THE DYNAMIC RESPONSE OF PILE FOUNDATIONS TO LATERAL FORCES

by

I. Alpan[I]

SYNOPSIS

A theoretical approach to compute the lateral spring constant of pile foundations is presented and a method is proposed for evaluating the requisite soil parameters.

The experimental programme, to be described in extenso in a future publication, included the dynamic testing of a prestressed R. C. pile, fixed at its top against rotation, by inducing free oscillations from the records of which the damped natural frequency and the logarithmic decrement were obtained. In addition, slow lateral load tests with inclinometer measurements along the pile axis were conducted in order to enable the comparison of the measured dynamic response with that predicted on the basis of calculations utilizing the soil deformation parameters obtained from in-situ tests.

The soil exploration, carried out quite near the pile location, included pressiometer tests which furnished representative values of the horizontal elastic soil modulus with depth.

The conclusions, drawn from the field test data and their analysis were:

a. Free oscillation tests of piles in-situ are likely to furnish useful data for the evaluation of their dynamic response in general;
b. In cohesive soils the elastic modulus, as obtained with the pressiometer technique, may be used with reasonable confidence in the analysis of dynamic problems;
c. The theory of beams on elastic supports (the so-called Winkler model) appears to be sufficiently valid to be used in the dynamic analysis of laterally loaded piles subject to obvious displacement restrictions.

INTRODUCTION

The investigation, discussed in this paper, was carried out in connection with a project which involved the evaluation of the seismic response of a typical structure already built and forming part of a large industrial complex. The structure in question could be idealized as a closed box-like arrangement of frames with appropriately placed concentrated masses, supported by an array of vertical piles. The pile heads being thus connected by a horizontal frame of considerable stiffness, it is obvious that any horizontal movement of the box would produce a lateral displacement of fixed pile heads, that is with their rotation practically eliminated.

[I] Professor of Civil Engineering, Israel Institute of Technology, Haifa.

Now it is a well-established fact in engineering seismology that the degree to which a structure is likely to be affected when exposed to a horizontal disturbance is considerably influenced by the natural period of the structure (spectrum concept). More specifically, the shorter the period (concomitant with increased lateral stiffness of the structure) the greater the effect of the seismic disturbance as expressed by a higher value of the so-called seismic coefficient, widely used in Earthquake-Resistance design codes.

An excellent illustration of the trend mentioned has been furnished by Merritt and Housner (1954), as shown in Fig. 1, from which it is evident that as the foundation compliance decreases (i.e. with increasing stiffness of coupling) so does the natural period of the structure, T, approaching a value T_o corresponding to perfect fixity at the foundation level (ideal cantilever). One of the curves in the figure shows the variation of the period ratio for multi-story experimental models (5, 10 and 15 stories) whereas the other illustrates the case of a simple, one-degree-of-freedom model.

Returning to our structure, it is evident that an essential part of the investigation would have to be concerned with the determination of the lateral compliance or, conversely, of the lateral stiffness of typical piles fixed at their top against rotation.

The analytical model, selected to represent most satisfactorily the prevailing conditions, was essentially that of a beam on elastic supports as applied to our problem by Reese and Matlock (1956). We propose discussing the model in greater detail in the following section.

In order to check the validity of this selection, a fairly comprehensive field testing programme was carried out on a typical pile placed in soil strata known to approximate closely those below the structure under investigation. The basic data were furnished by the response of the pile to appropriate static and dynamic stimuli as well as by a suitably planned subsoil exploration. The discussion of the programme will, similarly, be presented in reasonable detail further on.

THE ANALYTICAL PILE MODEL

Consider an almost completely embedded pile subjected at its head to an increasing horizontal load which evidently produces increasing lateral deflections. Initially, the load-deflection relation is sensibly linear until, beyond a certain yield load, an increasingly significant deviation from linearity is registered.

The first loading stage may be termed elastic whereas the non-linear stage exhibits the departure of the stress-strain characteristics of the supporting soil from linearity. The test data by Mori (1964) on steel pipes, as shown in Fig. 2, are typical of the trend discussed above. In this particular case the soil was assumed to behave as an elastoplastic material, an often reasonable assumption particularly with fairly stiff cohesive soils.

However, the point to note is that, below certain load levels, the horizontal load-deflection relation may be taken as linear and this condition we assume to prevail in our case. No doubt, an extension of the analysis, including non-linear soil behaviour, would enhance the model's realism. Recent work in this direction has been published, e.g. by Siva Reddy and Valsangkar (1970) and by Madhav et al. (1971). All the same, it appears justified in our case to forgo the refinement involved in the use of an elasto-plastic soil model and limit ourselves to a linear medium. We find this view supported in notable contributions to the extensive literature on the subject such as the comprehensive papers by Matlock and Reese (1962) and Broms (1965) and, in particular, reports on experimental work such as have been published by Brandes (1953) and by Gaul (1958).

As already indicated, we shall use the model proposed by Reese and Matlock (1956) suitably adapted to our conditions. Their analysis includes the case of a pile fixed at the top, however, as far as the soil characteristics are concerned, the elastic modulus is assumed to be a linear function of the depth, z, below ground level:

$$E_s = fz \tag{1}$$

with f a subgrade constant. For cases such as the one of interest to us, in which the soil modulus must be assumed constant with depth, Reese and Matlock (1956) suggest an iteration procedure aimed at adapting the use of the constant f also for these cases. However, this procedure, which turns out to be not very convenient, can be obviated by means of a short-cut method which will be applied in the following to the case of a fixed head pile.

Let L be the embedded pile length, E the elastic modulus of the pile material, I the moment of inertia of the pile cross section (relative to the appropriate axis) and E_s the elastic modulus of the supporting soil. We now define a function

$$\psi(Z_{max}) = 2\frac{E_s L^4}{EI} \tag{2}$$

in which Z_{max} may be viewed as a dimensionless reference parameter (actually representing the ratio of the embedded pile length and the system stiffness). Having computed $\psi(Z_{max})$, we find Z_{max} from the curve shown in Fig. 3.

Let now the pile head be acted upon by a horizontal force, P_H, and let the resulting deflection be denoted by δ_H. We define the horizontal spring constant of the pile given as:

$$k_H = \frac{P_H}{\delta_H} = \varkappa \frac{EI}{T^3} \tag{3}$$

where

$$T = \frac{L}{Z_{max}} \tag{4}$$

Having determined Z_{max} and thus also T, we use Fig. 4 to obtain $\mathcal{X}$ and are now able to compute k_H from Eq. 3.

The theoretical deflection curve of a horizontally loaded pile has approximately the shape of a strongly attenuated wave of which the upper quarter wave length exhibits considerably larger deflections than at greater depth. For this reason it is often useful to be able to determine the depth at which the first deflection zero occurs. The curve shown in Fig. 5 enables this depth to be found, again as a function of the stiffness parameter Z_{max}.

In conclusion, we note that in the case of a pile group, assumed to act together, the overall stiffness or the collective spring constant may be approximated by virtue of the superposition principle as

$$k_{nH} = \sum_{1}^{n} k_{iH} \tag{5}$$

where n is the number of piles in the group. This approximation is on the safe side in the context of our problem and should be quite realistic provided the distance between individual pile is not less than, say, four pile diameters.

THE FIELD TESTS

The field tests were carried out on a prestressed R. C. pile of 30 by 30 cm in cross section and an embedded length of 4. 7 m. The upper part of the pile was encased in a rigid R. C. block which was provided with two knife pivots meant to ensure that, as far as possible, no rotation of the pile head take place during its horizontal motion. The arrangement is shown schematically in Fig. 6. As it turned out, rotation could not be completely prevented; nevertheless an appreciable degree of fixity (about 80%) was ensured.

The subsurface exploration programme, carried out quite near the pile location, revealed the upper 3. 5 m of the subsoil to consist of a highly plastic clay having an average undisturbed shear strength (determined by in-situ vane tests) ranging between 10 and 15 t/sq. m. Pressiometer tests showed this clay layer to have an elastic modulus, E_s, varying from 1300 to 1800 t/sq. m. Below the depth of about 3. 5 m fine sand layers were discovered. In view of our earlier remarks, the sand was unlikely to contribute much lateral resistance to the pile displacement lying, as it did, below the first zero deflection point.

In addition to appropriately placed deflectometers, enabling the measurement of the displacements of the head block, two accelerometers were mounted on the block which registered its horizontal motions following dynamic excitation. Inclinometer readings along the axis of the pile furnished the data required to compute the variation of the pile deflection with depth for static load applications.

Two types of tests were carried out on the pile:

a. Conventional horizontal load tests but including, as already mentioned, the measurement of the axial deflection curve and

b. Dynamic free vibration tests which furnished the damped natural frequency of the pile in horizontal oscillation as well as the logarithmic decrement (with respect to time) as a measure of the damping of the system.

The procedure of inducing free horizontal oscillation was, briefly, as follows: a cable with a calibrated shear pin was attached to the head block to which it transmitted a horizontal pull whose maximum evidently equalled the failure load of the pin. Upon the sudden failure of the latter the released pile head executed a number of steadily decreasing horizontal oscillations around its equilibrium position which were registered through the accelerometers as traces typically shown in Fig. 7.

TEST RESULTS AND EVALUATION

The dynamic tests furnished the damped natural frequency of the fixed pile as f_{nd} = 12.2 Hz (i.e. a period of T_{nd} = 0.082 s.). The logarithmic decrement, Δ, obtained from the registered attenuated response curves such as the one shown in Fig. 7, was found to be

$$\Delta = \ln \frac{A_n}{A_{n+1}} = 0.338 \qquad (6)$$

where A_n and A_{n+1} are two consecutive amplitudes occuring at the time interval T_{nd}. The damping ratio, D, representing the damping of the system relative to critical damping, was computed from the logarithmic decrement as

$$D = \left[\Delta^2/(4\pi^2 + \Delta^2)\right]^{1/2} = 0.054 \qquad (7)$$

i.e. the system exhibited 5.4% of critical damping, quite a small value which justifies neglecting the difference between the damped and the undamped natural frequencies.

The linear portion of the static loading curves furnished an average spring constant (cf. Eq. 3) of k_{HS} = 1300 t/m. As to the vibrating mass of our system, it consisted of the head block and essentially the previously mentioned upper quarter wave length of the pile. Taking into consideration that the upper portion of the pile vibrates with amplitudes decreasing with depth (and vanishing at the first deflection zero), the pile mass effectively participating in the vibration was computed as about 0.2 of the total pile mass. Thus the total vibrating mass was found to be M = 0.16 ts^2/m and hence the undamped natural frequency of the system is given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{HS}}{M}} = 14.3 \text{ Hz} \qquad (8)$$

which compares satisfactorily with the measured value of f_{nd}.

We shall now attempt to predict the natural frequency from the pressiometer data used with the method of Reese and Matlock (1956) for a fixed head pile as previously explained. Taking as a representative soil modulus value E_s = 1500 t/sq.m. and for the pile concrete the value of E = 3.8 (10^6) t/sq.m (based on laboratory tests on cored specimens) and, further, with L = 4.7 m and I = 6.75 x 10^{-4} m^4, we find: $\psi(Z_{max})$ = 570 (Eq. 2). Hence Z_{max} = 3.7 (Fig. 3) and thus T = 1.27 m (Eq. 4). Finally with $\varkappa$ = 1.03 (Fig. 4), we compute the horizontal spring constant in accordance with Eq. 3 obtaining k_H = 1290 t/m. We arrive thus at a natural frequency practically identical with that given by Eq. 8.

From Fig. 5 we obtain z_o/L = 0.75 and thus the depth of the first deflection zero as z_o = 3.5 m, that is at the bottom of the clay layer as previously mentioned. This result compares extremely well with the depth of the deflection zero as determined experimentally with the inclinometer.

CONCLUSIONS

Although our inferences from the work described in this paper are formulated in fairly general terms, it must be borne in mind that much of their empirical substantiation rests on our case history of an investigation which, however methodically planned, was of rather limited scope when viewed against the quite complex background of our problem. This stated, we shall present our conclusions as being, in our opinion, of reasonably general applicability and offering, within their limitations, useful guidelines for prediction in cases where, for one reason or another, in-situ measurement of dynamic response (always preferable in principle) turns out to be impracticable.

We recall that our problem is to determine, as realistically as possible, the horizontal spring constant of piles. Furthermore, our discussion deals specifically with almost completely embedded piles, fixed at their top against rotation. However, a study of, e.g., the relevent references mentioned earlier, should easily provide the tools necessary to perform entirely analogous analyses of free-end and partly embedded piles including other types of soil modulus functions, to which cases our comments are still pertinent in the main.

The following remarks are, then, offered in conclusion:

1. In cohesive and not too soft soils an analysis based on elastic subgrade theory may be used with reasonable confidence. The method developed by Reese and Matlock (1956) has proved quite satisfactory in my experience.

2. As a tentative and conservative criterion for the use of the linear theory for typical fixed-head piles, an upper limit for the anticipated horizontal load is suggested as follows

$$P_H \leq 3Z_{max} \qquad (9)$$

3. In cohesive soils of reasonable stiffness pressiometer data (although obtained under slow pressure application) may be considered to furnish reliable values of the horizontal elastic modulus of the supporting soil to be used in dynamic analyses. Our recommendation, as explained in the following paragraph, is conditional on a lower limit of the anticipated dynamic displacements.

4. The linear portion of conventional horizontal load tests on piles similarly furnishes acceptable data for the computation of the elastic spring constant to be used in the prediction of dynamic response. This approach appears justified in those cases where not too small deflections may be expected and has proved itself, in my experience, in connection with machine foundation problems. Where quite small deflections are likely to occur, it would be reasonable to obtain the requisite data from dynamic tests (e.g. wave propagation velocity measurements). We are not in a position, at this stage, to offer a quantitative criterion.

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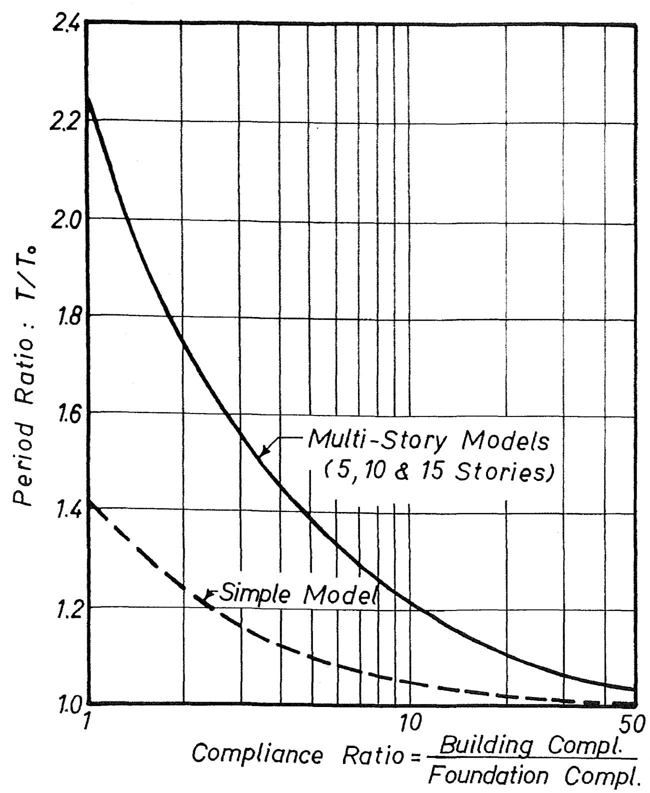

Fig.1. The Influence of Building and Foundation Compliance on the Natural Period of Buildings (after Merrit & Housner, 1954).

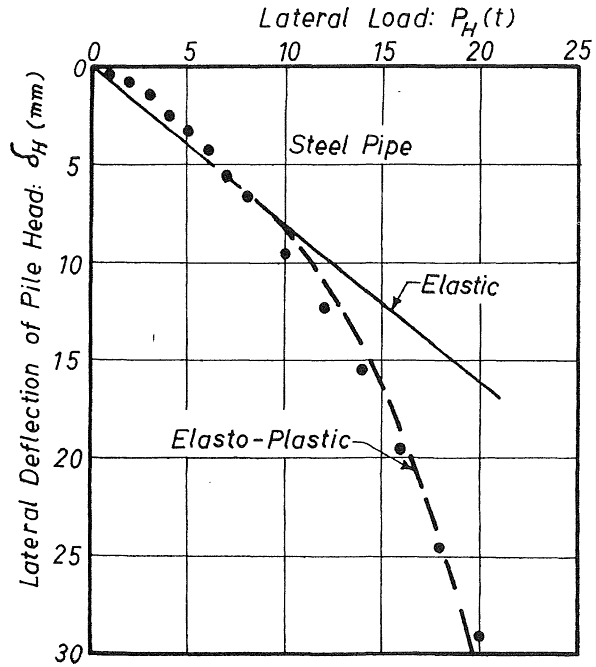

Fig.2. The Elastic and Elasto-Plastic Range in Horizontal Pile Loading Tests (after Mori, 1964).

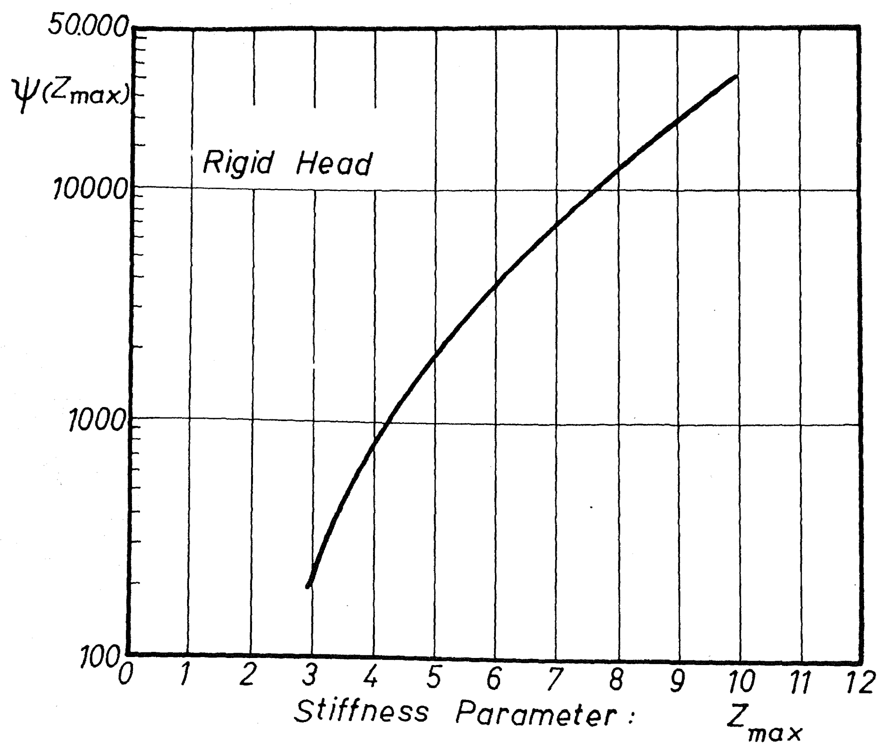

Fig. 3. The Stiffness Parameter Function $\psi(Z_{max})$.

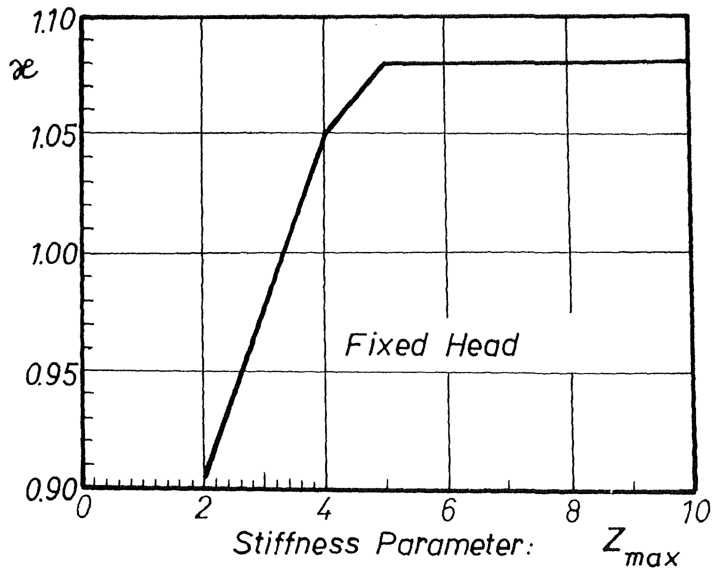

Fig. 4. The Influence of the Stiffness Parameter Z_{max} on the Deflection Coefficient $\varkappa$.

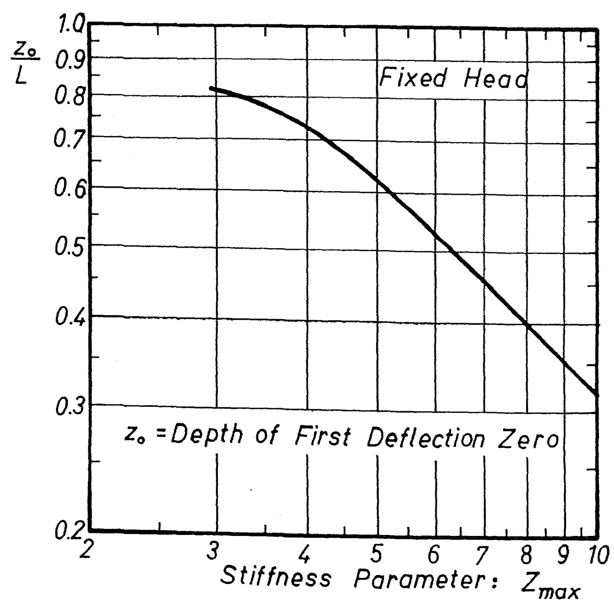

FIG. 5. The Relation between the Depth of the First Deflection Zero and the Stiffness Parameter Z_{max}.

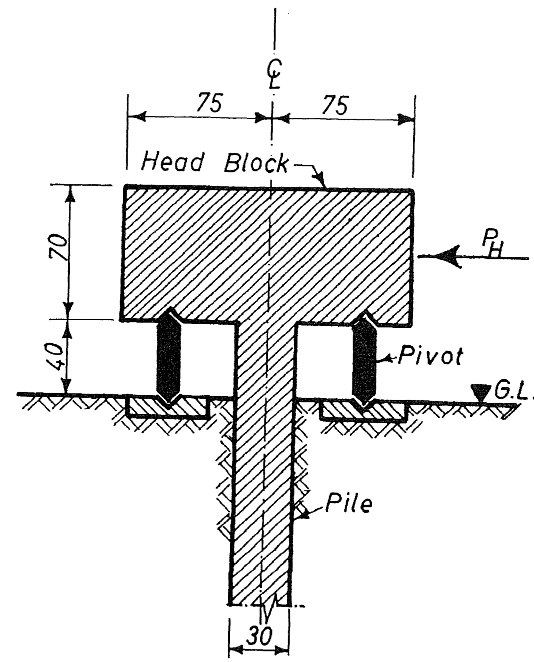

Fig. 6. Arrangement to prevent Pile Head Rotation (schematic).

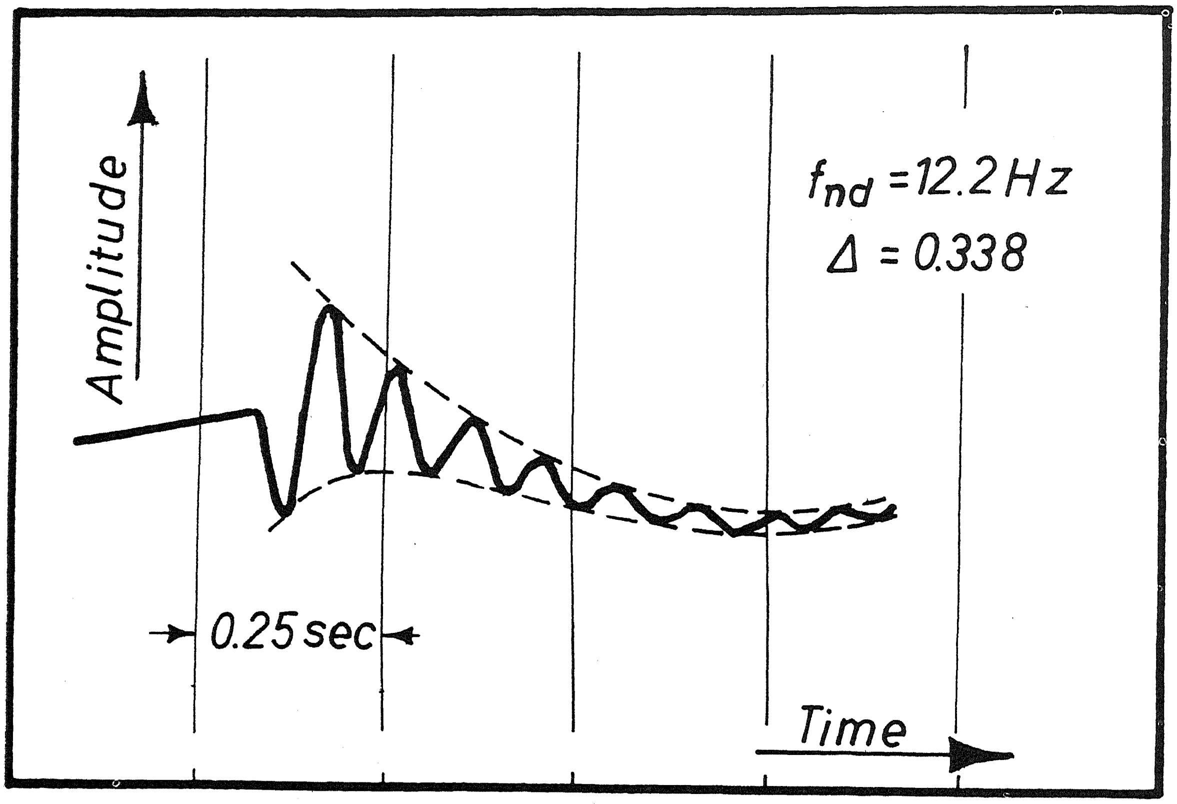

Fig. 7. Damped Free Horizontal Vibration of Fixed Head of Test Pile (Typical Response Curve).