DETERMINATION OF FOUNDATION FLEXIBILITIES OF STRUCTURES

by
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SYNOPSIS

A relatively simple and direct method is presented for determining quantitatively the foundation flexibilities under structures. Data required are the modal properties and masses. The procedure is illustrated by finding the rotational and horizontal foundation stiffnesses for a 15-storey concrete building on clay, a 9-storey concrete building on dense sand, and a bridge pier founded on clay. On the basis of the fixed-base frequencies obtained, an assessment is made of the importance of the foundation flexibilities on the modal characteristics of these structures.

INTRODUCTION

A knowledge of the foundation flexibility under structures is important for the followings reasons: 1. Flexibility in a foundation changes the dynamic characteristics of the structure, and consequently the response to dynamic disturbances such as wind, traffic and earthquakes. 2. Flexible foundations must be considered in the interpretation of experimental measurements of structural properties. 3. Quantitative values of foundation stiffnesses are required in dynamic structural design and analysis.

Although many theoretical studies on the effects of flexible foundations under dynamic loads have been reported, there is a scarcity of quantitative values of foundation stiffnesses associated with existing structures. In a previous study, (1) a method was presented for determining quantitatively the foundation stiffness of structures, which was based on matching measured and calculated structural modal properties. This paper presents a simpler and more direct method that requires only measured modal properties and the masses of the structure.

THEORY

The differential equation for lateral modal vibration x of a structure having rotational foundation stiffness K_φ is

$$\omega^2(\Sigma m_i \ddot{x}_i h_i + I_o \theta) = K_\varphi \theta \qquad (1)$$

where ω is the angular frequency of the mode, m_i are the masses at storey i, h_i is the height from the base, and θ^1 is the base rotation. The term I_o is the rotational inertia of the masses and can usually be neglected. For the lateral motion of a building foundation as well as vertical vibration of a bridge pier the forces acting on the soil-footing interface are the reaction forces R associated with a particular modal vibration of the structure. If x_b is the modal amplitude of the footing,

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then the footing stiffness K_h can be found from

$$K_h x_b = \omega^2 R \qquad (2)$$

where $R = \Sigma m_i x_i$ for a building. For vibrations of buildings as well as bridges, all terms can be determined from full-scale measurements and the masses can be calculated easily.

After the foundation stiffnesses are found, the uncoupled rocking period T_φ and translational period T_h can be computed from

$$T_\varphi = 2\pi\,(I/K_\varphi)^{1/2} = 2\pi[\,(\Sigma m_i h_i^2)/K_\varphi]^{1/2} \qquad (3)$$

$$T_h = 2\pi\,[(\Sigma m_i)/K_h]^{1/2}. \qquad (4)$$

The fixed-based period T_f is then determined from the well-known approximation

$$T^2 = T_\varphi^2 + T_h^2 + T_f^2 \qquad (5)$$

where T is the measured period of the structure with the flexible base.

RESULTS

The procedure is used to determine the foundation stiffnesses for three structures.

(1) 15-storey reinforced concrete shear wall building. - The building consists of 7-in. (17.8 cm) slabs, an 8-in. (20.3 cm) roof slab, and one basement, with a 3-ft (0.92 cm) thick foundation mat and 10-in. (25.4 cm) perimeter basement walls. A typical horizontal section is shown in Fig. 1. The soil profile beneath the raft consists of 40 ft (12 m) of clay, 10 ft (2.5 m) of till, then shale. Further soil details are given in Ref. (2).

The ambient movements of the structure were monitored and mode shapes and frequencies determined. Rocking motion was measured at opposite ends of the building, with a reference instrument at the 15th floor level. The fundamental mode shape in the E-W direction is shown in Fig. 2, and in the N-S direction in Fig. 3. Using the measured mode shapes, with interpolated values for locations without experimental amplitudes, the foundation stiffnesses were determined from Eqs. 1 and 2, and are presented in Table I. Also shown are the uncoupled rocking and translational periods and the fixed-based period T_f computed from Eqs. 3, 4 and 5. A comparison between the measured and the fixed-base periods shows that the foundation flexibility affects the fundamental period of the structure significantly, as the rocking period is appreciably larger than the translational period.

(2) 9-storey reinforced concrete building. - By way of contrast, a building founded on firm soil was investigated. From the results of a forced vibration test on the Millikan Library, California Institute of Technology,[3] foundation stiffnesses and uncoupled periods are obtained for the fundamental modes from Eqs. 1 to 5, and are presented in Table I. The modal amplitudes of floors not measured were estimated by interpolation. A

comparison between the measured and fixed base periods confirms quantitatively that the influence of the base flexibilities on the structural periods is negligible.

(3) Steel truss railway bridge. - The bridge consists of four simply supported, parallel-chord, through-truss spans carrying a single railway track. The solid masonry piers were constructed in the 1880's on a clay river bed; the original spans were replaced in the 1920's by the present 9-panel steel trusses, 204 ft 9 in. (63 m) long.

The vertical modes of vibration of one end span and pier and the adjacent span were obtained from wind-induced vibrations. The masses of the truss and pier were computed from construction drawings and the tributary weights lumped at each panel point. With the measured modal amplitudes for the first vertical mode, the foundation flexibility under the pier was determined to be 1.0×10^{-7} in./lb from Eq. 2.

From an eigenvalue formulation, the theoretical mode shapes and natural frequencies were computed using the above foundation flexibility. As shown in Table II, another mode has been inserted between the first and second fixed-based modes.

DISCUSSION

In this method, the masses can usually be computed within close tolerances; thus the accuracy of the answers depends primarily on the accuracy of the measurements of modal amplitude and the square of the frequency. These can be obtained to within less than 10% variation, and consequently the values of foundation stiffnesses are of the same order of accuracy. Equally important as the measurement accuracy is the placement of the transducers to measure foundation movement. The possibility of measuring purely local deformations has to be guarded against.

The answers obtained are applicable to the level of deformations undergone by the structure and foundation during the test. For ambient vibrations the strains are very small. The movements at strain levels associated with earthquakes could be obtained by placing two strong-motion seismographs at opposite sides in the basement to measure rocking, and at least one instrument near the top storey for translation. The relevant deformations at intermediate levels could be obtained from extrapolation of low level vibration tests or theoretical calculations.

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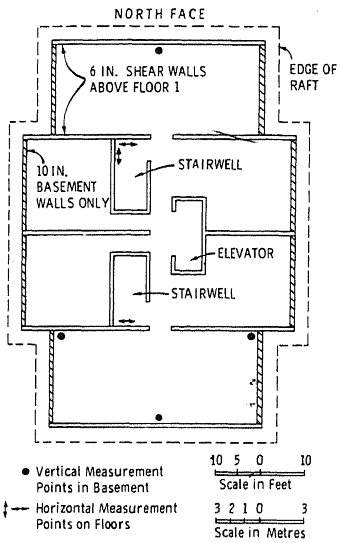

FIGURE 1
TYPICAL FLOOR PLAN OF 15 STOREY BUILDING

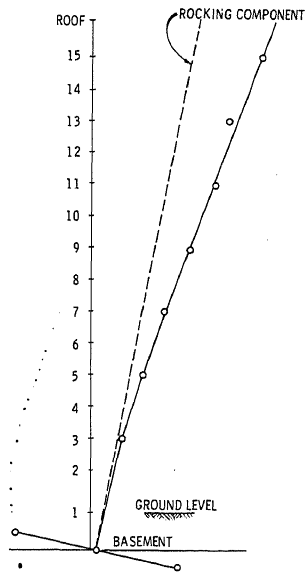

FIGURE 2
FUNDAMENTAL MODE SHAPE OF 15 STOREY BUILDING IN E-W DIRECTION

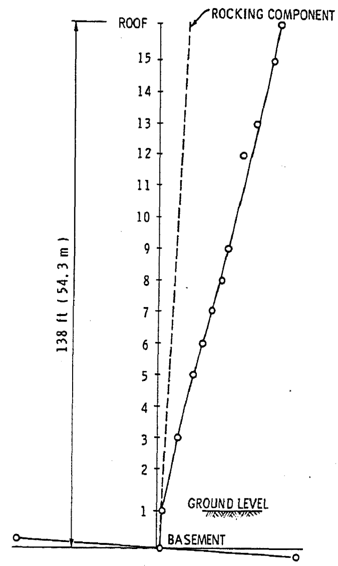

FIGURE 3
FUNDAMENTAL MODE SHAPE OF 15 STOREY BUILDING IN N-S DIRECTION

TABLE I. PROPERTIES OF THE 15-STOREY AND 9-STOREY BUILDINGS

Direction	Measured period, T, sec.	Foundation Stiffness rotation, K_φ in. lb/rad	Foundation Stiffness translation, K_h, lb/in.	Rocking period, T_φ, sec.	Transl. period, T_h, sec.	Fixed base, T_f, sec.
15-storey bldg.						
E-W	0.632	4.6×10^{12}	36×10^{6}	0.59	0.20	0.34
N-S	0.699	11.3×10^{12}	46×10^{6}	0.32	0.18	0.46
9-storey bldg.						
E-W	0.675	1.3×10^{15}	62×10^{7}	0.044	0.067	0.670
N-S	0.530	1.1×10^{15}	18×10^{7}	0.047	0.123	0.514

TABLE II. VERTICAL MODAL FREQUENCIES OF BRIDGE SPAN

Mode	1	2	3	4
From ambient vibrations, Hz	4.6	*	10.8	16.5
Calculations with flexible foundation, Hz	4.61	5.40	11.8	16.3
Calculations with fixed foundation, Hz	4.75	11.8	16.6	

*could not be identified with certainty from wind induced vibrations.