DYNAMICS OF ELASTIC AND YIELDING STRUCTURE-FOUNDATION SYSTEMS

by

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SYNOPSIS

Response spectra are presented for the maximum deformations of simple elastic and yielding structures which are supported at the surface of an elastic halfspace and are excited by horizontal ground motions. By comparing these spectra with those computed for similarly excited, rigidly mounted structures, the conditions of occurrence of major foundation-structure interaction are identified, and the effects of the most important parameters are assessed.

SYSTEM AND METHOD OF ANALYSIS

The structure-foundation system considered is modeled as a mass connected by a spring and a dashpot to a rigid weightless disk which, in turn, is supported at the surface of a homogeneous, perfectly elastic halfspace. This system may be viewed either as the direct model of a flexibly mounted one-story building frame with relative viscous damping, or more generally, as the one-mode approximation of a similarly supported multi-story, multi-mode structure.

The base excitation is specified by the free-field motion of the ground, and there is assumed to be only a horizontal component of excitation with displacement $y(t)$, velocity $\dot{y}(t)$, and acceleration $\ddot{y}(t)$. The characteristics of this motion are presumed to be independent of the soil properties. The input motions investigated include the half-cycle displacement pulse shown in the inset diagram of Fig. 1a, and the first 6.29 secs of the N-S component of the 1940 El Centro earthquake record, for which the traces of $y(t)$, $\dot{y}(t)$ and $\ddot{y}(t)$ are given in Ref. 3.

For the results presented herein, Poisson's ratio for the halfspace is taken as $\nu = 0.45$; the damping factor for the structure in its fixed-base condition is taken as $\zeta = 0.05$; and the mass ratio, $\delta = m/(\rho \pi r^2 h)$, is taken as 0.15. In the latter expression, m = the mass of the superstructure, assumed to be uniformly distributed over a circular area of radius r; r = the radius of the massless disk or foundation mat; ρ = the mass density of the soil; and h = the height of the structure.

The response of the system was computed by numerical integration of the governing equations of motion, making use of the impulse response functions for the foundation. These functions take due account of the frequency dependence of the foundation compliance. The details of the method are given in Ref. 3.

PRESENTATION AND ANALYSIS OF RESULTS

Spectra for Elastic Systems. In Fig. 1a are presented response spectra for the maximum deformation, u_m, for a family of elastic structures excited by the half-cycle displacement pulse. The value of $h/r = 1$ considered is representative of short structures. Each curve refers to a fixed value of the dimensionless wave

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parameter, $\sigma = c_s/(fh)$, where c_s = the speed of propagation of shear waves in the supporting medium, and $f = p/(2\pi)$ = the fixed-base natural frequency of the system in cps (hertz). This parameter, which may be viewed as a measure of the relative stiffness of the foundation and the structure, is the most important single parameter of the problem.

The results are plotted in the familiar tripartite form, with all scales non-dimensionalized, where the subscripts o on y, $\dot{y}$, and $\ddot{y}$ identify the maximum values of these quantities. The symbols a, b, c and d on the abscissa identify the boundaries of the various frequency regions of the spectrum, as defined in Ref. 1. The range ab corresponds to the moderately-low-frequency region, bc to the medium-frequency region, and cd to the moderately-high-frequency region. The widths of these regions are different for different inputs.

It is clear from this figure that the response of a flexibly mounted structure may be significantly different from the response of the corresponding fixed-base structure, $\sigma = \infty$, the difference being a function both of σ and of the spectral frequency region involved. The smaller the value of σ, i.e. the softer the foundation in comparison to the structure, the more pronounced is the interaction effect. A reduction in σ is associated with a decrease in u_m in all but the moderately-high-frequency regions of the spectrum. For a fixed value of σ, the interaction effect is greatest in and near the medium-frequency region, and decreases in importance as the low-frequency and high-frequency limits are approached.

In Fig. 1b are presented response spectra for the short structures considered in Fig. 1a when excited by the El Centro record. Comparison of these data with those presented in Fig. 1a shows that, within corresponding spectral regions, the relationships between the curves for the interacting systems and the associated fixed-base systems are similar for the two inputs. As a matter of fact, these results provide further confirmation of the conclusion in Ref. 1 that there is good correlation between the response of systems to earthquakes and pulse-type inputs, and that valuable insight into the seismic effects in structures may be gained from analyses of the effects of simpler excitations.

In Figs. 1a and 1b, it is important to note that the natural frequency f appears both on the abscissa and in the expression for σ. Since the spectra in these figures are for fixed values of σ and h/r, the ratio c_s/r which characterizes the foundation properties varies along each curve unless f is held constant. For the pulse-type input, it is therefore desirable to interpret ft_1 as a measure of the characteristic pulse duration, t_1, rather than as a measure of f. On the other hand, for the earthquake input, for which a similar interpretation is not as useful, it is more instructive to plot the spectra for fixed values of c_s/r rather than for fixed values of σ. Such plots are given in Figs. 2a and 2b for the El Centro record for two values of h/r, of which the larger is representative of tall, slender structures.

It is apparent from these plots that, for systems having the same foundation conditions, the interaction effect is most important for stiff, high-frequency structures, and practically negligible for flexible, low-frequency structures. Furthermore, the effect is significantly greater for tall structures than for short structures;

this is due to the increased rocking motion of the foundation of tall structures. For the short structures in Fig. 2a, the interaction effect always reduces the magnitudes of the peak deformations, u_m. By contrast, the deformations of the interacting tall structures in Fig. 2b are greater or smaller than those of the corresponding fixed-base structures depending on the values of f and c_s/r involved.

Some insight into the factors responsible for these trends may be gained from the inset diagrams of Figs. 2, which give the natural frequency, f*, and the damping factor, ζ^*, of the "equivalent" single-degree-of-freedom oscillator defined in Ref. 2. Over wide ranges of the parameters defining the problem, the maximum deformation of this oscillator may, for most practical purposes, be considered to be the same as that of the actual system. Note that the ratio f*/f decreases with decreasing σ, and that a reduction in σ is associated with an increase in ζ^* for short structures, and with a decrease in ζ^* for tall structures. The negligible effect of interaction observed in the lower frequency regions of the spectra in Figs. 2 is due to the fact that, for the values of c_s/r considered, the values of f* and ζ^* are practically equal to those of the fixed base structures.

Spectra for Yielding Structures. It is known (Ref. 1) that the principal effect of inelastic action is to reduce the apparent frequency of vibration of the system. Considering that a reduction in f is associated with an increase in σ, and that such an increase generally reduces the interaction effect, it may be expected that the interaction effect for yielding structures will not be as important as for elastic structures and that the effect will decrease with an increase in the degree of yielding.

These predictions are confirmed by the response spectra presented in Figs. 3a and 3b, which refer to elastoplastic structures subjected to the half-cycle displacement pulse. The vertical and diagonal scales in these figures are measures of the yield deformation of the system, u_y, rather than of the maximum deformation, u_m, and each curve refers to fixed values of σ, h/r, and of the ductility ratio, $\mu = u_m/u_y$. The quantities p and f are those associated with the low-amplitude motions of the fixed-base structures.

For the inelastic structures considered in Figs. 3, the differences between the spectra for the coupled systems and the associated fixed-base systems are significantly smaller than those for the elastic structures in Fig. 1a, the differences decreasing with increasing μ. It follows that consideration of the interaction effect is indeed not as important in the design of yielding structures as it is in the design of elastic structures. Similar results have also been obtained for the El Centro record.

REFERENCES AND ACKNOWLEDGMENT

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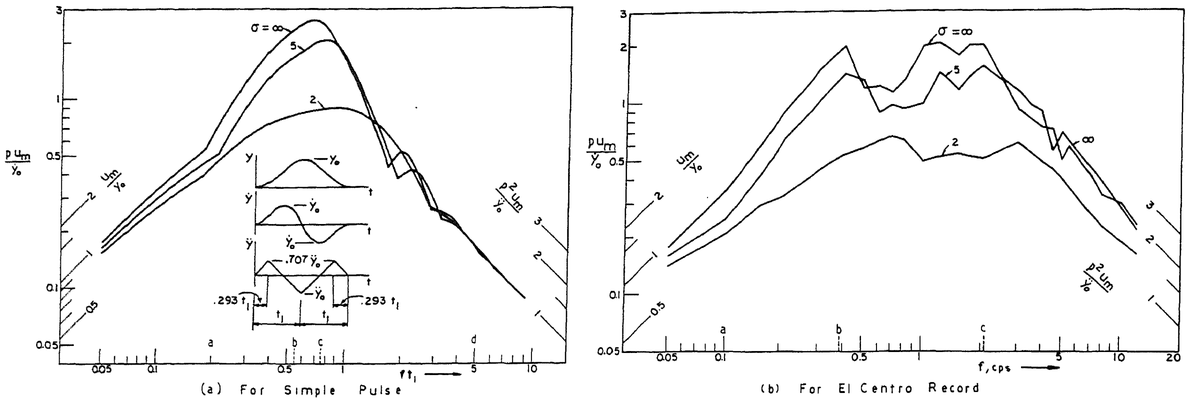
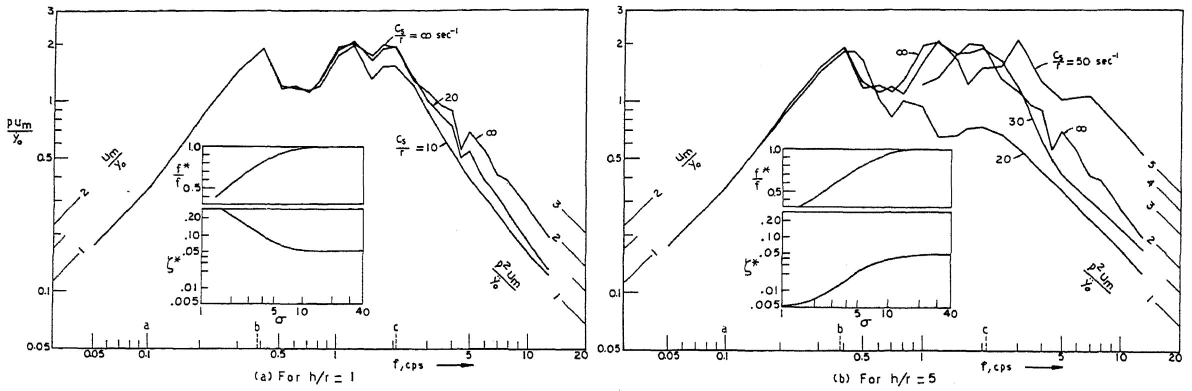


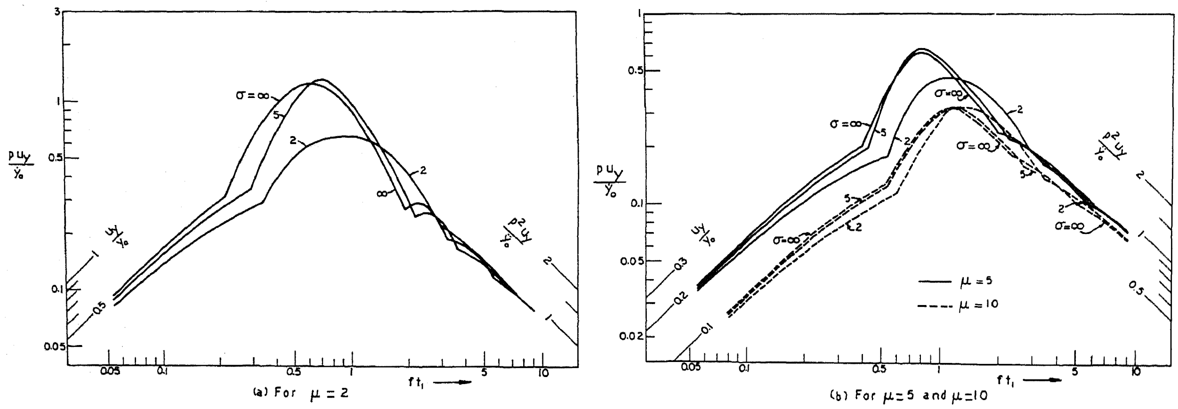
(a) For Simple Pulse (b) For El Centro Record

FIG. I RESPONSE SPECTRA FOR ELASTIC STRUCTURES WITH FIXED σ AND h/r = 1



(a) For h/r = 1 (b) For h/r = 5

FIG. 2 RESPONSE SPECTRA FOR ELASTIC STRUCTURES WITH FIXED $\frac{c_s}{r}$; EL CENTRO RECORD



(a) For μ = 2 (b) For μ = 5 and μ = 10

FIG. 3 RESPONSE SPECTRA FOR ELASTOPLASTIC STRUCTURES WITH h/r = 1 ; SIMPLE PULSE