PRELIMINARY SEISMIC ANALYSIS OF REINFORCED CONCRETE TALL BUILDINGS.

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SYNOPSIS.

A mathematical model representing a fictitious structure is analyzed for inertia forces. Actual structures can fit that mathematical model just by adjusting the so called stiffness parameters, and several examples are shown. The method permits an accurate estimation of deflections and stresses.

1.- Formulation of the Model.

The method of representing building frames with relatively wide walls by a continuous medium has been used by several authors : Chitty, Albiges and Goulet, Beck, Erikson, Rosman, Magnus, Coull and others. Monge[1] proposes an extension of this technique assuming that every building structure may be represented as a continuum. Normal modes satisfy the modal equation:

$$(C_2\ \phi'')'' - (C_1\ \phi')' = \mu\ \omega^2\phi \qquad (1)$$

where $C_1(x)$ and $C_2(x)$ are stiffness parameters depending on the mechanical and geometrical properties of the actual structure and can be easily evaluated for the type of structures analyzed herein. $\mu(x)$ is the mass of the structure per unit length, ω is the natural frequency of the mode ϕ, and x is the coordinate relative to the base.

In this paper, solutions are obtained for the case when C_1 and C_2 are positives for any x eventhough the technique remains the same when one of these parameters is negative, like towers and chimneys where shear deformations are considered.

The governing differential equation (1) is obtained for the fictitious structure represented in Fig. 1 consisting of a shear beam and a slender beam fixed at the base and linked together uniformly throughout the height . Denoting by $p_1(x)$ and $p_2(x)$ the lateral loads per unit length acting on the shear and slender beam respectively, the following equations can be written :

$$-(\frac{GA}{\kappa}\ y')' = p_1(x) \qquad (2)$$

$$(EI\ y'')'' = p_2(x)$$

If the lateral load per unit length on the structure is $p(x)$ from equilibrium,equation (3) is obtained

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$$(EI\ y'')'' - \left(\frac{GA}{\kappa}\ y'\right)' = p(x) \tag{3}$$

Substituting $p(x)$ by by the inertia forces per unit length and expressing $GA/\kappa = C_1(x)$ and $EI = C_2(x)$ the equation of motion is obtained

$$\frac{\partial^2}{\partial_x{}^2}\left[C_2(x)\ \frac{\partial^2 y}{\partial_x{}^2}\right] - \frac{\partial}{\partial x}\left[C_1(x)\ \frac{\partial y}{\partial x}\right] = -\ \mu\ \frac{\partial^2 y}{\partial t^2} \tag{4}$$

Using separation of variables $y(x,t) = \phi(x)T(t)$ the modal equation (1) is obtained. Boundary conditions are

$$\left|\phi(x)\right|_{x=o} = 0;\ \left|\phi'(x)\right|_{x=o} = 0;\ \left|\phi''(x)\right|_{x=H} = 0; \tag{5}$$

$$\left|\{C_2(x)\phi''(x)\}' - C_1(x)\phi'(x)\right|_{x=H} = 0$$

Solution of this modal equation is performed defining the state vectors $|\phi,\ \phi',\ \phi'',\ (C_2\phi'')' - C_1\phi'|$ and using transfer matrices.

The first mode parameters for the case C_1, C_2, μ constant are shown in Fig. 2,3,4,5,6 where

$$\alpha^2 = \frac{C_1 H^2}{C_2} \tag{6}$$

and

$$\omega^2 = \frac{\delta^2 C_2}{\mu\ H^4}$$

2.- Representation of Actual Structures.

In order to fit the mathematical model just defined for the case of actual structures, it is required to determine the specific stiffness parameters $C_1(x)$ and $C_2(x)$. An actual building consists of a set of substructures such as walls, frames, etc. This paper deals only with substructures fixed at the base. Substructure j has the differential equation

$$(C_{2j}\ \xi''_j)'' - (C_{1j}\ \xi'_j)' = p_j(x)$$

Geometric compatibility gives $\xi_j = \phi$ and equilibrium requires $\sum_j p_j(x) = p(x)$. If $C_1 = \sum C_{1j}$ and $C_2 = \sum C_{2j}$,

$$(C_2\ \phi'')'' - (C_1\ \phi')' = p(x) \tag{7}$$

which fits the model for $p(x) = \mu\omega^2\phi$

As an example of actual structural elements, some cases are shown:

- Slender wall $C_{1j} = 0$ $C_{2j} = (EI)_j$
- Frame with relatively stiff beams (axial deformations not included)

$$C_{1j} = h\ \sum(aK_c)_i \qquad C_{2j} = 0$$

where h is the story height, $(aK_c)_i$ is the reduced shear stiffness of column i of the j frame, computed accordingly to Muto's method[2]. For prismatic columns, $1/K_c = \kappa h/GA + h^3/12\,EI$

- Frame with relatively slender beams[3] (axial deformations not included).

$$C_{1j} = (1/h)\sum (aK_b\, L^2)_i \qquad C_{2j} = \sum (EI)_i$$

where h is the story height, $(aK_b)_i$ is the reduced shear stiffness of the beam at span i, L_i is the length of the span i measured between column axis. For prismatic beams, $1/K_b = \kappa L_1/GA + L_1^3/12EI$ where L_1 is the free length of the span i. Parameter a is computed by Muto's method, interchanging beams and columns. If axial deformations have to be considered, as it is the case of two walls, C_{1j} and C_{2j} change to incorporate some contribution of the area in the moment of inertia of the cross section[3].

- Structural elements (axial and shear deformations considered). Following Monge's idea[1], $(EI)_j$ and $(GA/\kappa)_j$ are obtained from an equivalent beam representation of the actual structural element j. As an example, for a slender frame the hypothesis of the cantilever method can be used for defining the equivalent beam. For $(EI)_j$, $(GA/\kappa)_j$ constant,

$$C_{1j} = -\frac{\delta^2}{\alpha_j^4 - \alpha_j^2\,\alpha^2 - \delta^2} \cdot \left(\frac{GA}{\kappa}\right)_j \; ; \; C_{2j} = \frac{\alpha_j^4}{\alpha_j^4 - \alpha_j^2\,\alpha^2 - \delta^2}\,(EI)_j \; ; \; \alpha_j^2 = H^2\left(\frac{GA}{\kappa EI}\right)_j$$

In order to compute C_{1j} and C_{2j} an iterative procedure assuming an initial value of α^2 is performed until the computed $\alpha^2 = H^2 \sum C_{1j} / \sum C_{2j}$ converges.

3.- Computation of Design Stresses.

From the design point of view it is necessary to obtain distribution of lateral forces among the resisting elements, and internal stresses for the individual members, According to the mathematical model, the lateral loads acting on an element j are $p_j(x) = (C_{2j}\phi'')'' - (C_{1j}\phi')'$ where ϕ has to be scaled in order to include the participation factor and to satisfy the design base shear. Shear forces are

$$Q_j(x) = \Big|(C_{2j}\phi'')' - C_{1j}\phi'\Big|_x^H$$

In order to determine internal stresses, several simple techniques are available and the accuracy of results depend mainly on the estimation of deflections and acting lateral forces. The model presented in this paper gives a good evaluation of both deflections and forces.

For a preliminary design it is possible to assume that the structure has constant properties and that the acting lateral load on the structure has a triangular distribution of the type $p(x) = 2Qx/H^2$. The deflection of the structure $\phi(x)$ could be considered as a good estimation of the first mode shape and has the following analytical expression :

$$\phi(x) = (QH/C_1)\left[a(\cosh\alpha z - 1) + b(\operatorname{senh}\alpha z - \alpha z) - z^3/3\right] \qquad \text{where}$$

$z = x/H$; $a = \left[2/\alpha^2 - (2/\alpha^2 - 1)\operatorname{senh}\alpha/\alpha\right]/\cosh\alpha$; $b = (2/\alpha^2 - 1)/\alpha$

From the expressions, $\phi'(x)$, $\phi''(x)$ and $\phi'''(x)$ can be determined and therefore a simple way of evaluating the $Q_j(x)$ is provided. For better results it is recommended to use the values of $\phi'(x)$ and $\phi''(x)$ obtained from the state vectors used in connection with the transfer matrix techniques.

4. References.

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FIGURES

FIG. 1- MATHEMATICAL MODEL

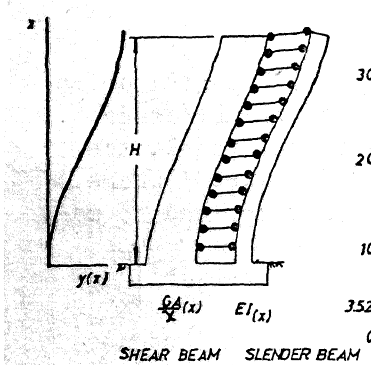

FIG. 2.- FIRST NATURAL FREQUENCY: ω1

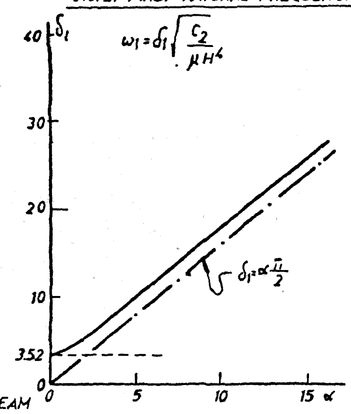

FIG. 3.- EQUIVALENT MASS, FIRST MODE: M_1^*

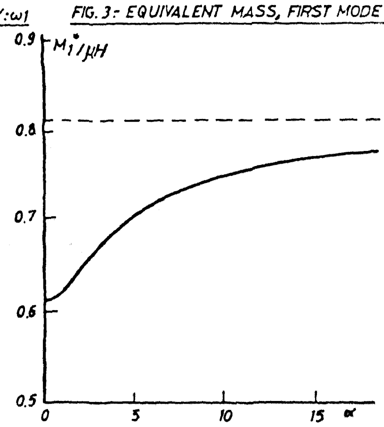

FIG. 4.- FIRST MODE SHAPE: φ_1

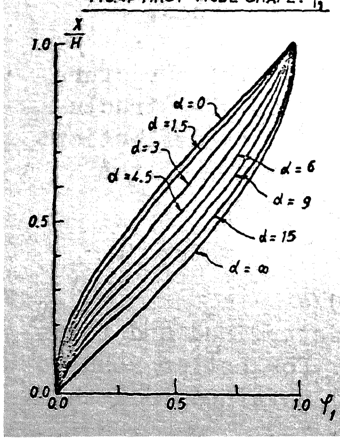

FIG. 5.- FIRST MODE SHEAR DIAGRAM: $q_1(x)$

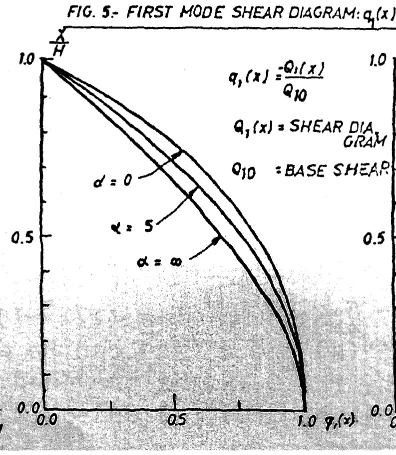

FIG. 6.- FIRST MODE OVERTURNING MOMENT DIAGRAM: $m_1(x)$