DYNAMICAL INTERACTION OF MULTIPLE STRUCTURAL SYSTEMS ON A SOIL MEDIUM

by

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SYNOPSIS

This paper is concerned with the cross-interaction among the multiple structural systems on a visco-elastic soil medium. In order to investigate the effect of such the interaction on the earthquake response of a building structure with radiation damping, the stochastic process of the interaction is theoretically developed by discussing formulation and power flow in the matrix forms, and the typical numerical examples are presented for the case of the simple interaction configuration system.

INTRODUCTION

In recent years there has been an increasing interest in the dynamical interaction between a single building structure and its surrounding soil ground as one of the most important problems in earthquake engineering. The objective of this paper is to investigate on the cross-interaction effect among the multiple structural systems on a visco-elastic soil ground. According to the fact that considerable radiation energy is carried away from the vicinity of the basement mass of a building structure, the interaction between one and another building must occur through the soil ground if a building structure is built so close to another building as in a big city. Under such circumstances the dynamical characteristics of a building structure is unable to be independent of that of the near building structure, and then it may be necessary for evaluation of the earthquake response of a building structure to determine the dynamical characteristics of its building taking account of the interaction of the nearby buildings.

In order to analyze such a cross-interaction problem, each building structure on the soil ground may be simplified by a lumped spring mass structural system resting on a visco-elastic medium. For this case the coupling interaction should be described by the power flow among multiple lumped spring mass systems through the visco-elastic waves propagated at soil ground. The stochastic nonstationary process of those structural systems is theoretically developed by discussing formulation and power flow expressed in the matrix forms of such the interaction configuration system. Without loss of generality, the multi-rigid masses (like the rigid structures) on a visco-elastic medium are assumed as a typical numerical example which may be essential to the above-mentioned more complex structural system. And the interaction among the multi-rigid masses on a visco-elastic stratum along the only one axis is, then, analyzed in detail as one-dimensional cross-interaction problem. Furthermore the two identical square masses or two simple structural systems on a visco-elastic stratum are excited by a single horizontal hormonic force

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applied through one of the masses or basements. From these analyses it may be found that the divergent waves travelling through the soil medium from one rigid mass or basement do exert on the other rigid mass or basement mutually.

DYNAMICAL CHARACTERISTICS

The dynamical characteristics of a soil-structure system consisting of L-number of elastic structural systems which are along the x-axis on the surface of a visco-elastic soil medium on a bed rock are considered here 1),2),3). The degrees of freedom of the i-th structural system is assumed to be N_i+1 including its basement mass. Two kinds of input excitations to the soil-structure system are supposed, namely, the displacement excitation at the soil-rock interface and the force excitation at the basement mass of each structural system. For the sake of simplicity, only the x-direction horizontal components of excitations and responses are considered. All the dynamical characteristics of the soil-structure system are discussed in the form of dimensionless transfer functions. In defining dimensionless quantities, b_0 , ρb_0^3 and $b_0/V_S = b_0\sqrt{\rho/\mu}$ are taken as the reference values of length, mass and time, respectively, where ρ, μ and V_S are density, shear modulus and S-wave velocity of the elastic soil medium. The symbol $\sim$ denotes the Fourier transform with respect to dimensionless frequency $a_0 = \omega b_0/V_S$ in which ω is frequency.

The basic responses of the soil-structure system are the displacement vector $\{u\}^o$ and reaction vector $\{r\}^o$ of the connecting points between the multiple structural systems and the soil medium. The relationship between the Fourier transforms of these basic vectors and those of $\{u\}^S$ which is the displacement vector of the connecting points at the free surface of the soil medium without the structural systems and of the force vector $\{f\}$ on the basement masses are expressed as follows:

$$\{\tilde{u}\}^o = [G]^U_D\{\tilde{u}\}^S + [G]^U_F\{\tilde{f}\}, \qquad \{\tilde{r}\}^o = [G]^R_D\{\tilde{u}\}^S + [G]^R_F\{\tilde{f}\} \tag{1}$$

$$[G]^U_D = ([I] - {}_G[J]\,{}_S[R])^{-1}, \qquad [G]^R_D = {}_G[R]([G]^U_D - [I]) \tag{2}$$

$$[G]^U_F = [G]^U_D\,{}_G[J], \qquad [G]^R_F = {}_G[R][G]^U_F \tag{3}$$

In Eqs.(1) to(3) all the quantities are functions of ja_0 , ($j=\sqrt{-1}$) except for the identity matrix $[I]$. Also, in Eqs.(2) and (3), ${}_G[J]$ means the transfer matrix of the displacements of connecting points to the force excitations applied to the points, namely, the ground compliance matrix 4),5),6), ${}_G[R]$ is the inverse matrix of ${}_G[J]$, namely, the ground stiffness matrix, and ${}_S[R]$ is the transfer matrix, which is diagonal in this case, of the reactions from the structural systems to displacement excitations at their connecting points. If the basic vectors $\{\tilde{u}\}^o$ and $\{\tilde{r}\}^o$ are known, all responses of the soil-structure system can be determined. For instance, the displacement vector ${}_S\{u\}$ of the structural system is given by

$${}_S\{\tilde{u}\} = {}_S[G]^U_D\{\tilde{u}\}^o \tag{4}$$

where ${}_S[G]^U_D$ consists of sub-vector $\{G\}_i$ which is the transfer vector of the displacements of the i-th structural system to the displacement excitation at its connecting point.

In particular, in the case where the soil medium is of the Voigt type visco-elastic stratum welded on the rigid bed rock, the ground compliance ${}_G[J]$ is given by

$$ {}_G[J] \equiv [J_\ell^k] = [J(a_0, x_k - x_\ell, 0, 0)] \tag{5} $$

in referring to Appendix. Also, the displacement vector $\{u\}^s$ at the free surface is expressed as

$$ \{\tilde{u}\}^s = \tilde{u}^s\{1\}, \qquad \tilde{u}^s = G_B^s\, \tilde{u}^B \tag{6} $$

in which $\tilde{u}^B$ is the displacement excitation at the soil-rock interface and the transfer function G_B^s is given by

$$ G_B^s = \frac{2\exp\{-ja_0h\sqrt{1/(1+j\eta_s a_0)}\}}{1+\exp\{-2ja_0h\sqrt{1/(1+j\eta_s a_0)}\}} \tag{7} $$

where

$$ h = H/b_0, \quad \eta_s = (V_s/b_0)(\mu'/\mu) = \eta_p = (V_s/b_0)\{(\lambda'+2\mu')/(\lambda+2\mu)\} \tag{8} $$

In the above equation, H is the thickness of stratum, λ' and μ' are viscous constants of the Voigt solid. In view of Eq.(6), the transfer matrices $[G]_D^U$ and $[G]_D^R$ can be replaced by the transfer vectors $\{G\}_D^U$ and $\{G\}_D^R$ which are

$$ \{G\}_D^U \equiv \{{}_iG_s^o\} = \{\tilde{u}_i^o\}/\tilde{u}^s = [G]_D^U\{1\}, \qquad \{G\}_D^R = \{\tilde{r}_i^o\}/\tilde{u}^s = [G]_D^R\{1\} \tag{9} $$

When each structure is of shear type as shown in Fig.1, the sub-vector $\{G\}_i$ of ${}_s[G]_D^U$ in Eq.(4) is expressed as

$$ \{G\}_i \equiv \{{}_iG_o^j\} = \{\tilde{u}_i^j\}/\tilde{u}_i^o = ([\hat{k}]_i - a_0^2[m]_i)^{-1}\{\hat{k}_i^1 \; 0 \cdots 0\}^T $$
$$ \hat{k}_i^j = k_i^j + ja_0c_i^j, \quad i = 1, 2, \cdots, L, \quad j = 1, 2, \cdots, N_i \tag{10} $$

in which $[m]_i$ and $[\hat{k}]_i$ are the mass and complex stiffness matrices of the i-th structural system and their elements are given by dimensionless masses $m_i^j = M_i^j/\rho b_0^3$, rigidities $k_i^j = K_i^j/b_0\mu$ and viscous damping coefficients $c_i^j = C_i^j/b_0^2\sqrt{\rho\mu}$, and superscript T denots the transposed matrix.
The diagonal matrix ${}_s[R]$ appearing in Eq.(2) is characterized by the i-th diagonal element,

$$ a_0^2 m_i^o + \hat{k}_i^1({}_iG_o^1 - 1) \tag{11} $$

where m_i^o is the dimensionless basement mass of the i-th structural system and ${}_iG_o^1$, $\hat{k}_i^1$ are given in Eq.(10).
Then the transfer vectors $\{G\}_D^U$ and $\{G\}_D^R$ in Eq.(9) as well as the transfer matrices $[G]_F^U \equiv [{}_iG_\ell^o]$ and $[G]_F^R$ in Eq.(3) are determined by using Eqs.(5) and (11). Hence the overall transfer vector of displacement $\{u\}_i$ of the i-th structural system ${}_i\{G\}_{\ell F}^U$ to the force f_ℓ at the ℓ-th basement mass, ${}_i\{G\}_D^U$ to the displacement u^s at the free surface of soil medium and ${}_i\{G\}_B^U$ to the displacement u^B at the soil-rock interface are respectively written as follows:

$$ {}_i\{G\}_{\ell F}^U \equiv \{{}_iG_\ell^j\} = {}_iG_\ell^o\{{}_iG_o^j\}, \qquad {}_i\{G\}_D^U \equiv \{{}_iG_s^j\} = {}_iG_s^o\{{}_iG_o^j\} $$

and

$$ {}_i\{G\}_B^U \equiv \{{}_iG_B^j\} = G_B^s\{{}_iG_s^j\} \tag{12} $$

STATISTICS OF POWER FLOW

Statistical properties of the random power flow to the i-th structural system from the others of the multiple structural systems through the soil medium are discussed here 7). For this purpose, the reaction vector $\{r\}'$ defined by

$$ \{\tilde{r}\}' = {}_G[R]'(\{\tilde{u}\}^o - \{\tilde{u}\}^s) \tag{13} $$

is considered. The matrix ${}_G[R]'$ is composed of only the off-diagonal elements of the ground stiffness matrix ${}_G[R]$. The transfer matrices $[G]_D^{R'}$ and $[G]_F^{R'}$ associated with $\{\tilde{r}\}'$ are obtained by replacing ${}_G[R]$ by ${}_G[R]'$ in Eqs.(2) and (3).

The power flow Π_i to the i-th structural system through the soil medium is defined as

$$\Pi_i = -\dot{u}_i^o \, r_i' \tag{14}$$

in which • denotes the differentiation with respect to dimensionless time $\tau = t V_s / b_o$ where t is time.
Therefore, the following output vector $\{v\}$ is to be considered:

$$\{v\} = \{ \{u\}^{oT} \{r\}'^{T} \}^T \tag{15}$$

In what follows, only displacement random excitation $\{u\}^s$ at the surface is taken into consideration, and it is assumed that the excitation belongs to the zero-mean, non-stationary, Gaussian process. Hence the basic statistical quantities of the output vector $\{v\}$ in Eq. (15) are the co-variance matrix $[K_v(\tau_1, \tau_2)]$ and the corresponding generalized spectral density matrix $[S_v(a_{01}, a_{02})]$ which are respectively defined as

$$[K_v(\tau_1,\tau_2)] \equiv E(\{v(\tau_1)\}\{v(\tau_2)\}^T) = [K_{pq}(\tau_1,\tau_2)] \tag{16}$$

$$[S_v(a_{01},a_{02})] = E(\{\tilde{v}(ja_{01})\}\{\tilde{v}(ja_{02})\}^*) = [S_{pq}(a_{01},a_{02})] \qquad p, q = U \text{ and } R' \tag{17}$$

in which superscript $*$ denotes the complex conjugate matrix. The sub-matrices in Eqs. (16) and (17) are related by the double conjugate Fourier transform, namely,

$$[K_{pq}(\tau_1,\tau_2)] = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} [S_{pq}(a_{01},a_{02})] \exp(j(a_{01}\tau_1 - a_{02}\tau_2)) \, da_{01} \, da_{02} \tag{18}$$

From Eqs. (1), (2) and (13), $[S_{pq}(a_{01},a_{02})]$ in Eq. (17) is expressed in terms of the generalized spectral density matrix $[S_{DD}(a_{01},a_{02})]$ of the free surface displacement vector $\{u\}^s$ and the transfer matrices $[G]_D^U$ and $[G]_D^{R'}$ as follows:

$$[S_{pq}(a_{01},a_{02})] = [G(ja_{01})]_D^p \, [S_{DD}(a_{01},a_{02})][G(ja_{02})]_D^{q*} \tag{19}$$

In the case of the real-valued, zero-mean, Gaussian process, the mean ${}_\pi M_i(\tau)$ of the power flow $\Pi_i(\tau)$ and the co-variances ${}_\pi K_\ell^k(\tau_1,\tau_2)$ between $\Pi_k(\tau_1)$ and $\Pi_\ell(\tau_2)$ are respectively expressed as follows:

$${}_\pi M_i(\tau) = E(\Pi_i(\tau)) = -K_{\dot{U}R'}{}_i^i(\tau,\tau) \tag{20}$$

$$\begin{aligned} {}_\pi K_\ell^k(\tau_1,\tau_2) &= E(\Pi_k(\tau_1)\,\Pi_\ell(\tau_2)) \\ &= K_{\dot{U}\dot{U}}{}_\ell^k(\tau_1,\tau_2) K_{R'R'}{}_\ell^k(\tau_1,\tau_2) + K_{\dot{U}R'}{}_\ell^k(\tau_1,\tau_2) \, K_{R'\dot{U}}{}_\ell^k(\tau_1,\tau_2) \end{aligned} \tag{21}$$

By making use of Eq. (18), ${}_\pi M_i(\tau)$ and ${}_\pi K_\ell^k(\tau_1,\tau_2)$ are expressed in terms of the elements of $[S_{pq}(a_{01},a_{02})]$ in Eq. (17) as follows:

$${}_\pi M_i(\tau) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} a_{01} \, Im\,(S_{UR'}{}_i^i(a_{01},a_{02}) \exp(j(a_{01}-a_{02})\tau)) \, da_{01} \, da_{02} \tag{22}$$

$$\begin{aligned} {}_\pi K_\ell^k(\tau_1,\tau_2) = \frac{1}{(2\pi)^4} \Big(& \iint_{-\infty}^{\infty} a_{01} a_{02} \, Re\,(S_{UU}{}_\ell^k(a_{01},a_{02}) \exp(j(a_{01}\tau_1 - a_{02}\tau_2))) \, da_{01} \, da_{02} \\ & \cdot \iint_{-\infty}^{\infty} Re\,(S_{R'R'}{}_\ell^k(a_{01},a_{02}) \exp(j(a_{01}\tau_1 - a_{02}\tau_2))) \, da_{01} \, da_{02} \\ & + \iint_{-\infty}^{\infty} a_{01} \, Im\,(S_{UR'}{}_\ell^k(a_{01},a_{02}) \exp(j(a_{01}\tau_1 - a_{02}\tau_2))) \, da_{01} \, da_{02} \\ & \cdot \iint_{-\infty}^{\infty} a_{01} \, Im\,(S_{UR'}{}_k^\ell(a_{01},a_{02}) \exp(j(a_{01}\tau_2 - a_{02}\tau_1))) \, da_{01} \, da_{02} \Big) \end{aligned} \tag{23}$$

where Re and Im represent the real and immaginary parts of a complex number, respectively.

NUMERICAL EXAMPLES

In order to obtain the basic dynamical characteristics of the interaction configuration system, the transfer functions Eq. (12) developed in the foregoing mathematical analysis are computed for the case of two or several simplified structural systems on a visco-elastic stratum over a bed rock half space. In the numerical analyses, the horizontal components of the displacement amplitude responses are only evaluated herein, because the horizontal component of the earthquake response of the structural system has been regarded as one of the most important measures for the aseismic design. And a visco-elastic soil medium is idealized as a Voigt solid with Poisson's ratio $\nu=1/4$, the viscosity coefficients $\eta_p=\eta_s=0.1$ and the dimensionless thickness h=2 or 4. And also the every dimensionless distance x among the multiple simplified structures on a soil stratum is equal to 4.

First of all, we are going to take up the simplest cross-interaction problem that two identical square masses on a visco-elastic stratum shown in Fig. 2 are excited by a single horizontal harmonic force applied through the center of gravity of one of the masses. The transfer functions $|{}_1A_1|$ and $|{}_2A_1|$, i.e., the relation of the displacement amplitude of either mass (active or passive mass) to the applied harmonic force, are shown in the solid curves of Figs. 3(a)-3(d) using the abscissa $a_0=\omega b_0/V_s$, and also the broken curves in these figures indicate the transfer functions in the 'non-cross-interaction' case that means the case of a single square active mass on a visco-elastic stratum. The interaction effect of the case h=2 is a little more sensitive than that of the case h=4.

Secondary, we consider the advanced case that seven identical square masses resting on a visco-elastic stratum in a x direction (like the multi-rigid masses in Fig. 4) are excited by the horizontal harmonic motion arriving at the free surface of a soil stratum. For this case the transfer functions $|{}_iG_S|$ are shown in Figs. 5(a)-5(c) for h=2 and Figs. 6(a)-6(c) for h=4. The transfer functions for the 'non-cross-interaction' case are given by the broken curves in these figures. It is pointed out that the difference among the amplitude characteristics of each ith mass remarkably increases as the value of each mass ratio becomes large, and noted that the each solid curve (e.g. i=4) in Figs. 5 and 6 is considerably different from the corresponding broken curve. The transfer functions $|{}_4G_B|$ shown in Figs. 7(a) and 7(b) are the displacement amplitude characteristics of the 4th mass that is the central mass of seven square masses on a visco-elastic soil stratum excited by the horizontal harmonic motion arriving at the soil-rock interface, especially the broken curves $(m_{1\text{-}7}=0)$ in the same figures correspond to the free surface characteristics of a visco-elastic stratum where no rigid mass exists. The solid curves of the transfer functions $|{}_iG_B|$ shown in Figs. 7(c) and 7(d) using the ordinary ordinate scale are the same curves as in Figs. 7(a) and 7(b) except dotted curves concerned with the displacement amplitude characteristics of 1st or 7th mass. The displacement amplitude responses for the shallow stratum case are quite different from those for the deep stratum case in relation to the fundamental natural frequency at each peak of the broken curves, and the amplitude characteristics of the central mass also differ from those of the end-located mass.

Finally, we pay attention to an essential interaction example in which two identical simple lumped mass systems on a visco-elastic stratum shown in Fig. 8 are excited by a single horizontal harmonic force applied to one of the basements of this systems with the distinct frequency characteristics. For the convenience of the numerical analyses, we define the lumped mass ratio m_i^1, the basement mass ratio m_i^0 and the natural frequency ratio $\lambda_i = {}_i a_{os}/a_{og}$, where the natural frequency of a lumped mass system is ${}_i a_{os} = (k_i^1/m_i^1)^{1/2}$ and the fundamental natural frequency of a stratum denotes $a_{og} = \pi/2h$. And we choose the three types of lumped mass systems with viscous damping ratio $\zeta_i = 0.01$, found in the table of S_i in Fig. 8. The transfer functions $|{}_1A_1^j|$ and $|{}_2A_1^j|$ are presented in Fig. 9 for h=2 and Fig. 10 for h=4 as the displacement amplitude characteristics of the lumped masses (solid curves) or the basement masses (broken curves) of the simple structural systems. Fig. 9(a) or Fig. 10(a) shows the interaction effect of the active structure on the passive structure, and Fig. 9(b) or Fig. 10(b) represents the interaction effect of the passive structure on the active structure. The active structure means the structural system excited by a harmonic force applied to its own basement and the passive structure indicates another structure except the active one. Fig. 9(c) and Fig. 10(c) show the displacement amplitude characteristics of the only active structure that is in the 'non-cross-interaction' case. According to the above results, the displacement amplitude responses of the active structure are somewhat affected at the neighbourhood of the natural frequency of the passive structure as the frequency characteristics of the passive structure vary. On the contrary, the displacement amplitude responses of the passive structure are considerably influenced by the frequency characteristics of the active structure.

In addition we observe the transfer functions $|{}_1G_s^j|$ shown in Fig. 11 for the case where two simple lumped mass systems on a soil stratum as in Fig. 8 are excited by the horizontal harmonic motion arriving at the free surface of a stratum. It may be noted from Fig. 11 that if the transfer characteristics of two simple lumped mass systems are similar, the interaction of the displacement amplitude characteristics between two systems is rather demonstrative over the wide broad frequency range involving the resonant natural frequencies of two systems, otherwise the interaction phenomenon seems to appear only in the neighbourhood of the natural frequencies in these configuration systems.

CONCLUSION

In the foregoing sections we have discussed on the structure-soil-structure interaction, which is called 'cross-interaction' among the multiple structural systems resting on a visco-elastic stratum over a bed rock. Throughout this paper, the cross-interaction would be equally emphasized as the soil-structure interaction is important in earthquake engineering. The knowledge of whether the dynamical characteristic of the individual structure in multiple structural systems should be of the similar or different type is believed to be very important to determine the aseismic design philosophy of a building structure built so close to another building as in a big population city of Japan.

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REFERENCES

1) Mac Calden, P.B., "Transmission of Steady-State Vibrations between Rigid Circular Foundations," Ph.D. Thesis, Univ. of Calif., Los Angeles, 1969.
2) Warburton, G.B. et al., "Forced Vibrations of Two Masses on an Elastic Half-Space," Journal of Appl. Mech., Vol.37, No.1, 1971.
3) Kobori, T., "Random Vibrations of Structure-Foundation Interaction System," Proc. of the 3rd U.S.-Japan Joint Seminar on Stochastic Methods in Dynamical Problems, 1971.
4) Thomson, W.T. and T. Kobori, "Dynamical Compliance of Rectangular Foundations on an Elastic Half-Space," Journal of Appl. Mech., Vol.30, 1963.
5) Kobori, T., R. Minai, T. Suzuki and K. Kusakabe, "Vibrational Characteristics of Semi-Infinite Viscoelastic Medium to Surface Excitations on a Rectangular Area," Annuals, Disas. Prev. Res. Inst., Kyoto Univ., No.12A, 1969, (in Japanese).
6) Kobori, T., R. Minai and T. Suzuki, "The Dynamical Ground Compliance of a Rectangular Foundation on a Viscoelastic Stratum," Bull. of the Disas. Prev. Res. Inst., Kyoto Univ., Vol.20, 1971.
7) Lyon, R.H. an G. Maidanik, "Power Flow between Linearly Coupled Oscillators," Journal of the Acous. Soci. of America, Vol.34, 1962.

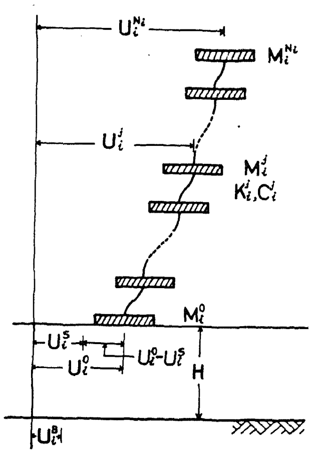
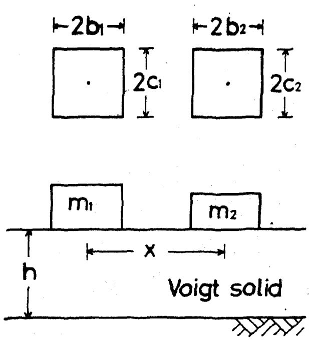
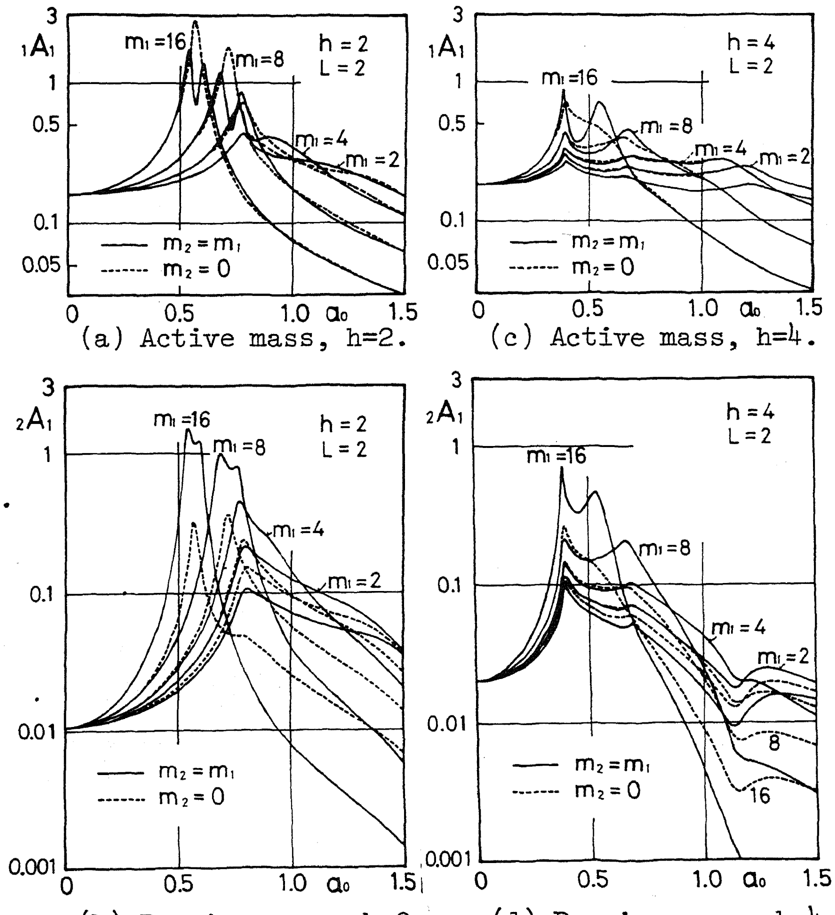


Fig. 1 Soil-structure system considered.



$\nu = 1/4$ $\eta_p = \eta_s = 0.1$ $b_1 = b_2 = b_0$ $x = 4$

Fig. 2 Two-rigid-mass system.



(a) Active mass, h=2. (c) Active mass, h=4.

(b) Passive mass, h=2. (d) Passive mass, h=4.

Fig. 3 Displacement amplitude characteristics of two-rigid-mass system to force excitation.

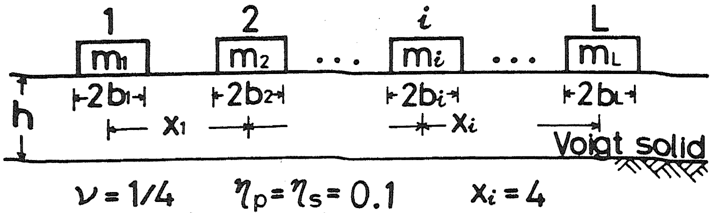
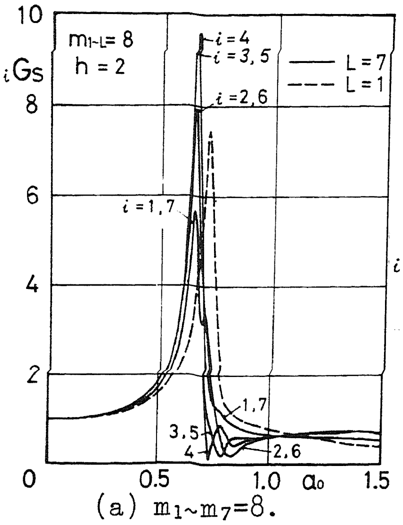
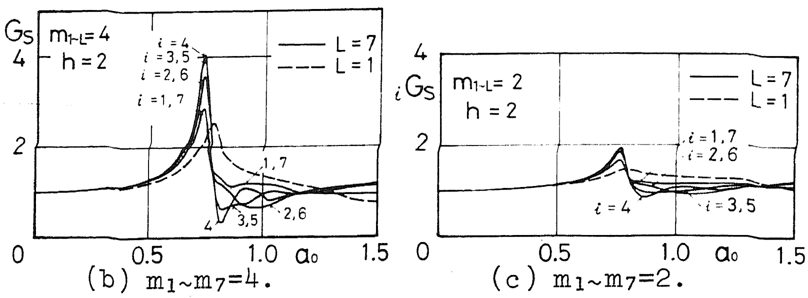


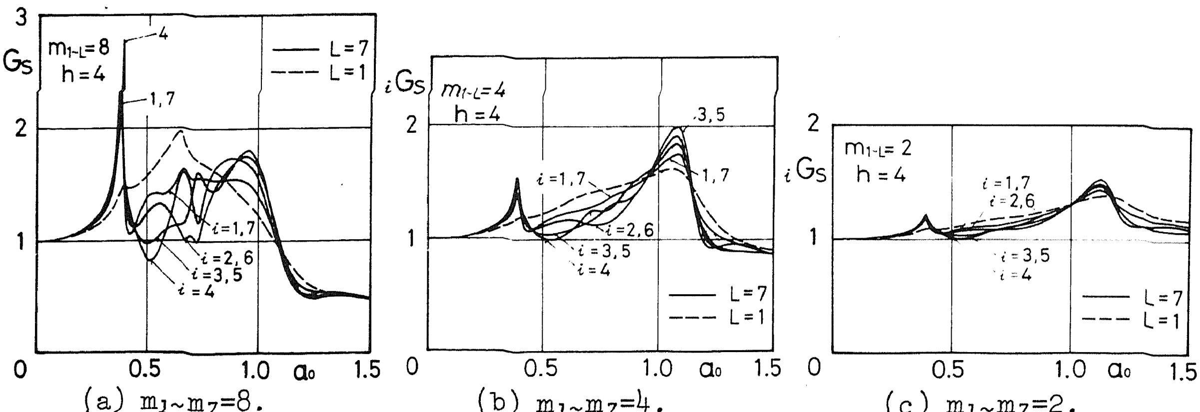
Fig. 4 L-rigid-mass system.





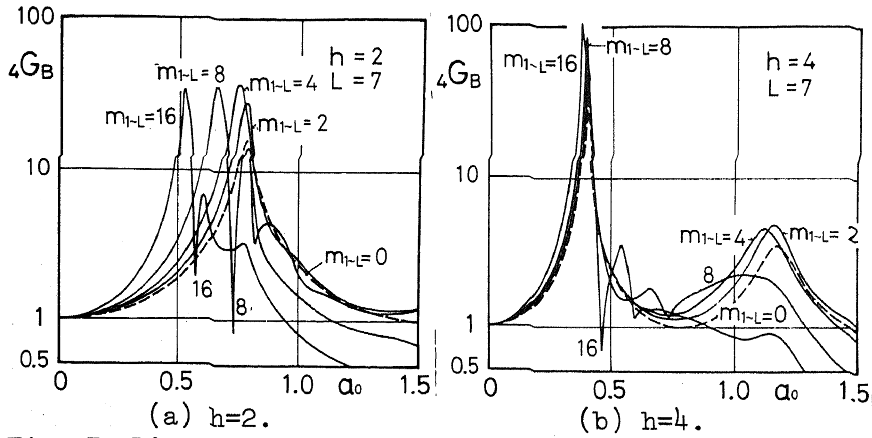
(a) $m_1 \sim m_7 = 8$. (b) $m_1 \sim m_7 = 4$. (c) $m_1 \sim m_7 = 2$.

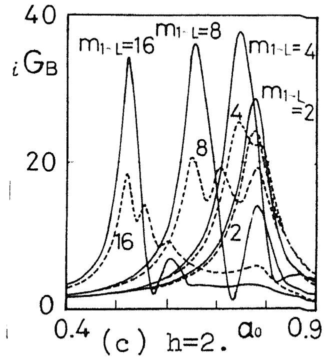
Fig. 5 Displacement amplitude characteristics of seven-rigid-mass system to surface displacement excitation, h=2.



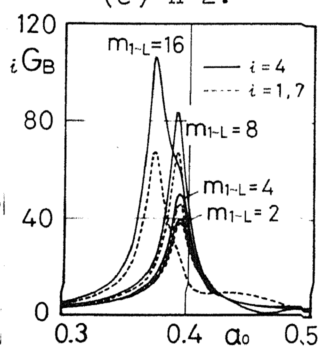
(a) $m_1 \sim m_7 = 8$. (b) $m_1 \sim m_7 = 4$. (c) $m_1 \sim m_7 = 2$.

Fig. 6 Displacement amplitude characteristics of seven-rigid-mass system to surface displacement excitation, h=4.





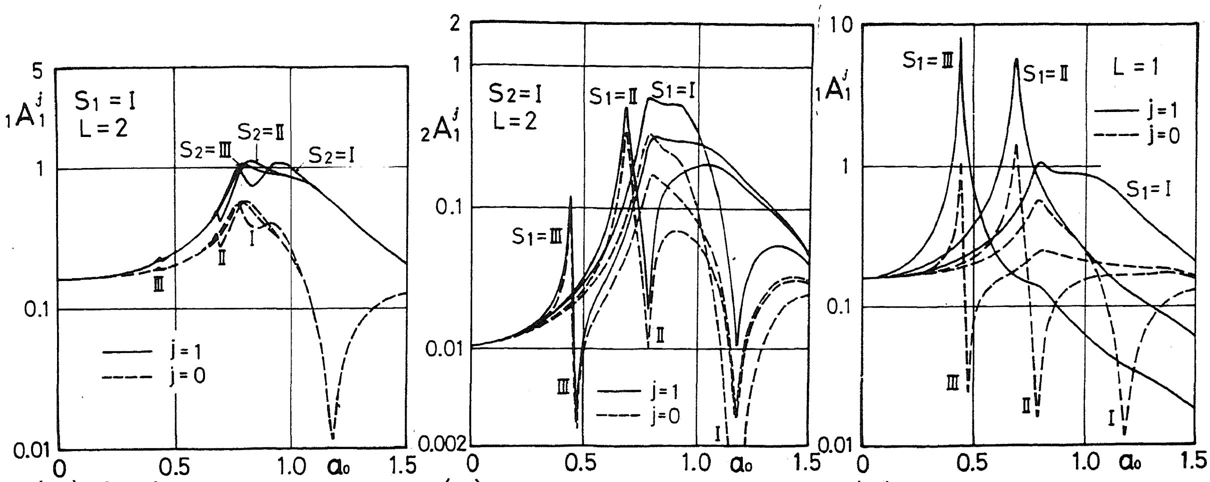
(c) h=2.



(d) h=4.

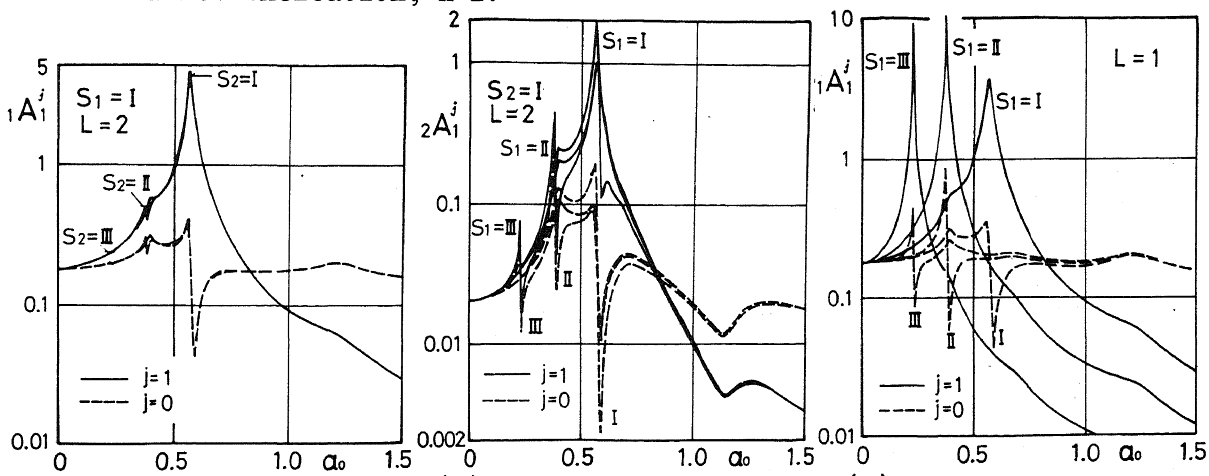
(a) h=2. (b) h=4.

Fig. 7 Displacement amplitude characteristics of *seven-rigid-mass system to bed rock displacement excitation.*



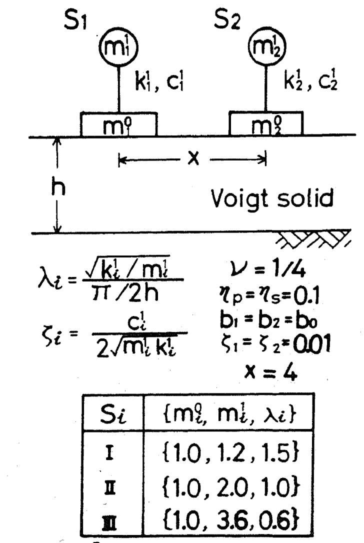
(a) Active struc., L=2. (b) Passive struc., L=2. (c) Active struc., L=1.

Fig. 9 Displacement amplitude characteristics of two structural systems to force excitation, h=2.



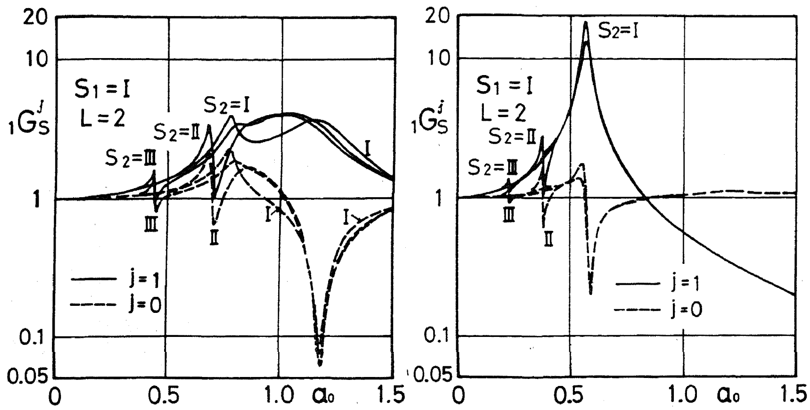
(a) Active struc., L=2. (b) Passive struc., L=2. (c) Active struc., L=1.

Fig. 10 Displacement amplitude characteristics of two structural systems to force excitation, h=4.



S_i	$\{m_i^0, m_i^1, \lambda_i\}$
I	{1.0, 1.2, 1.5}
II	{1.0, 2.0, 1.0}
III	{1.0, 3.6, 0.6}

Fig. 8 Two-structure system.



(a) h=2. (b) h=4.

Fig. 11 Displacement amplitude characteristics of two structural systems to surface displacement excitation.

APPENDIX GROUND COMPLIANCE OF A RECTANGULAR FOUNDATION

The ground compliance of a rectangular foundation is defined as the dimensionless force-displacement transfer function which means the complex amplitude ratio of the average Fourier transformed displacement weighted by the stress distribution in a rectangular area on the surface of the ground to the total transformed force applied on another rectangular area on the same surface. In the case of uniform stress distributions and the Voigt type visco-elastic stratum welded on the rigid bed rock, the ground compliance associated with the horizontal displacement at X_2 to the force in the same direction at X_1, where X_i's (i=1,2) are the co-ordinates of each center of the two rectangular areas, is expressed as follows:

$$J(a_0, X_2-X_1)=\frac{\tilde{u}_{ave}(a_0, X_2-X_1)}{P\,\tilde{q}(a_0)}\,b_0\mu \equiv Re\,J_1^2+j\,Im\,J_1^2$$
$$=\frac{a_0}{\pi^2(1+j\eta_s a_0)^2}\int_0^{\pi/2}d\theta\int_0^{\infty}\Big[\frac{\xi\alpha_s}{F(\xi)}\{\alpha_p\alpha_s\coth a_0h\alpha_p-\xi^2\coth a_0h\alpha_s\}\cos^2\theta$$
$$+\frac{(1+j\eta_s a_0)\xi}{\alpha_s}\cdot\frac{\sinh a_0h\alpha_s}{\cosh a_0h\alpha_s}\sin^2\theta\Big]\cdot S(a_0\xi,\theta,X_2-X_1)\,d\xi \qquad \text{(a-1)}$$

where

$$F(\xi)=4\alpha_p\alpha_s\xi^2\{2\xi^2-1/(1+j\eta_s a_0)\}\,\mathrm{cosech}\,a_0h\alpha_p\,\mathrm{cosech}\,a_0h\alpha_s-\alpha_p\alpha_s\,[\{2\xi^2-1/(1+j\eta_s a_0)\}^2+4\xi^4]\coth a_0h\alpha_p\coth a_0h\alpha_s+\xi^2[\{2\xi^2-1/(1+j\eta_s a_0)\}^2+4\alpha_p^2\alpha_s^2]$$

$$S(a_0\xi,\theta,X_2-X_1)=\frac{\sin(\frac{b_1}{b_0}a_0\xi\cos\theta)}{\frac{b_1}{b_0}a_0\xi\cos\theta}\cdot\frac{\sin(\frac{c_1}{b_0}a_0\xi\sin\theta)}{\frac{c_1}{b_0}a_0\xi\sin\theta}\cdot\frac{\sin(\frac{b_2}{b_0}a_0\xi\cos\theta)}{\frac{b_2}{b_0}a_0\xi\cos\theta}$$
$$\cdot\frac{\sin(\frac{c_2}{b_0}a_0\xi\sin\theta)}{\frac{c_2}{b_0}a_0\xi\sin\theta}\cdot\cos\Big(\frac{X_2-X_1}{b_0}a_0\xi\cos\theta\Big)\cdot\cos\Big(\frac{Y_2-Y_1}{b_0}a_0\xi\sin\theta\Big) \qquad \text{(a-2)}$$

and

$$\alpha_p=\sqrt{\xi^2-\frac{n^2}{1+j\eta_p a_0}},\quad \alpha_s=\sqrt{\xi^2-\frac{1}{1+j\eta_s a_0}},\quad n^2=\Big(\frac{V_s}{V_p}\Big)^2=\frac{1-2\nu}{2(1-\nu)}$$

In the above equations, b_i and c_i (i=1,2) are the half-length of a side of the i-th rectangular foundation parallel to the horizontal direction considered and that of the other side, respectively, and ν means Poisson's ratio of the stratum. In Eq. (a-1), $\tilde{u}_{ave}(a_0, X_2-X_1)$ is given by

$$\tilde{u}_{ave}(a_0,X_2-X_1)=\iint_{B^2}q_2(X)\,\tilde{u}(a_0,X-X_1)\,dx\,dy\Big/\iint_{B^2}q_2(X)\,dx\,dy \qquad \text{(a-3)}$$

in which B^2 means the rectangular area at X_2 and $q_2(X)$ is the stress distribution in B^2.

Figs. A-1 and A-2 show respectively the ground compliance of the square area at X_2=(0,0,0) and X_2=(4,0,0,) where the reference value $b_0=b_1=b_2$ and $q_2(X)$ is uniform stress distribution.

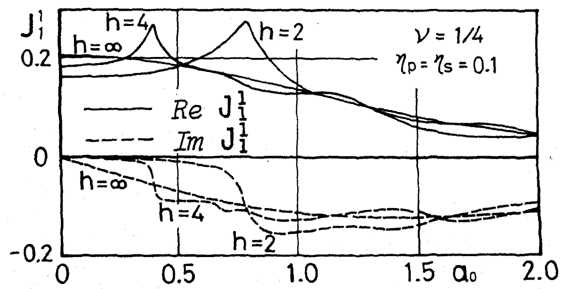


Fig. A-1 Ground compliance, $X_1=X_2$=(0,0,0).

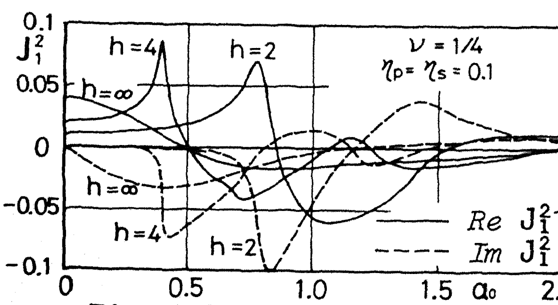


Fig. A-2 Ground compliance, X_1=(0,0,0), X_2=(4,0,0).

ERRATA of S.6A-256.

"DYNAMICAL INTERACTION OF MULTIPLE STRUCTURAL SYSTEMS ON A SOIL MEDIUM"

by T. Kobori and R. Minai, Japan

Page	Line	Reads	Should Read
2061	17	one and another building	one and another buildings
	29	propagated at	propagated into
	40	hormonic	harmonic
2062	9	is assumed	are assumed
	20	with respect to	into the range of
	24	the relationship	the relationships
2063	21	denots	denotes
2064	14	$E(\{v(\tau_1)\}\{v(\tau_2)\}^T)$	$E(\{v(\tau_1)\}\{v(\tau_2)\}^*)$
2065	11	among the multiple simplified	between the adjacent
	30	each ith mass	each mass
	51	this systems	these systems
2066	5	denotes	is denoted by
	11	effect of the active structure on the passive structure	effect of the passive structure on the active structure
	12	effect of the passive structure on the active structure	effect of the active structure on the passive structure
	46	thank to	thank
2070	29	the ground compliance of the square area	the ground compliances of the square area