INTERACTION BETWEEN A LARGE STRUCTURE AND THE GROUND

by

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SYNOPSIS

Recently, dinamic analysis for a conbined system of the ground and a structure on it became possible through development of the finite element method. But, it will be not practical to analyse as a whole the system including a large structure on the widely extended ground, because of large memory and long computing time. Usually, the structural part is divided from the ground and solved independently. In this case, the effect of the ground on the structural part should be taken into account. This paper presents a method of introduction for the effect of the ground, and an explisit formula for the equivalent mass, damping and stiffness matrix.

EQUATIONS OF MOTION FOR A STRUCTURE CONSIDERED GROUND EFFECT

If we use absolute displacements for independent variables, the equations of motion may be written as follows.

$$\underline{M}\,\underline{\ddot{X}} + \underline{C}\,\underline{\dot{X}} + \underline{K}\,\underline{X} = \underline{0} \qquad (1)$$

in which $\underline{M}$, $\underline{C}$ and $\underline{K}$ are mass, damping and stiffness matrix, respectively. $\underline{X}$ is absolute displacement vector, dot at the top denotes differentiation with respect to time and bar at the bottom expresses matrix or vector.

Let us consider the case in which the ground and a structure on it are divided into finite elements and the nodes of them are classified in three parts, namely, structure, ground and support as shown in Fig.1. Then the stiffness matrix, $\underline{K}$, and the displacement vector, $\underline{X}$, of Eq.1 could be expressed as follows.

$$\underline{K} = \begin{bmatrix} \underline{Kss} & \underline{Ksg} & \underline{0} \\ \underline{Kgs} & \underline{Kgg} & \underline{Kge} \\ \underline{0} & \underline{Keg} & \underline{Kee} \end{bmatrix}, \quad \underline{X} = \begin{Bmatrix} \underline{Xs} \\ \underline{Xg} \\ \underline{Xe} \end{Bmatrix}$$

The suffix s, g and e designate structural part, ground part and support, respectively. The same forms can be led for the mass matrix, M, and the damping matrix, C, in Eq.1. As the support displacements are seismic waves themselves and are known values, the equations of motion with respect to the support nodes need not be considered. Then Eq.1 becomes

$$\begin{bmatrix} \underline{Mss} & \underline{Msg} \\ \underline{Mgs} & \underline{Mgg} \end{bmatrix} \begin{Bmatrix} \underline{\ddot{Xs}} \\ \underline{\ddot{Xg}} \end{Bmatrix} + \begin{bmatrix} \underline{Css} & \underline{Csg} \\ \underline{Cgs} & \underline{Cgg} \end{bmatrix} \begin{Bmatrix} \underline{\dot{Xs}} \\ \underline{\dot{Xg}} \end{Bmatrix} + \begin{bmatrix} \underline{Kss} & \underline{Ksg} \\ \underline{Kgs} & \underline{Kgg} \end{bmatrix} \begin{Bmatrix} \underline{Xs} \\ \underline{Xg} \end{Bmatrix} = - \begin{Bmatrix} \underline{0} \\ \underline{F} \end{Bmatrix} \qquad (2)$$

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in which $\underline{F} = \underline{Mge}\,\ddot{\underline{Xe}} + \underline{Cge}\,\dot{\underline{Xe}} + \underline{Kge}\,Xe$

Especially, if there is no interaction of mass and damping between the ground and the structure, $\underline{Msg}$, $\underline{Mgs} = \underline{0}$ and $\underline{Csg}$, $\underline{Cgs} = \underline{0}$ will be concluded. Then Eq.2 may be expressed

$$\underline{Mss}\,\ddot{\underline{Xs}} + \underline{Css}\,\dot{\underline{Xs}} + \underline{Kss}\,\underline{Xs} = -\underline{Ksg}\,\underline{Xg} \qquad (3)$$
$$\underline{Mgg}\,\ddot{\underline{Xg}} + \underline{Cgg}\,\dot{Xg} + \underline{Kgg}\,\underline{Xg} = -\underline{Kgs}\,\underline{Xs} - \underline{F}$$

Performing Fourier's transformation to Eq.3 and then eliminating $\underline{\chi g}$ leads to

$$\left\{-w^2\underline{Mss} + i\underline{Css} + \left[\underline{Kss} - \underline{Ksg}\,\underline{A(w)}^{-1}\,\underline{Kgs}\right]\right\}\underline{\chi s} = \underline{Ksg}\,\underline{\chi go} \qquad (4)$$

in which $\underline{A(w)} = -w^2\underline{Mgg} + iw\underline{Cgg} + \underline{Kgg}$

$$\underline{\chi go} = \underline{A(w)}^{-1}\,(-w^2\underline{Mge} + iw\underline{Cge} + \underline{Kge})\underline{\chi e}$$

$\underline{\chi s}$, $\underline{\chi g}$ and $\underline{\chi e}$ in Eq.4 and followings are Fourier's transformation of $\underline{Xs}$, $\underline{Xg}$ and $\underline{Xe}$, respectively, and i is imaginary unit.

The third term of left hand side of Eq.4 is the Fourier's transformation of dynamic stiffness matrix of the structure. Expanding inverse matrix, $A(w)^{-1}$, it follows that,

$$\begin{aligned}\underline{A(w)}^{-1} &= \underline{Kgg}^{-1} - iw\underline{A(w)}^{-1}(\underline{Cgg} + iw\underline{Mgg})\,\underline{Kgg}^{-1} \\ &= \underline{Kgg}^{-1} + w^2\underline{Kgg}^{-1}\,\underline{Mgg}\,\underline{Kgg}^{-1} - iw\underline{A(w)}^{-1}(\underline{Cgg} + \\ &\quad w^2\underline{Cgg}\,\underline{Kgg}^{-1}\,\underline{Mgg} + iw^3\underline{Mgg}\,\underline{Kgg}^{-1}\,\underline{Mgg})\,\underline{Kgg}^{-1}\end{aligned} \qquad (5)$$

Substituting into Eq. 4 and then performing reverse transformation of Fourier's expression, the equations of motion for the structural part could be obtained

$$(\underline{Mss}^{(1)} + \overline{\underline{M}})\,\ddot{\underline{Xs}} + (\underline{Css}^{(1)} + \overline{\underline{C}})\,\dot{\underline{Xs}} + (\underline{Kss}^{(1)} + \overline{\underline{K}})\,Xs = \underline{Ksg}\,\underline{Xgo} \qquad (6)$$

in which

$\overline{\underline{K}} = \underline{Kss}^{(2)} - \underline{Ksg}\,\underline{Kgg}^{-1}\,\underline{Kgs}$ -------- equivalent stiffness matrix

$\overline{\underline{C}} = \underline{Css}^{(2)} + \underline{Ksg}\,\underline{Kgg}^{-1}\,\underline{Css^{*}}\underline{Kgg}^{-1}\underline{Kgs}$ - - -equivalent damping matrix

$\overline{\underline{M}} = \underline{Mss}^{(2)} + \underline{Ksg}\,\underline{Kgg}^{-1}\underline{Mgg}\,\underline{Kgg}^{-1}\underline{Kgs}$ - - -equivalent mass matrix

Xgo = Displacements of the ground fixed at the boundary with the structure and subjected to seismic waves at the supports.

$\underline{Kss}$, $\underline{Css}$ and $\underline{Mss}$ are divided into two parts in Eq.6, namely, the one concerned with the structural part itself ($Kss^{(1)}$, $Css^{(1)}$ and $Mss^{(1)}$) and the other with the ground at the boundary ($Kss^{(2)}$, $Css^{(2)}$ and $Mss^{(2)}$). And Cgg^{*} is a damping coefficient matrix equivalent with the damping forces obtained from the reverse Fourier's transformation of

$$-iw\underline{A(w)}^{-1}\,(\underline{Cgg} + w^2\underline{Cgg}\,\underline{Kgg}^{-1}\,\underline{Mgg} + iw^3\underline{Mgg}\,\underline{Kgg}^{-1}\,\underline{Mgg})\chi \qquad (7)$$

ALTERNATE EXPRESSION OF EQUIVALENT MATRICES

The second one of Eq.3 represents equations of motion for the ground fixed at the boundary with the structure and subjected to seismic waves at the supports, if the first term of the right hand side, $\underline{Kgs}\,\underline{Xs}$, is ignored. And the solution of this equation is $\underline{Xgo}$.

Fig.2 shows this system. If the diagonal matrix of natural circular frequencies and the mode matrix are denoted as $\underline{P}$ and $\underline{\Phi}$, there exist next relations among them and the mass damping and stiffness matrix.

$$\underline{\Phi}^T \underline{Mgg}\, \underline{\Phi} = \underline{E} \quad , \quad \underline{\Phi}^T \underline{Cgg}\, \underline{\Phi} = 2\underline{hP} \tag{8}$$

$$\underline{\Phi}^T \underline{Kgg}\, \underline{\Phi} = \underline{P}^2 (\text{or } Kgg^{-1} = \underline{\Phi}\, \underline{P}^{-2}\, \underline{\Phi}^T)$$

Using this relations, Eq.5 may be expressed as follows

$$\underline{A(w)}^{-1} = \underline{\Phi} \left\{ \underline{P}^{-2} - iw\underline{B(w)}^{-1} (2\underline{hP} + iw\underline{E})\ \underline{P}^{-2} \right\} \underline{\Phi}^T$$

$$= \underline{\Phi} \left\{ \underline{P}^{-2} + w^2 \underline{P}^{-4} - iw\underline{B(w)}^{-1} (2\underline{hP}^3 + w^2 2\underline{hP} + iw^3 \underline{E})\ \underline{P}^{-4} \right\} \underline{\Phi}^T \tag{9}$$

in which $\underline{B}(w) = -w^2\underline{E} + iw2\underline{hP} + \underline{P}^2$

And finally, the following alternate expressions for equivalent matrices are obtained.

$$\overline{\underline{K}} = \underline{Kss}^{(2)} - \underline{Ksg}\ \underline{\Phi}\ \underline{P}^{-2}\ \underline{\Phi}^T\ \underline{Kgs}$$

$$\overline{\underline{C}} = \underline{Css}^{(2)} + \underline{Ksg}\ \underline{\Phi}\ 2\underline{h}^*\underline{P}^{-3}\underline{\Phi}^T\ \underline{Kgs} \tag{10}$$

$$\overline{\underline{M}} = \underline{Mss}^{(2)} + \underline{Ksg}\ \underline{\Phi}\ \underline{P}^{-4}\ \underline{\Phi}^T\ \underline{Kgs}$$

in which $2\underline{h}^*\underline{P}$ is a diagonal damping coefficient matrix equivalent with the damping forces obtained from the reverse Fourier's transformation of

$$-iw\ \underline{B(w)}^{-1}\ (2\underline{hP}^3 + w^2 2\underline{hP} + iw^3\underline{E})\ \underline{P}^{-2} (Mgg\, \Phi^T \chi)$$

If the natural frequencies and the modes of the system shown in Fig.2 are obtained from one of the available methods, equivalent matrices will be calculated by Eq.10. The dominant periods of the ground may be considered to serve approximately for the natural frequencies of this system. The vector, $\underline{Xgo}$, in Eq.6 is the absolute nodal displacements of the system of Fig.2 subjected to seismic waves. Among the elements of $\underline{Xgo}$, only the displacements near the boundary with the structure are needed for the calculation of the term, $\underline{Ksg}\ \underline{Xgo}$. The observed seismic records near the foundation of the structure may be used for the approximate nodal displacements, $\underline{Xgo}$. If the equivalent matrices and the ground displacement are obtained, the response of the structure will be calculated by Eq.6, independently from the ground.

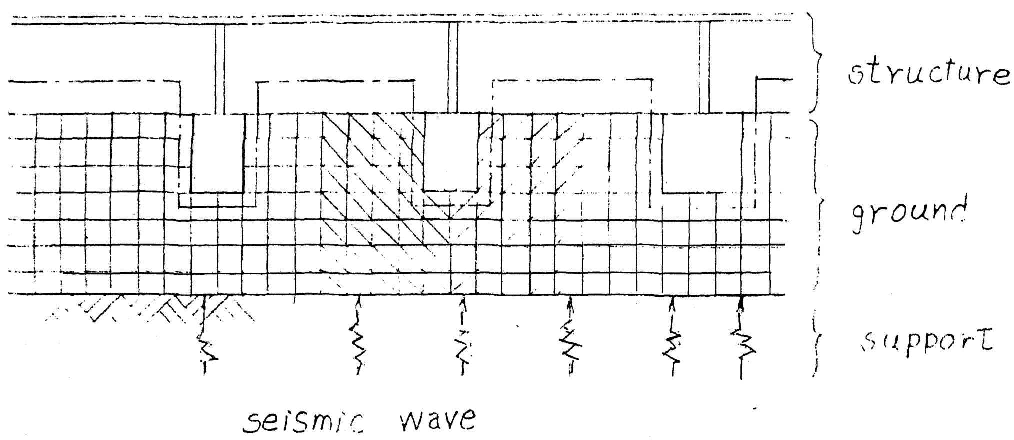
structure
ground
support
seismic wave

Fig 1

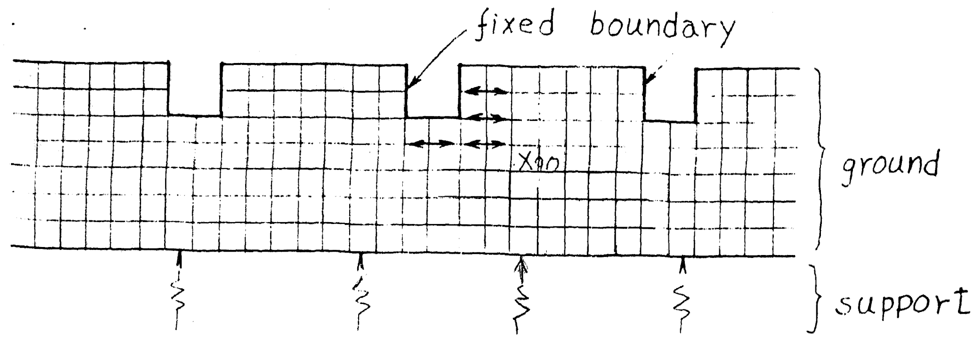
fixed boundary
ground
support

Fig. 2