CUMULATIVE DAMAGE EFFECTS IN SEISMIC STRUCTURES

by

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ABSTRACT

Structural failure due to low-cycle fatigue when structural components operate in the plastic range of action of the material is considered. The criteria that cumulative damage effects in plastic zones may be represented by the hysteretic dissipated energy, which has been shown to lead to reasonable predictions of low-cycle fatigue life, is adopted. The objective of the investigation is the development of a relatively simple technique for determination of dissipated energy.

INTRODUCTION

Structural failure due to strong motion earthquakes may occur in either of several failure mechanisms: (a) The exceeding of an assigned level of some stress, deflection or other parameter of the system, and (b) Progressive collapse wherein failure occurs due to some cumulative effect of a number of excursions of some system parameter past an assigned level.

In the present investigation consideration is given to applicability of low-cycle fatigue mechanisms to prediction of failure of simple structural components operating in the plastic range of action of the material. Recent studies by Dowling [1] as well as Matsuishi and Endo [2] have indicated that even very approximate techniques for evaluating cumulative damage effects in plastic hinges based upon hysteretic dissipated energy lead to reasonably accurate predictions of low-cycle fatigue life. This technique was recently applied by Suidan [3] to investigate cumulative fatigue damage in simple single-degree-of-freedom systems.

In fatigue considerations, energy is dissipated because of plastic deformation. Most of the plastic strain energy is converted to thermal energy and dissipated as heat to the surroundings. The plastic strain energy is useful in predicting the accumulation of damage before final failure [4]. Under certain conditions of constant stress amplitude applied to test coupons, plastic strain energy concepts have been applied to predict low-cycle fatigue lives involving at most several thousand reversals of loading [5]. Thus, resistance of a metal to repeated loading may be characterized in terms of its capacity to absorb and dissipate plastic strain energy.

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For structural components subject to seismic excitation it is necessary to consider the accumulation of fatigue damage under the complex loading pattern taking into account all of the irregular cyclic plastic strains during the excitation process. Serensen and Shneiderovitch [6] have done some preliminary work in this direction and evolved the concept of a "damage number." The present investigation offers a relatively simple approximate method for determination of the accumulated plastic strains in such structural components.

ANALYSIS

A straight-forward, albeit tedious procedure for determination of accumulated plastic strains in a structural component subject to seismic excitation is as follows. Artificially generated earthquake accelerograms can readily be formed digitally by the state variable technique described by Franklin [7]. Such accelerograms correspond to a Gaussian stationary process having a power spectral density function given by

$$S_a(\omega) = \frac{S_o\,[1 + 4\xi_g^{\,2}(\frac{\omega}{\omega_g})^2]}{[1-(\frac{\omega}{\omega_g})^2] + 4\xi_g^{\,2}(\frac{\omega}{\omega_g})^2} \tag{1}$$

where S_o is a constant power spectral density, ω_g is a characteristic ground frequency, and ξ_g is a characteristic damping ratio. Responses of a viscously damped linear, single-degree-of-freedom system to these support accelerations may be established by simple numerical integration on a time-dependent basis by digital techniques to yield displacements.

At this point of the investigation it is necessary to specify a specific configuration and the example selected is that of a vertical cantilever bar of rectangular cross-section. This system is approximated by a lumped mass concentrated at the tip of the bar. The time-dependent displacements determined earlier may readily be interpreted in terms of curvatures.

For a bar of rectangular cross-section elementary plastic analysis indicates that the curvature at which yield initiates at the outer fibers is given by $2\sigma_{yp}/Eh$ where σ_{yp} denotes the yield point of the material, E represents Young's modulus, and h is the depth of the cross-section. Thus, it is possible to trace the spread of plastic flow from the outer fibers toward the centroid by considering on the curvature-time record those peak-to-peak changes greater in magnitude than $2\sigma_{yp}/Eh$. These data may also be interpreted in terms of bar lateral displacement instead of curvature. The accumulated plastic effects after any specified time may be determined by having the digital computer collect the significant over-shoots of the above parameter. In the presnet investigation a total of thirty sample functions representing accelerograms were generated, each of 30 seconds duration. The following parameters were employed:

$S_o = 1\ ft^2/sec^3$ (chosen in the interest of simplicity, since only a comparison of the two methods of finding plastic effects is desired), ω_g = 15.6 rad/sec and $\xi_g = 0.6$, which are taken as being representative of firm soil conditions [8]. The mean of the accumulated plastic effects is represented in the figure by the solid line, with the results from various records falling within the regions bounded by the vertical lines. In that figure the abscissa represents percent of critical damping of the structure.

A much more rapid approximate method for determination of accumulated plastic strains is as follows. The motion of the linear, damped single-degree-of-freedom system to support acceleration $\alpha(t)$ is described by the equation

$$m\ddot{x} + \beta\dot{x} + kx = m\alpha(t) \tag{2}$$

If we let

$$\beta/m = 2\xi_o\omega_o \quad ; \quad k/m = \omega_o^2$$

then the above equation becomes

$$\ddot{x} + 2\omega_o\xi_o\dot{x} + \omega_o^2 x = \alpha(t) \tag{3}$$

The spectral density $S(\omega)$ of the displacement of the lumped mass may be found to be

$$S(\omega) = \frac{\phi(\omega)}{(\omega-\omega_o^2)^2 + (2\xi_o\omega_o\omega)^2} \tag{4}$$

where ω_o is the natural frequency of the structure. However, when $\omega_o<\omega_g$ and $0<\xi<0.1$, the displacement may be approximated as a narrow band process having central frequency $\omega = \omega_o$. Thus, the spectral density may be replaced by $\tilde{S}(\omega)$ which may be represented as two rectangles centered respectively at $\pm\ \omega_o$, i.e.

$$\tilde{S}(\omega) = \begin{cases} K \ ; & 0<\omega_a<|\omega|<\omega_b \\ 0 \ ; & \text{otherwise} \end{cases} \tag{5}$$

where

$$K = \frac{\phi(\omega_o)}{4\xi_o\omega_o^4}$$

$$\omega_a = \omega_o - \frac{\pi\xi o}{2}$$

$$\omega_b = \omega_o + \frac{\pi \xi_o}{2}$$

Let $\tilde{x}(t)$ be a narrow band process which has the spectral density $\tilde{S}(\omega)$. Further, the random variable h represents the peak-to-peak distance at any point on the sample function. Let N_{d+} be the average number of positive crossings per second of $\tilde{x}(t)$ at level d and ${}^{H}N_{\rho}$ be the average number of rises per unit time which exceed ρ in height. We state the relation between these quantities as:

$$\underset{\omega_a \to \omega_b}{\text{l.i.m.}} \; {}^{H}N_{2d} = 2N_{d+} \tag{6}$$

where l.i.m. denotes limit in the mean, i.e. as $\omega_a \to \omega_b$, the average number of rises which exceed 2d is equal to twice the average number of positive crossings of $\tilde{x}(t)$ at level d. Space does not permit the inclusion of a proof of (6).

Once a sample function of a narrow band process crosses a given level d, it tends to make several excursions over this level, i.e. the level crossings of $\tilde{x}(t)$ occur in "clumps." The average clump size may be computed from the relation

$$\langle cs \rangle = \frac{N_{d+}}{{}^{A}N_{d+}} \tag{7}$$

where ${}^{A}N_{d+}$ denotes the average number of positive crossings of the envelope A(t) of $\tilde{x}(t)$ at the level d per second. Thus the average sum of overshoots of $\tilde{x}(t)$ above d and below (-d) in one clump becomes

$$E[W_c] = (\bar{h}_{2d} - 2d)\ \{2\langle cs \rangle + \tfrac{1}{4}\} \tag{8}$$

where

$$\bar{h}_{2d} = E[h \mid h \geqslant 2d]$$

i.e. the mean of the excesses of h over 2d,

where the factor $(\bar{h}_{2d} - 2d)/4$ arises because of the first and last peaks of the clump. Since the average number of clumps per second is ${}^{A}N_{d}$ the average sum of overshoots is given by

$$E[W_\rho] = E[W_c]\,{}^{A}N_{d+}$$

$$= (\bar{h}_{2d} - 2d)(2N_{d+} + \tfrac{1}{4}\,{}^{A}N_{d+}) \tag{9}$$

For the narrow band process $\tilde{x}(t)$:

$$\bar{h}_{2d} = (2\sigma_x^2 e^{\frac{d^2}{2\sigma_x^2}})(\sqrt{2\pi} - \int_0^{d/\sigma_x} \gamma^2 e^{-\gamma^2/2} d\gamma)$$

where γ is a dummy variable of integration and

$$N_{d+} = \frac{1}{2\pi} \frac{\sigma_{\dot{x}}}{\sigma_x} \exp(-\frac{d^2}{2\sigma_x^2})$$

$$A_{N_{d+}} = \frac{d\sigma_1}{\sqrt{2\pi}\ \sigma_x^2} \exp(-\frac{d^2}{2\sigma_x^2})$$

$$\sigma_x^2 = 2K(\omega_b - \omega_a)$$

$$\sigma_{\dot{x}}^2 = \frac{2}{3} K(\omega_b^3 - \omega_a^3)$$

$$\sigma_1 = \frac{K}{6}(\omega_b - \omega_a)^3$$

The accumulated plastic strain effects as indicated by Equation (9) using the narrow band approximation are evaluated for the bar under consideration and the results for various values of damping are indicated by the dotted line in the figure. It is evident that the approximation given by (9) offers a relatively simple method for determination of accumulated plastic strain effects which for a given structural shape may be related to energy dissipated which in turn may be employed to offer predictions of low-cycle fatigue behavior.

ACKNOWLEDGMENT

The authors express their thanks to the Air Force Office of Scientific Research for their support of this work under Grant AFOSR 68-1527.

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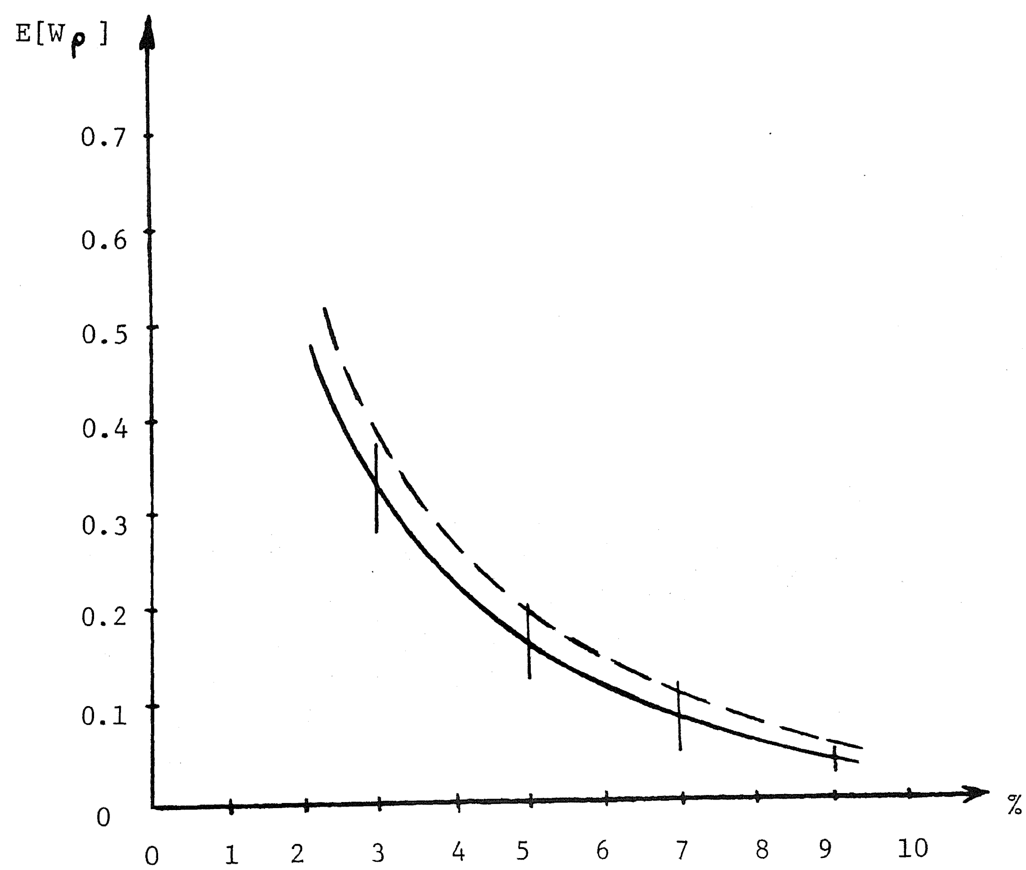

ACCUMULATED PLASTIC STRAIN VS. DAMPING