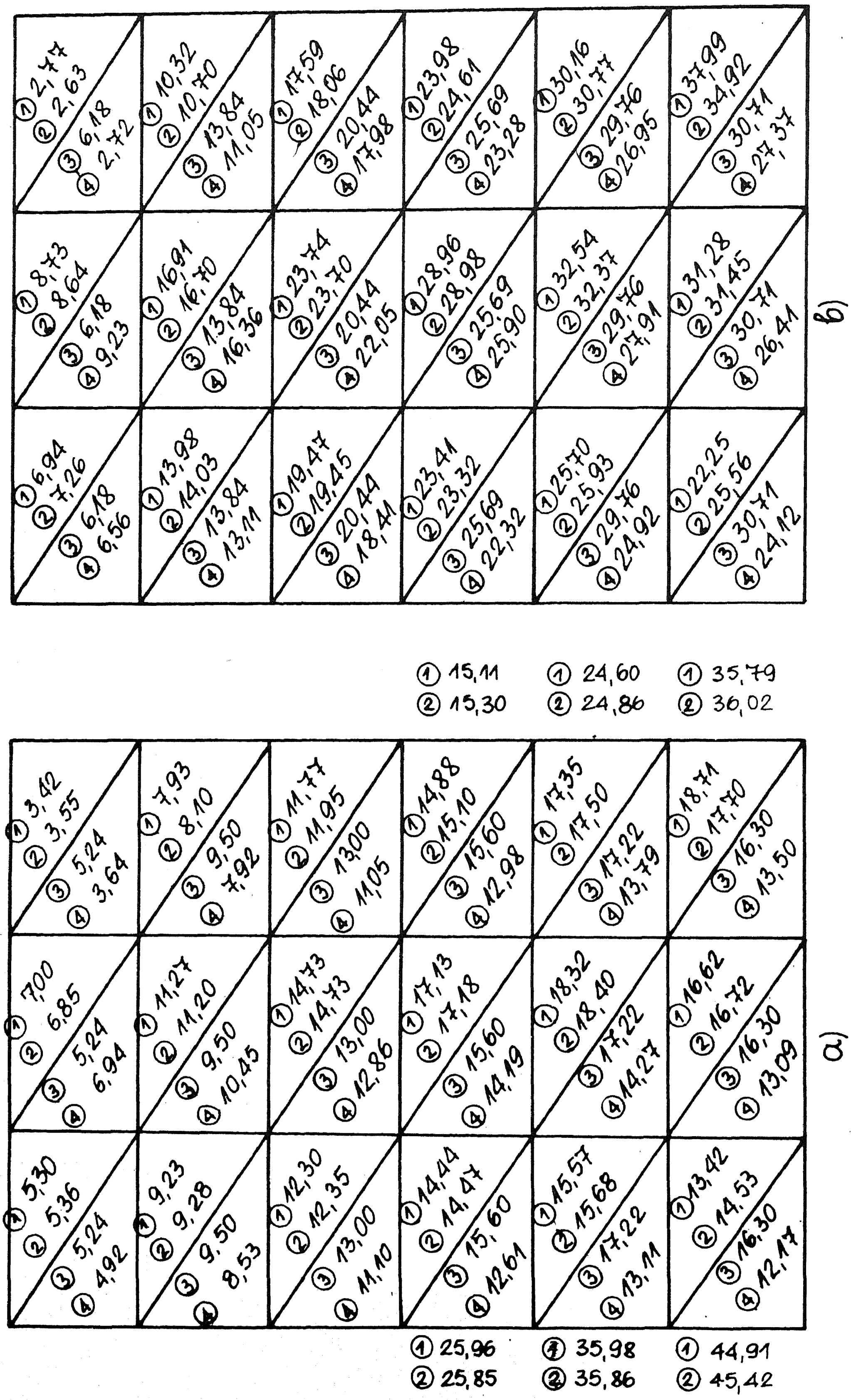
b)
① 2,77 ② 2,63 ③ 6,18 ④ 2,72
① 10,32 ② 10,70 ③ 13,84 ④ 11,05
① 17,59 ② 18,06 ③ 20,44 ④ 17,98
① 23,98 ② 24,61 ③ 25,69 ④ 23,28
① 30,16 ② 30,77 ③ 29,76 ④ 26,95
① 37,99 ② 34,92 ③ 30,71 ④ 27,37
① 8,73 ② 8,64 ③ 6,18 ④ 9,23
① 16,91 ② 16,70 ③ 13,84 ④ 16,36
① 23,74 ② 23,70 ③ 20,44 ④ 22,05
① 28,96 ② 28,98 ③ 25,69 ④ 25,90
① 32,54 ② 32,37 ③ 29,76 ④ 27,91
① 31,28 ② 31,45 ③ 30,71 ④ 26,41
① 6,94 ② 7,26 ③ 6,18 ④ 6,56
① 13,98 ② 14,03 ③ 13,84 ④ 13,11
① 19,47 ② 19,45 ③ 20,44 ④ 18,41
① 23,41 ② 23,32 ③ 25,69 ④ 22,32
① 25,70 ② 25,93 ③ 29,76 ④ 24,92
① 22,25 ② 25,56 ③ 30,71 ④ 24,12
① 15,11 ② 15,30
① 24,60 ② 24,86
① 35,79 ② 36,02
a)
① 3,42 ② 3,55 ③ 5,24 ④ 3,64
① 7,93 ② 8,10 ③ 9,50 ④ 7,92
① 11,77 ② 11,95 ③ 13,00 ④ 11,05
① 14,88 ② 15,10 ③ 15,60 ④ 12,98
① 17,35 ② 17,50 ③ 17,22 ④ 13,79
① 18,71 ② 17,70 ③ 16,30 ④ 13,50
① 7,00 ② 6,85 ③ 5,24 ④ 6,94
① 11,27 ② 11,20 ③ 9,50 ④ 10,45
① 14,73 ② 14,73 ③ 13,00 ④ 12,86
① 17,13 ② 17,18 ③ 15,60 ④ 14,19
① 18,32 ② 18,40 ③ 17,22 ④ 14,27
① 16,62 ② 16,72 ③ 16,30 ④ 13,09
① 5,30 ② 5,36 ③ 5,24 ④ 4,92
① 9,23 ② 9,28 ③ 9,50 ④ 8,53
① 12,30 ② 12,35 ③ 13,00 ④ 11,10
① 14,44 ② 14,47 ③ 15,60 ④ 12,61
① 15,57 ② 15,68 ③ 17,22 ④ 13,11
① 13,42 ② 14,53 ③ 16,30 ④ 12,17
① 25,96 ② 25,85
① 35,98 ② 35,86
① 44,91 ② 45,42

fig.6