DISCUSSION OF PAPER No. 249

["Preliminary Seismic Analysis of Reinforced Concrete Tall Building" by T. Guendelman and J. Monge]

by Arthur C. Heidebrecht[I]

The authors have pursued a mathematical model for a linked frame -shear wall building which has the advantages of representing the system by a single differential equation. The results shown for calculating the first natural frequency (Fig.2) would appear not to be sufficiently general, based on some work which has recently appeared in the literature (1). This work shows that the first five natural frequencies can be computed approximately (with maximum errors of less than ten percent for a practical range of the parameter α) using the following formulation for the frequency parameter δ

$$\delta_i = \lambda_i^2 \sqrt{1 + \frac{\alpha^2}{\lambda_i^2}} \qquad (i = 1 \; .... \; 5)$$

in which λ_1 is the frequency parameter for mode i for an ordinary cantilever beam. Values of this frequency parameter are given by Timoshenko (2) and take on the following values for the first five modes: 1.875, 4.694, 7.855, 10.996, and 14.137.

It should also be noted that reference (1) contains a transfer matrix approach which allows for relatively easy static analysis of non-uniform structures of this type.

Bibliography

1. Heidebrecht, A.C. and Stafford Smith, B., "Approximate Analysis of Tall Wall-Frame Structures", Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 99, No. ST2, February 1973, pp. 199-221.

2. Timoshenko, S., "Vibration Problems in Engineering", D. Van Nostrand Co. Inc., New York, N.Y., 1953, pp. 337-338.

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ANSWER TO PREPARED DISCUSSION OF PAPER N° 249
by Dr. Arthur C. Heidebrecht

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The authors acknowledge the interesting discussion by Dr. Heidebrecht. At the oral presentation of the paper, Figures 7 and 8 describing the computation of dynamic properties for the first five modes of a model of uniform properties were shown. We add now in Fig. 7 the curves (1) that correspond to formula (') proposed by Drs. Heidebrecht and Stafford Smith :

$$\delta_i = \lambda_i^2 \sqrt{1 + \frac{\alpha^2}{\lambda_i^2}} \qquad (i=1....5) \qquad (1)$$

This formula gives a very good approximation for δ_1, provided $\alpha < 10$; and it gives a very good approximation for $\delta_i (i>1)$ in the whole range of α.

We have used in our paper the following formula (2) :

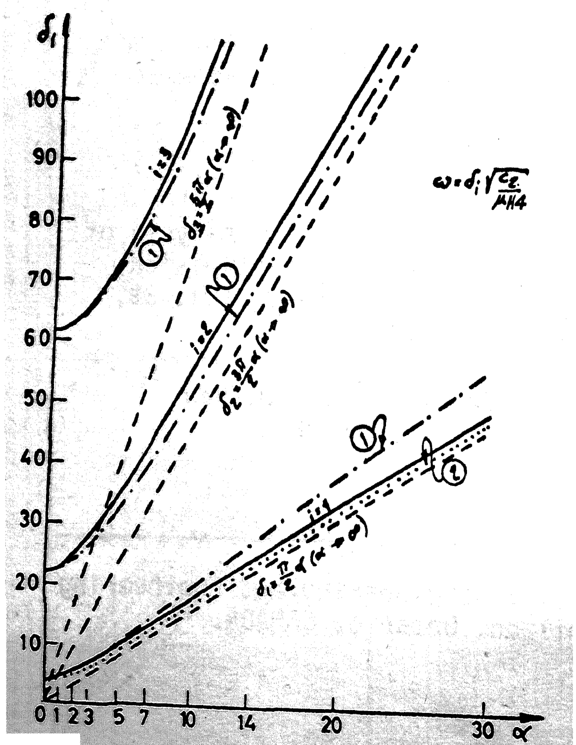

FIG. 7- FREQUENCY FACTORS δ_i

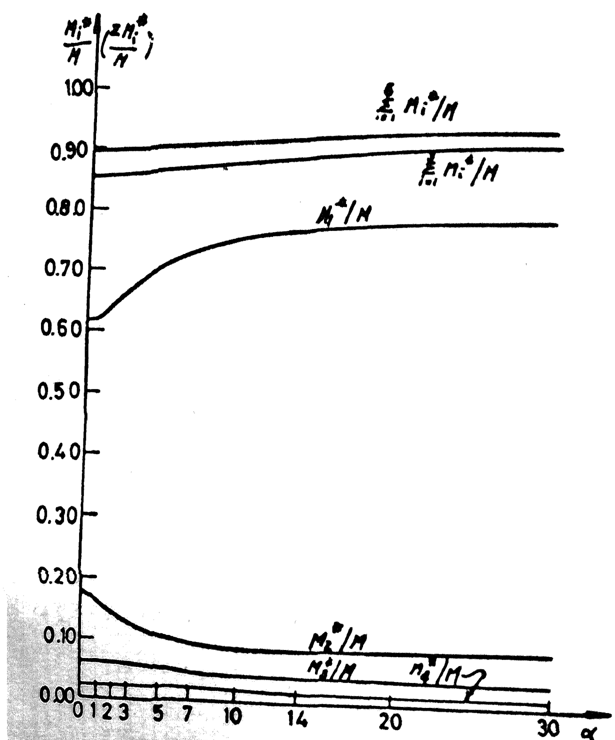

FIG. 8- EQUIVALENT MASSES M_i^* FOR BASE SHEAR

$$\alpha_i = \lambda_i^2 \sqrt{1 + \frac{(2i-1)^2\pi^2}{4} \quad \frac{\alpha^2}{\lambda_i^4}} \quad (i=1....5) \qquad (2)$$

that gives a better approximation than formula (1) for δ_1 when $\alpha > 10$. For $i > 1$, this formula coincides for practical purposes with formula (1), since then λ_i approaches closely the value $(2i-1)\ \pi/2$. Curve (2) in Fig. 7 represents δ_1 by formula (2).

Formula (2) was deducted for i=1 by using a lower bound theorem due to Southwell[1]. The system with stiffness parameters (C_1, C_2) and mass μ is divided in two subsystems $(C_1,\ \theta;\ \mu)$ and $(0,\ C_2;\ \mu)$. The theorem states that

$$\omega^2 > \omega_1^2 + \omega_2^2$$

where ω is the fundamental frequency of the system and ω_1, ω_2 are the fundamental frequencies of the subsystems 1 and 2. By replacing

$$\omega^2 = \delta_1^2 \frac{C2}{\mu H^4} \qquad \omega_1^2 = \frac{\pi^2}{4} \alpha^2 \frac{C2}{\mu H^4} \qquad \omega_2^2 = \lambda_1^4 \frac{C2}{\mu H^4},$$

formula (1) is found for i=1. The formula was generalized for i > 1 without a formal proof.

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[1] N.M. Newmark and E. Rosenblueth
"Fundamentals of Earthquake Engineering", Chapter 4, Sec. 4.7
Prentice Hall, 1971.