EARTHQUAKE RESPONSE OF SINGLE-DEGREE-OF-FREEDOM HYSTERETIC STRUCTURES[I]

by

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Synopsis

Two different linearization techniques were used to find the general properties of earthquake response of nonlinear hysteretic structures. The relation between two techniques was investigated and it was found that they conclude the same expression of the equivalent linear structure. By using these two methods, not only stationary response but also nonstationary response of structures with hysteresis were predicted. Moreover from a numerical simulation performed on a digital computer, the applicability of the equivalent linearization techniques used in this study were verified.

Equivalent Linearization Techniques

One of the linearization techniques used herein is the least mean-square error method first developed by T. K. Caughey(1). This method minimizes the mean-square error term due to linearization of nonlinear hysteretic structures. The other is what we may call an energy balance method first proposed by L. S. Jacobsen(2). This method replaces the hysteretic energy dissipation by the equivalent linear viscous damping. These two methods have been discussed by many research workers separately and their relation with each other has never been investigated.

By using the slowly varying parameter method which assumes that the response is a sinusoidal time function with a slowly varying random amplitude and a random phase angle, it has been found that the final expressions of the equivalent damping factor h_{eq} and of the equivalent natural frequency ω_{eq} obtained from these two methods have the same form. It is considered very interesting from the physical viewpoint that the quite different methods conclude the same result. That is to say the equivalent linear structure determined from the least mean-square error method has the same resonant frequency and moreover dissipates the same amount of energy as that of hysteretic structures. Therefore we can conclude that the least-square error method is physically well-grounded in linearizing hysteretic structures.

Equivalent linearization parameters h_{eq} and ω_{eq} in stationary

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random response were obtained as a function of standard deviation σ_μ of ductility factor μ by using the peak amplitude distribution obtained by S. O. Rice.(3) In the case of nonstationary random response, the peak amplitude which was obtained by T. Kobori and R. Minai(4) as a function of standard deviation of μ and $\dot{\mu}$ and the correlation coefficient $\zeta_{\mu\dot{\mu}}$ between them was used.

Stationary Response of Structures with Bilinear Hysteresis

An iterative method was used to predict the stationary r. m. s. response σ_μ of structures with bilinear hysteresis, of which natural frequency in infinitesimal vibration is ω_o, and of which second spring constant is $(1-n)$, subjected to the Gaussian excitation with the predominant frequency ω_f and the parameter r_s of its intensity. This method determines the optimum parameters $h_{eq}(\sigma_\mu)$, $\omega_{eq}(\sigma_\mu)$ of the equivalent linear structure, and also the r. m. s. response $\sigma_\mu(\omega_{eq}, h_{eq})$ of the structure iteratively.

In order to make theoretical discussion about the numerical results shown in Fig. 1, the variation of h_{eq} and ω_{eq} were also investigated. The equivalent frequency ω_{eq} is always less than ω_o because of the softening type spring characteristics, and the equivalent damping factor h_{eq} is necessarily greater than h_o in consequence of hysteretic energy dissipation. The effect of such properties of ω_{eq} and h_{eq} in the stationary r. m. s. response would be explained from the concept of the transition of the receptance of the structure due to its hysteretic properties schematically illustrated in Fig. 2 in terms of the spectrum coordinate. That is to say, as the structure softens due to yielding and consequently ω_{eq} decreases, the receptance of a relatively "rigid" structure ($\eta = 0.5$; $\omega_o > \omega_f$) moves closer to the peak of the spectrum of the excitation and tends to increase the response. On the contrary, such a shift of the receptance tends to suppress the response of a relatively "soft" structure ($\eta = 2.0$; $\omega_o < \omega_f$). Besides, an increase in h_{eq} limits the response over the whole frequency range, and the compound effect of ω_{eq} and h_{eq} results in Fig. 1.

Nonstationary Response of Structures with Bilinear Hysteresis

The step-by-step linearization technique(4), (5) has been adopted for the divided segments of excitation to predict the time depending variance of response subjected to an earthquake-type nonstationary Gaussian process represented by the product of a nonstationary deterministic function shown in Fig. 3 and a stationary random process. This technique consists of two procedures; one is to estimate nonstationary reponse of linear structure during one segment subjected to stationary random excitation under arbitrary initial conditions, the other is to seek the parameters h_{eq}, ω_{eq} of the equivalent linear structure varying with the response level of the linear structure of the previous segment. In order to check the accuracy of the first procedure, the nonstationary response of linear structure obtained theoretically by H. Kameda(6) and the results of the step-by-step method are shown in Fig. 3. Both results seem to coincide well with each other except the first and second step.

The time depending variance $\sigma_\mu^2(t)$ of ductility factor μ and parameters $heq(t)$, $\omega_{eq}(t)$ of the equivalent linear structure obtained from the step-by-step linearization technique are shown in Fig. 4. It may be observed that the maximum value of the r. m. s. response of structure with bilinear hysteresis is limited and the time lag between the time of the maximum response and that of the maximum excitation is shortened as the nonlinearity of hysteresis becomes strong. It is obvious that these results are mainly attributable to the additional damping due to the hysteresis. As could be expected, the equivalent damping factor h_{eq} grows greater and the equivalent natural frequency ω_{eq} smaller as the nonlinearity of the hysteresis becomes stronger.

Numerical Simulation on a Digital Computer

To check the accuracy of the analytical results obtained in the previous section, a numerical simulation of the nonstationary mean-square response has been carried out on a digital computer. The sample size for each parameter shown in Fig. 5 was taken as 50. It is obvious that the analytical and the simulated results agree rather well for the parameters of $n = 0.50$ and 0.75. On the contrary, the discrepancy between two results was quite clear for the structure with perfect elasto-plastic hysteresis ($n = 1.0$). So it can be said that the equivalent linearization techniques dealt with herein are applicable to the prediction of earthquake response of structures with bilinear hysteresis within the parameters $0.00 \leq n \leq 0.75$ and $0.0 \leq r_s \leq 1.0$.

However, these methods cannot be applied to the structure with perfect elasto-plastic hysteresis due to the growth of the plastic deformation which could be considered as the process of the structural failure in strong earthquakes. From this viewpoint, the authors are planning to investigate the amount of the plastic deformation through both theoretical and experimental methods.

Acknowledgement

The authors wish to express their sincere thanks to Dr. H. Kameda for his profitable discussion and also to Dr. P. C. Jennings for his sending them some references.

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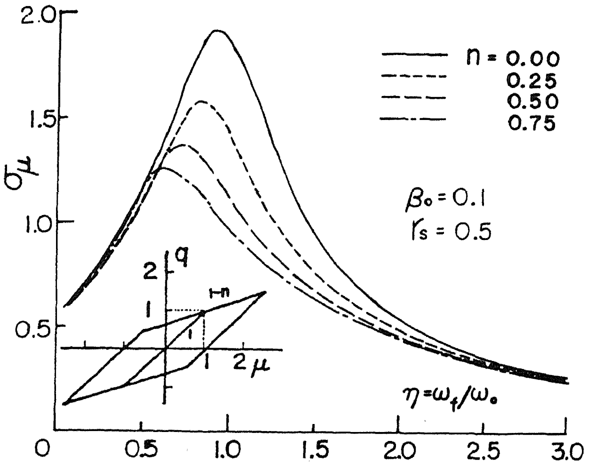


Fig. 1. Stationary r.m.s. Response of Structures with bi-linear Hysteresis.

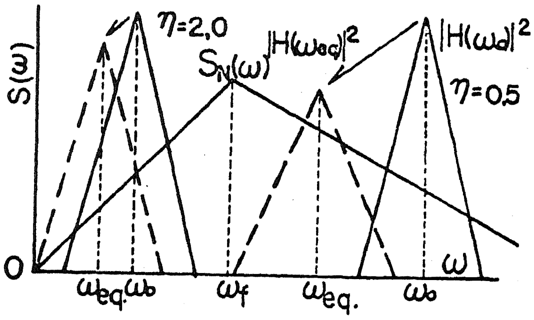
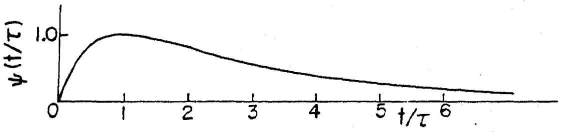


Fig. 2. Transition of the Receptance Due to Nonlinearity and Hysteresis.



Shape Function of the Excitation

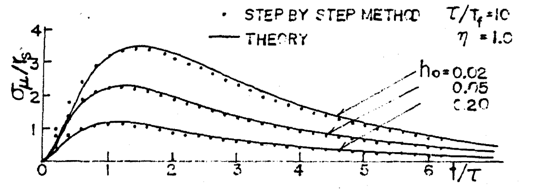
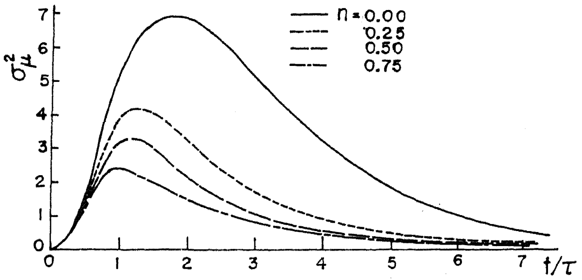
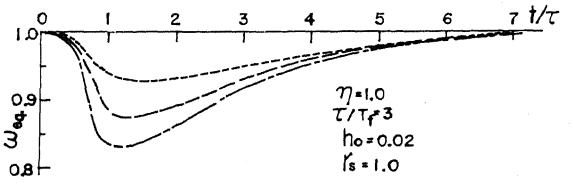


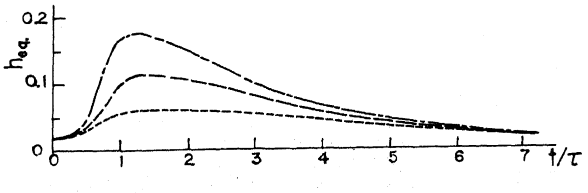
Fig. 3. Shape Function of the Excitation and Comparison between Step-by-Step Method and Theoretical Results.



Variance of Ductility Factor



Equivalent Natural Frequency



Equivalent Damping Factor

Fig. 4. Nonstationary Response of Variance of Ductility Factor and Transition of h_{eq} and ω_{eq}.

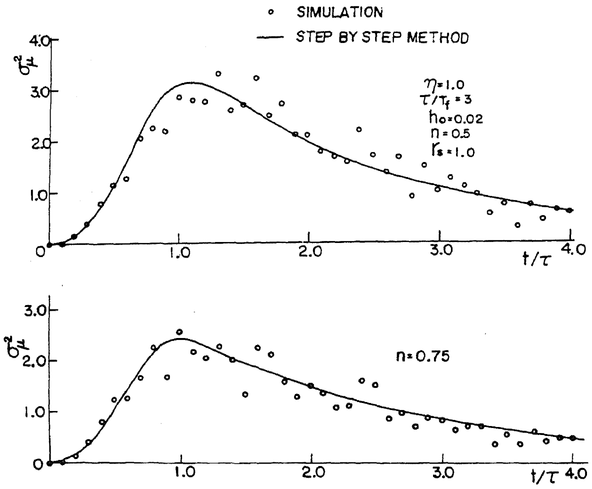


Fig. 5. Comparison between Simulated and Theoretical Results.