SEISMIC ANALYSIS OF LIGHTWEIGHT ELASTIC SYSTEMS IN NUCLEAR POWER PLANTS

by

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SYNOPSIS

The response spectrum technique is used in the development of a method for seismic analysis of lightweight elastic systems supported in buildings of nuclear power plants. The lightweight systems may be supported in one structure at several locations and elevations or interconnected between two or more structures. The method can be applied only to the systems which have classical normal modes. Methods of combining of modal responses are suggested.

NOMENCLATURE

ABS	=	Sumation of absolute responses as in Eq. (13)
ACC1	=	Combination of responses as given by Eq. (16)
ACC2	=	Combination of responses as given by Eq. (17)
A_n, $\ddot{A}_n$	=	Modal displacement and acceleration, respectively, of the n^{th} mode of a secondary system
$\ddot{A}_{nm_A}$	=	Modal acceleration of the n^{th} mode of a secondary system, when the support A has all directional responses of the m^{th} mode of the primary system
a_r	=	Displacements of the r^{th} mass of a secondary system due to a unit displacement of the support A
$\ddot{A}_{rnm}$	=	Acceleration response of the r^{th} mass of a secondary system in the n^{th} mode, when both supports are responding to the m^{th} mode of primary system
$\ddot{A}_{rnm_A}$	=	Same as $\ddot{A}_{rnm}$ but due to support A only
$\ddot{A}_{rnm_B}$	=	Same as $\ddot{A}_{rnm}$ but due to support B only
AVE	=	Average of ABS and SRSS responses

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(DLF) (t) = Dynamic Load Factor

f(t) = Nondimensional time function

J = Number of masses of the secondary system

KE = Kinetic energy in the secondary system

k_s = Spring stiffness, secondary system

L_A = Number of considered components of motion of support A

M = Number of modes, primary system

M_r = The r^{th} mass, secondary system

M_r' = Modified mass equal to $(a_r\, M_r)$

N = Number of modes, secondary system

S = Number of springs, secondary system

SRSS = Square root of the sum of the squares of responses as given by Eq. (14)

U = Strain energy, secondary system

$u_{no}(t)$ = Displacement function of SDF system having natural circular frequency w_n

$\ddot{u}_{no\ max}$ = Spectrum acceleration

$u_{rn}, \dot{u}_{rn}, \ddot{u}_{rn}$ = Relative displacement, velocity, and acceleration functions, respectively, of the r^{th} mass in the n^{th} mode of the secondary system

w_n = Natural circular frequency of the n^{th} mode of a secondary system

$X_A, \dot{X}_A, \ddot{X}_A$ = Displacement, velocity, and acceleration functions, respectively, of the support A, secondary system

$\ddot{X}_{Ao}$ = Amplitude of the acceleration function of support A

Γ_n = Participation factor of the n^{th} mode, secondary system

θ = Rotational degree of freedom

$\phi_{rn}, \phi_{\Delta_{sn}}$ = Constants for the mode n, secondary system (normal mode shape values)

INTRODUCTION

Lightweight piping and equipment (secondary systems) constructed in nuclear power plant structures (primary systems) must be designed to resist earthquake forces. Secondary systems are supported at several locations within a primary system and may be interconnected between two or more primary systems. The problem of analysing a secondary system which is located in one primary system has been studied, Refs., (1) (2) (3) (4) (5). However, to the writers' knowledge the problem of a secondary system supported by two or more primary systems has not been addressed. In this papaer the writers propose an approximate but general method which can accomodate both of these situations. In the application of this method it is assumed that both the primary and secondary systems have classical normal modes.

In this method the secondary systems are analysed by the response spectrum technique. Therefore, the modal acceleration response spectra of the primary systems at the support points of the secondary systems are required to implement this method. These spectra are defined as the response spectra for each individual mode of the primary systems and can be calculated by either the time history or response spectrum technique.

Included in this paper is an example of a secondary system analysed by the proposed method. These results are compared to a rigorous time history analysis of a secondary system coupled to a primary system. In this comparison some methods of combining modal responses are tested.

DEVELOPMENT

Consider the undamped multiple-degree-of-freedom (MDF) secondary system with two supports, A and B, Fig. 1. Each mass point may have six degrees of freedom. However, for simplicity of this development only motion in the XY plane is considered. If support A is displaced in the x-direction by an amount X_A, all masses will undergo translation and rotation in the XY plane. The displacement of the r^{th} mass may be expressed as $a_{rx} X_A$, $a_{ry} X_A$, and $a_{r\theta} X_A$, where a_{rx}, a_{ry} , and $a_{r\theta}$ are the translations and rotation of mass r due to a unit displacement of support A in the x-direction. Again, for simplicity in the presentation a_{rx}, a_{ry}, and $a_{r\theta}$ are replaced by the notation a_r. The system is further defined to have J masses, S springs, and N normal modes. When the support A, only, is in motion in the x-direction, Fig. 1, the kinetic energy, KE, of the system is

$$KE = \sum_{r=1}^{J} \tfrac{1}{2} M_r (\dot{X}_A a_r + \sum_{n=1}^{N} \dot{u}_{rn})^2 \qquad \ldots (1)$$

where M_r represents the translational and rotational mass properties, $\dot{X}_A a_r$ is the velocity of the mass r due to the motion of support A, and $\dot{u}_{rn}$ is the relative velocity of mass r for the n^{th} mode of the system. The total strain energy, U of the system is

$$U = \sum_{s=1}^{S} \tfrac{1}{2} k_s (\sum_{n=1}^{N} \Delta_{sn})^2 \qquad \ldots (2)$$

where k_s is the stiffness of spring s and Δ_{sn} is the distortion of spring s in the n^{th} mode of the system.

Expanding Eqs. (1) and (2), and then as in Ref. (6) employing the orthogonality of normal modes and introducing the notion of modal displacement A_n by expressing u_{rn} and Δ_{sn} in the form

$$u_{rn} = A_n\,(u_{rn}/A_n) = A_n\,\phi_{rn}$$
$$\Delta_{sn} = A_n(\Delta_{sn}/A_n) = A_n\,\phi\Delta_{sn} \qquad \ldots (3)$$

the kinetic and strain energies may finally be expressed as

$$KE = \sum_{r=1}^{J} \tfrac{1}{2} M_r(\dot{X}_A^2 a_r^2 + 2\,\dot{X}_A a_r \sum_{n=1}^{N} \dot{A}_n \phi_{rn} + \sum_{n=1}^{N} \dot{A}_n^2 \phi_{rn}^2)$$

$$U = \sum_{s=1}^{S} \tfrac{1}{2}\, k_s \sum_{n=1}^{N} A_n^2 \phi_{\Delta_{sn}}^2$$

The reduced form of Lagrange's equation, then, yields the modal equation of motion

$$\ddot{A}_n + w_n^2\, A_n = \ddot{X}_A\, \Gamma_n \qquad \ldots (4)$$

where, the square of the natural circular frequency and the participation factor of the n^{th} mode, respectively, are

$$w_n^2 = \frac{\sum_{s=1}^{S} k_s\, \phi_{\Delta_{sn}}^2}{\sum_{r=1}^{J} M_r \phi_{rn}^2}\;;\; \Gamma_n = \frac{\sum_{r=1}^{J} M_r'\, \phi_{rn}}{\sum_{r=1}^{J} M_r\, \phi_{rn}^2} \qquad \ldots (5)$$

It is noted that M_r' is called the modified mass and is expressed as $M_r' = a_r\, M_r$.

Defining $\ddot{X}_A = \ddot{X}_{Ao}\, f(t)$ the solution of Eq. (4) is

$$A_n\,(t) = -\ddot{X}_{Ao}/w_n^2\;(DLF)(t)\;\;\Gamma_n = u_{no}(t)\Gamma_n \qquad \ldots (6)$$

where $u_{no}(t)$ is the solution for a single-degree-of-freedom (SDF) system having a natural circular frequency w_n, when its support motion is defined as $\ddot{X}_{Ao}$ f (t), and (DLF) (t) is the dynamic load factor.

When the (DLF)(t) becomes a maximum, $u_{no}(t)$ is also maximum, $u_{no\ max}$ Introducing the concept of response spectrum, Ref. (6), then $u_{no\ max}$ is the value of the response spectrum of a forcing function, $\ddot{X}_A$, for an undamped SDF oscillator having a natural circular frequency w_n. Since there is a frequency relationship between spectrum displacement and spectrum acceleration, then the spectrum acceleration is

$$\ddot{u}_{no\ max} = w_n^2\ u_{no\ max} \qquad \ldots (7)$$

and the modal acceleration $\ddot{A}_n$ of the n^{th} mode of the system becomes

$$\ddot{A}_n = \ddot{u}_{no\ max}\ \Gamma_n \qquad \ldots (8)$$

To expand the development to include all possible motions of the support A, it is noted that for any given mode of vibration of the primary system, m, all motions of support A have the same time function f(t). This is based upon the premise that the primary system has classical normal modes. Consequently, the algebraic addition of these responses is required and can be expressed as

$$\ddot{A}_{nm_A} = \sum_{\ell_A=1}^{L_A} \ddot{A}_{nm\ell_A} \qquad \ldots (9)$$

where L_A represents all components of motion of support A which are to be considered. The acceleration of the r^{th} mass of the secondary system is then

$$\ddot{A}_{rnm_A} = \ddot{A}_{nm_A}\phi_{rn} \qquad \ldots (10)$$

By expanding this development to account for motion of the support B, $\ddot{X}_B$ (t), the expression for the acceleration of the r^{th} mass is similarly derived to be

$$\ddot{A}_{rnm_B} = \ddot{A}_{nm_B}\phi_{rn} \qquad \ldots (11)$$

When both supports A and B of the secondary system are attached to the same primary system the responses of the r^{th} mass due to both support motions should be added algebraicly as

$$\ddot{A}_{rnm} = \ddot{A}_{rnm_A} + \ddot{A}_{rnm_B} \qquad \ldots (12)$$

Eq. (12) yields the response of the r^{th} mass of the secondary system in mode n, when the primary system is vibrating in mode m. It follows that there are N times M values obtained by Eq. (12) and the combination of the separate responses must be considered. There are many ways to do this. In this paper the following combinations were considered:

a) The summation of the absolute values of all modal responses of both systems as

$$ABS = \sum_{m=1}^{M} \sum_{n=1}^{N} |\ddot{A}_{rnm}| \qquad \ldots (13)$$

b) The square root of the sum of the squares summation of the modal responses of both systems as

$$SRSS = \left(\sum_{m=1}^{M} \sum_{n=1}^{N} \ddot{A}_{rnm}^2\right)^{1/2} \qquad \ldots (14)$$

c) The average of a) and b) as

$$AVE = \frac{1}{2} (ABS + SRSS) \quad \ldots (15)$$

d) The square root of the sum of the squares of N responses of the secondary system, obtained from the algebraic addition of the values of Eq. (12) which are summed over M modes of the primary system, Ref. 1, as

$$ACC1 = [\sum_{n=1}^{N} (\sum_{m=1}^{M} A_{rnm})^2]^{\frac{1}{2}} \quad \ldots (16)$$

e) The square root of the sum of the squares of M responses of the primary system obtained from the absolute sum of the values of Eq. (12), summed over N modes of the secondary system, as

$$ACC2 = [\sum_{m=1}^{M} (\sum_{n=1}^{N} |A_{rnm}|)^2]^{\frac{1}{2}} \quad \ldots (17)$$

EXAMPLES OF APPLICATION

In Fig. 2 is shown a simplified dynamic model of a nuclear containment structure supporting a lightweight flexible system. The primary system is described by masses 1 through 13, while the masses 14 through 20 represent the lightweight secondary system. The primary and secondary systems were coupled for a time history analysis of responses using the Helena E-W earthquake record normalized to 0.06g maximum ground acceleration. Maximum responses (accelerations) in the horizontal direction were calculated for all masses of the secondary system. The modal acceleration response spectra values, required for the proposed method, were also obtained by time history analysis using the same earthquake record.

The calculations were extended to include a variation in the flexibility of the secondary system with respect to the primary system. This variation is described as follows:

Case 1, all frequencies of the secondary system are greater than all frequencies of significant modes of primary system, i.e., $w_n > w_m$.

Case 2, all frequencies of the secondary system are in the range of the frequencies of significant modes of primary system.

Case 3, all frequencies of significant modes of the secondary system are less than the frequencies of significant modes of primary system, i.e., $w_n < w_m$.

Table I shows results of analyses for the combinations of acceleration responses as obtained from Eqs. (13) through (17) for the cases 1,2, and 3. The values in parentheses in this table are the ratios of results obtained by this method to the result obtained from the time history analysis of the coupled primary and secondary systems.

The results show that in case 1 and 2 the best agreement is given by Eq. (16), while for case 3 the best agreement is obtained by Eq. (14).

SUPPORTS ON DIFFERENT PRIMARY SYSTEMS

If the supports A and B of a secondary system are on different primary systems uncertainties are introduced, because the motions of the supports have different time functions. Accordingly, a conservative engineering solution to this problem is suggested. To determine the response of r^{th} mass of the secondary system add the responses, obtained separately, from the modes of each of the primary systems. The phasing of the most significant modes of the primary system must be considered. Refering to Fig. 3.a, the r^{th} mass will have maximum horizontal response when the motions of supports A and B have an assumed phase angle of 0° as is shown when the supports move to position A′ and B′. Similarly, the maximum vertical acceleration of the r^{th} mass will occur when the assumed phase angle is 180° as shown in Fig. 3.b. It follows then, that the maximum response of the r^{th} mass can be expressed as

$$\ddot{A}_{rnm} = \ddot{A}_{rnm_A} \pm \ddot{A}_{rnm_B} \qquad \ldots (18)$$

Referring to the results of the investigation of a secondary system supported on one primary system, it is concluded that the combination of modal responses should correspond to Eqs. (14) and (16). Namely,

$$\text{SRSS} = \Big[\sum_{n=1}^{N} \Big(\sum_{m_A=1}^{M_A} \ddot{A}^2_{rnm_A} \pm \sum_{m_B=1}^{M_B} \ddot{A}^2_{rnm_B} \Big) \Big]^{1/2} \qquad \ldots (19)$$

$$\text{ACC1} = \Big[\sum_{n=1}^{N} \Big(\sum_{m_A=1}^{M_A} \ddot{A}_{rnm_A} \pm \sum_{m_B=1}^{M_B} \ddot{A}_{rnm_B} \Big)^2 \Big]^{1/2} \qquad \ldots (20)$$

where, M_A and M_B are the significant modes of primary systems.

CONCLUSIONS

It has been demonstrated that the response spectrum technique is generally applicable for the analysis of secondary systems. In the examples, methods of combining the modal responses have been suggested. More experience with the method and comparisons with rigorous solutions of coupled systems is required before firm recommendations can be made regarding the combination of modal responses. This method is particularly attractive when the modal response spectra of the primary system are calculated by the response spectrum technique.

REFERENCES

(1) Amin, M., Hall, W.J., Newmark, N.M., and Kassawara, R.P.,"Earthquake Responses of Multiply Connected Light Secondary Systems by Spectrum Methods", Proc. of ASME 1st National Congress of Pressure Vessel and Piping Technology, San Francisco, 1971.

(2) Kassawara, R.P., "Earthquake Response of Light Multi-Degree-of-Freedom Secondary systems by Spectrum Technique". Ph,D. Thesis, Department of Civil Engineering, University of Illinois, Urbana, Ill., 1970.

(3) Penzien, J., and Chopra, A.K., "Earthquake Response of Appendage on a Multi-Story Building", Proc. Third World Conf. on Earthquake Engineering, Vol. II, New Zealand, 1965.

(4) Biggs, J.M., and Roesset, J.M., "Seismic Analysis of Equipment Mounted on a Massive Structures", Seismic Design of Nuclear Power Plants, Edited by R.J. Hansen, The MIT Press, 1970.

(5) Jankov, Z.D., and Reeves, C.F., "Amplification of Seismic Ground Response Spectra", Boston, Ma. 1971, Unpublished.

(6) Biggs, John M., Introduction to Structural Dynamics, McGraw-Hill Co., 1964.

TABLE 1 - ACCELERATIONS OF SECONDARY SYSTEM IN HORIZONTAL DIRECTION

Natural Periods of 6 significant modes of primary system: .6674, .262, .1757, .066, .0502, .0363

CASE 1 - Natural Periods of Secondary System:
0.0855, 0.0846, 0.0308, 0.0178,0.0103,0.0094,0.0074,. . . . .

MASS	SRSS	ABS	AVE	ACC1	ACC2
14	0.061(0.75)	0.159(1.96)	0.120(1.36)	0.085(1.05)	0.083(1.02)
15	0.093(1.14)	0.204(2.51)	0.149(1.83)	0.085(1.05)	0.126(1.56)
16	0.064(0.90)	0.152(2.13)	0.108(1.52)	0.074(1.05)	0.089(1.25)
17	0.089(1.32)	0.143(2.13)	0.116(1.72)	0.069(1.02)	0.118(1.75)
18	0.064(0.90)	0.152(2.14)	0.108(1.52)	0.072(1.02)	0.089(1.26)
19	0.093(1.15)	0.204(2.52)	0.148(1.83)	0.080(0.99)	0.126(1.56)
20	0.061(0.75)	0.159(1.97)	0.110(1.36)	0.080(0.99)	0.083(1.02)

CASE 2 - Natural Periods of Secondary System:
0.6032,0.5969,0.2175,0.1255,0.0725,0.0663,0.0536,. . . . .

MASS	SRSS	ABS	AVE	ACC1	ACC2
14	0.061(0.74)	0.164(1.99)	0.113(1.37)	0.086(1.05)	0.084(1.02)
15	0.093(1.12)	0.213(2.55)	0.153(1.84)	0.088(1.05)	0.129(1.55)
16	0.244(0.93)	0.450(1.71)	0.347(1.32)	0.253(0.96)	0.294(1.12)
17	0.309(1.35)	0.355(1.55)	0.332(1.45)	0.290(1.27)	0.323(1.41)
18	0.244(1.07)	0.450(1.98)	0.347(1.52)	0.254(1.12)	0.294(1.29)
19	0.093(1.10)	0.213(2.52)	0.153(1.81)	0.084(1.00)	0.129(1.52)
20	0.061(0.73)	0.134(1.98)	0.113(1.36)	0.082(0.99)	0.084(1.01)

CASE 3 - Natural Periods of Secondary System:
1.9075, 1.8876, 0.6879, 0.3969, 0.2291,. . . . .

MASS	SRSS	ABS	AVE	ACC1	ACC2
14	0.089(0.92)	0.242(2.50)	0.166(1.71)	0.103(1.07)	0.137(1.42)
15	0.145(1.34)	0.342(3.17)	0.244(2.26)	0.118(1.09)	0.224(2.08)
16	0.062(1.50)	0.152(3.69)	0.107(2.59)	0.098(2.39)	0.083(2.01)
17	0.059(0.97)	0.101(1.65)	0.080(1.31)	0.007(0.12)	0.067(1.10)
18	0.062(1.38)	0.152(3.40)	0.107(2.39)	0.028(0.63)	0.083(1.85)
19	0.145(1.23)	0.342(2.90)	0.244(2.07)	0.153(1.30)	0.224(1.90)
20	0.089(0.86)	0.242(2.33)	0.166(1.60)	0.121(1.17)	0.137(1.32)

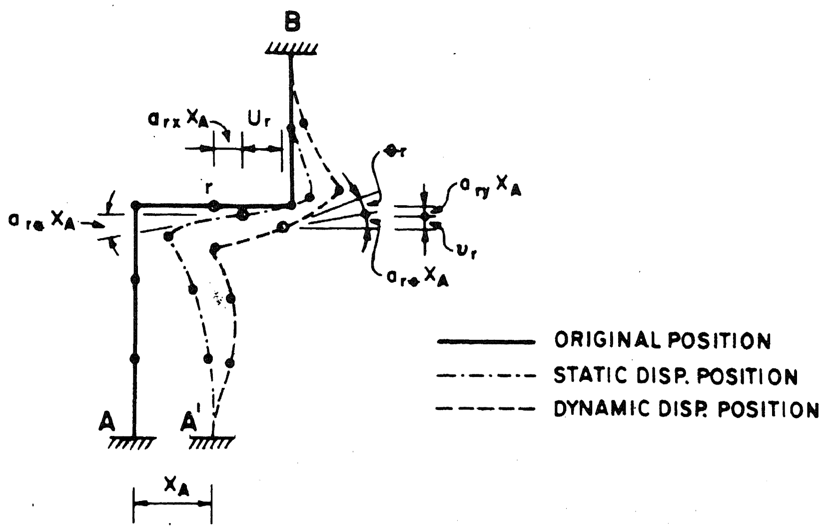

FIG. 1 - SECONDARY SYSTEM HAVING SUPPORT MOTION X_A

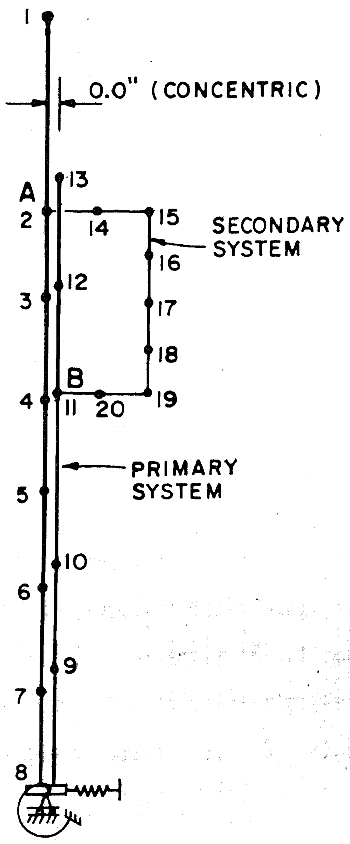

FIG. 2 - COUPLED PRIMARY AND SECONDARY SYSTEMS

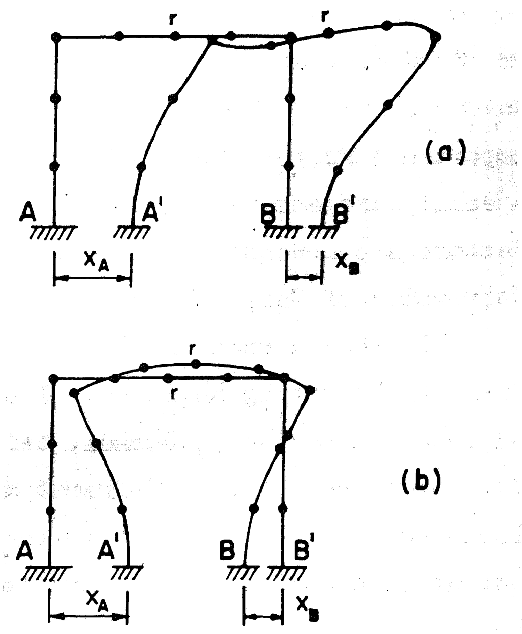

FIG. 3 - SECONDARY SYSTEM SUPPORTED ON TWO PRIMARY SYSTEMS