A COMPARISON OF LINEAR AND EXACT NONLINEAR ANALYSES OF SOIL AMPLIFICATION

by

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SYNOPSIS

The effect of soil properties on the amplification of earthquake motions is usually studied by an iterative viscoelastic analysis. To investigate the validity of this approach direct calculations of soil amplification were made using a Ramberg-Osgood model for the soil properties. The same profiles were also studied by the iterative technique using variations of stiffness and damping derived from the same Ramberg-Osgood model. Comparison of the results of the two techniques shows that the response spectra are quite similar for the range of frequencies of engineering interest. There is, however, evidence that the iterative approach tends to underestimate the displacements and to overestimate the accelerations.

INTRODUCTION

Experimental data show that the stress-strain relationships for soils at the level of strains that might be induced by moderate to strong earthquakes are nonlinear (1). On the other hand, the great majority of the practical methods of analysis for soil amplification of earthquake motions requires the soil to be linearly viscoelastic. The difficulty of simulating nonlinear behavior with a linear analysis has been overcome traditionally by obtaining the secant moduli and damping ratios of the soil as functions of the strain level in a cyclic loading test. The linear viscoelastic analyses are then carried out iteratively, values of modulus and damping ratio being changed in successive cycles until they correspond to the levels of strain computed.

Obviously numerous questions arise as to the validity of this approach. While the process appears to converge for most practical problems, it is not clear how the characteristic strain that defines the value of the modulus and damping is to be chosen for a transient motion rather than a harmonic steady state condition. One common procedure is to use between 60 and 70% of the maximum strain. A more fundamental question is how closely the motion calculated by the iterative procedure resembles the motion predicted by an exact nonlinear analysis.

In order to investigate some of these questions the authors undertook to examine the behavior of a particular nonlinear model for soil behavior. The soil was modelled as a series of lumped masses, springs, and dashpots. In one set of analyses the springs were defined by the nonlinear Ramberg-

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Osgood relationship described in the following section, and the solution was carried out by direct numerical integration in the time domain. In the other set of analyses the iterative linear viscoelastic procedure was followed, the spring and dashpot constants being related to the strain levels directly from the Ramberg-Osgood formula. Thus, the same fundamental stress-strain relation obtained for both sets of results; the differences lay in the way the solutions were computed.

SELECTION OF PARAMETERS FOR THE RAMBERG-OSGOOD MODEL

The constitutive relations of soil are too complicated to be fully described by a single equation. Several nonlinear models, from elasto-plastic to multilinear, hyperbolic, or Ramberg-Osgood relationships, have been often used, however, to approximate the stress-strain behavior of soil. For the purposes of this study, since the same characteristics will be used for both sets of analysis, the exact reproduction of any particular soil is not important. It is, however, desirable to have overall variations for moduli and damping ratios, as functions of strain, similar to those of most soils.

Ramberg-Osgood models have been extensively studied by Jennings (3). The corresponding load deflection relationship is of the form

$$\frac{x-x_i}{cx_y} = \frac{p-p_i}{cp_y} + \alpha\left(\frac{p-p_i}{cp_y}\right)^r$$

where

- x is the shear distortion of the spring
- p is the force developed in the spring
- x_y is a shear distortion constant, characteristic of the spring
- p_y is a force constant, characteristic of the spring
- $x_i p_i$ are the distortion and force representing the most recent point at which there was a load reversal
- α is a positive constant, characteristic of the spring
- r is a positive number, odd and integer in the original formulation. With proper handling of the sign of $p-p_i/cp_y$ it can be however any positive real number.
- c = 1 for virgin loading path
  = 2 for unloading or reloading.

  c has again a value of 1 if during unloading or reloading the virgin curve is reached.

Fig. 1 shows the general form of the load-deflection paths as defined by the above equation. With proper selection of the parameters α and r, a wide variety of physical behaviors can be reproduced, from that of a linear elastic to that of an elastic - perfectly plastic material. For a cyclic loading with a fixed amplitude of strain, a unique hysteresis loop is defined and a value of secant modulus and damping ratio can be obtained as illustrated in Fig. 2.

Figs. 3 and 5 show ranges of variation of modulus and damping as functions of strain, which have been suggested as representative of typical soils (2). These ranges are indicative of the amount of scatter in experimental data. Similar curves were theoretically derived for Ramberg-Osgood systems with different values of α and r. Although a perfect match with any of the bounding curves, or an average curve, was not possible, it was found that values of α of the order of 0.05 and values of r from 2 to 2.5 provided reasonable agreement in the overall trends. Values of $\alpha = 0.05$ and $r = 2$ were selected for the study and the corresponding modulus and damping curves are shown in Figs. 4 and 6.

ORGANIZATION OF THE STUDY

The "initial natural period" of a system was defined as that period derived from the initial stiffness of the Ramberg-Osgood relation. A set of single degree of freedom systems, with initial natural periods of 0.25, 0.5, 1 and 2 seconds and Ramberg-Osgood relationships with $r = 2$ and $\alpha = 0.05$, were first analyzed under a base motion corresponding to the N69W component of the Taft record of the 1952 Kern County earthquake, scaled to different intensities. For each system, and each intensity of motion, the dynamic response was computed by direct integration in the time domain of the nonlinear equations of motion, using the fourth order Runge-Kutta method. The analysis was then repeated using the iterative procedure, assuming at each cycle a linear model with stiffness and damping coefficients obtained from the curves of Figs. 5 and 6, and a characteristic strain equal to 2/3 of the maximum strain reached in the previous cycle.

Comparison of results included time histories and maximum values of both accelerations and displacements, response spectra for the motion of the system, and ratios of the response spectra for the motion of the mass and to that for the input motion. In addition a predominant period and a damping ratio were obtained from the motion of the mass in the exact analysis (4). These values were compared to the corresponding ones resulting from the last cycle of the iterative procedure (when a tolerance of 1% in the maximum strain had been reached).

The same basic approach was applied to a set of two close-coupled multidegree of freedom systems, each one with nine masses and springs. The first system had a uniform initial stiffness with depth, while the second one had stiffness increasing with the square root of depth. In these cases the iterative linear analyses were performed in the time domain, assuming normal modes and modal dampings computed by a weighting formula (5), and in the frequency domain without any assumption as to the existence of normal modes. For each of these two cases two different hypotheses were tried: the damping in each spring was made viscous or linearly hysteretic.

SINGLE DEGREE OF FREEDOM SYSTEMS

For each system the variation of maximum displacement with earthquake intensity was plotted, using the dimensionless variables $\lambda = x_{max}/x_y$ and $M\ddot{u}_G/f_y$, where x_{max} is the maximum displacement, M is the mass, $\ddot{u}_G$ is the peak ground acceleration, x_y and f_y are the Ramberg-Osgood parameters defined earlier. Figure 7 shows a typical plot for an initial natural period of 0.5 seconds. It can be seen that the iterative procedure, with a factor of 2/3 for the characteristic strain, consistently underestimates the maximum strain (or displacement). The ratio of maximum displacements obtained by both procedures is shown in Fig. 8 for all systems. The exact solutions are 14% to 50% higher than the approximate results, the discrepancy increasing with the ductility ratio λ (and therefore with the level of excitation and the system response).

Similar plots were obtained for the maximum acceleration. Fig. 9 shows the ratios of the exact to the approximate results for all systems. For accelerations, the iterative procedure overestimates the response, the exact value being from 80% to 100% of the approximate answer. There is not, however, a clear trend as a function of the ductility λ.

The combined effect is better illustrated by plotting f_{max}/f_y (or $M\ddot{u}_{max}/f_y$) versus x_{max}/x_y for both approaches (Fig. 10). In the exact case the resulting curve is simply given by the Ramberg-Osgood relationship and approximates closely a straight line in a log-log plot for moderate to large values of λ. Results from the iterative approach also fall essentially on a straight line, parallel to the exact one and somewhat higher and to the left.

While displacements are underestimated and accelerations overestimated by the iterative procedure, the values of natural period and damping derived from the exact response compare reasonably well with those resulting from the last cycle of iteration, except for long initial natural periods, where the estimating procedure is itself suspect (6). This would suggest that the discrepancies are not due to lack of convergence to an appropriate equivalent linear system, but rather to the impossibility of reproducing all aspects of the nonlinear response by a linear model, especially when only a few cycles of response will occur.

What is more important, response spectra and ratios of response spectra (for 2% damping) as shown in Figs. 11 to 14 for a system with an initial period of 0.5 seconds, are very similar. The differences increase again with period and level of excitation but, in view of the need to smooth these spectra for design purposes, it can be concluded in general that the approximate method gives adequate results in the calculation of response spectra.

Variations in the value of the strain factor that defines the characteristic strain, from 0.5 to 1, were also investigated. While values somewhat larger than 2/3 seemed to improve the agreement in several cases, there was no consistent trend and it would seem that this factor itself should be a function of the system characteristics and the level of excitation. The introduction of this refinement may not be justified considering all the uncertainties involved in the estimation of soil properties and the selection of appropriate motions.

MULTIDEGREE OF FREEDOM SYSTEMS

Results for the multidegree of freedom systems confirmed the trends already reported. The iterative procedure underestimated displacements and strains and overestimated accelerations. The agreement between both methods was much better when the nature of the damping in the individual springs was considered as hysteretic than when it was taken as viscous. The assumption of normal modes and the use of weighted modal damping seemed to introduce very little error in the cases studied.

Differences between both solutions increased again with the level of excitation and were larger for the variable than for the uniform profile. Over all, however, response spectra for the motion of the top mass and ratios of response spectra were comparable as illustrated by Figs. 15 to 22.

CONCLUSIONS

The iterative method traditionally used in soil amplification analyses cannot entirely reproduce the behavior of a nonlinear system. In general, for a characteristic strain of 2/3 of the maximum strain, it underestimates displacements (and strains), and it overestimates accelerations. The departures from the exact solution are not, however, of a large magnitude, and for moderate levels of excitation and a range of initial natural periods of 0.25 to 1 or even 2 seconds, resulting design spectra are reasonably close to the exact ones. Because of all the other uncertainties involved in usual analyses, use of this procedure within this range seems entirely appropriate. For multidegree of freedom systems it is better to assume the damping in each component to be of a hysteretic rather than of a viscous nature.

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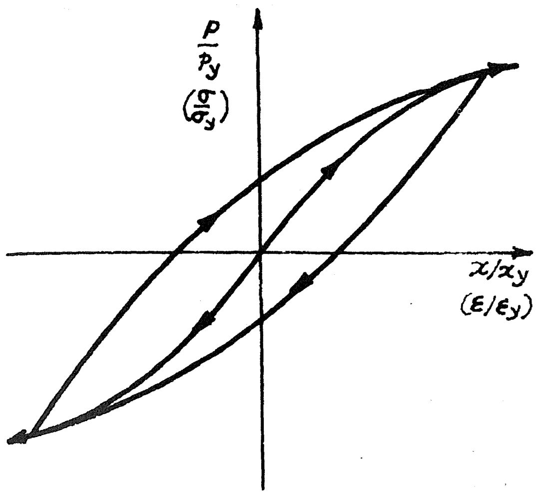


Fig. 1: Force vs Distortion for Ramberg-Osgood model under cyclic loading.

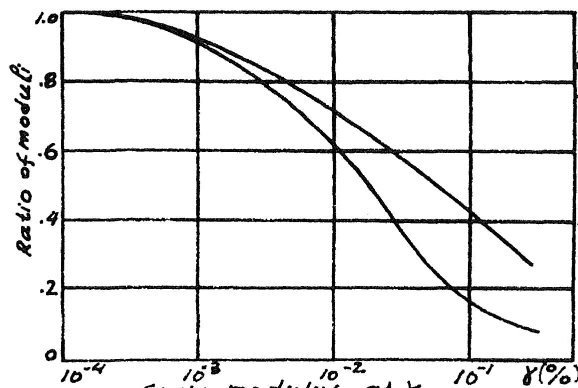


Fig.3: $\frac{\text{Equiv. modulus at } \gamma}{\text{Equiv. modulus for } \gamma = 10^{-4}\%}$ versus strain γ (Ref. 2).

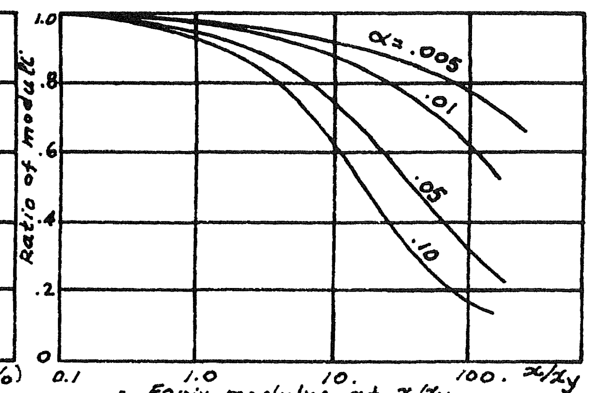


Fig. 4: $\frac{\text{Equiv. modulus at } x/x_y}{\text{Equiv. modulus for } x/x_y = 0.0}$ versus x/x_y for Ramberg-Osgood (r=2).

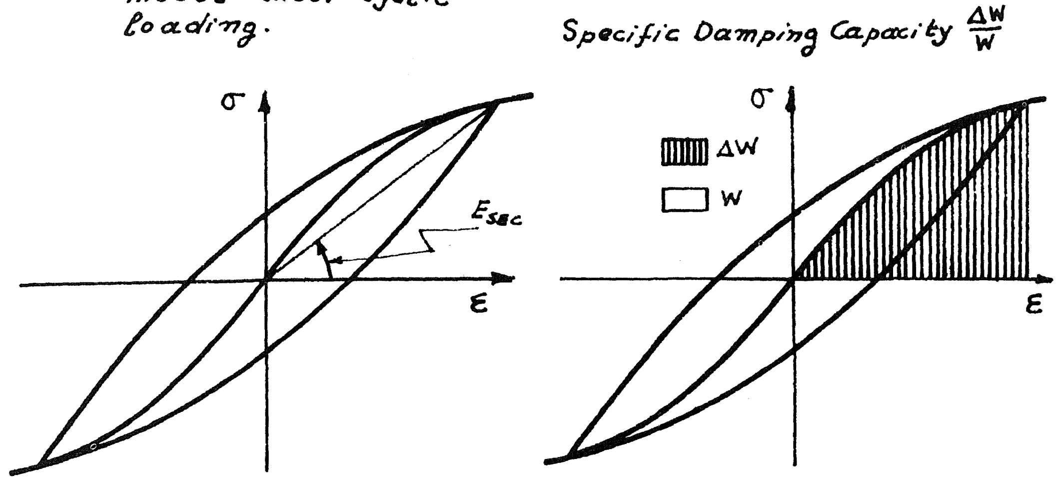


Fig. 2: Definition of Secant modulus and Specific damping.

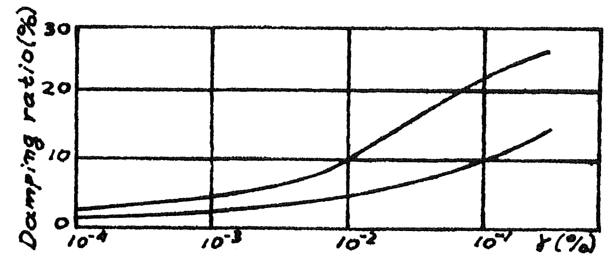


Fig.5: Damping ratio vs strain γ (Ref. 2).

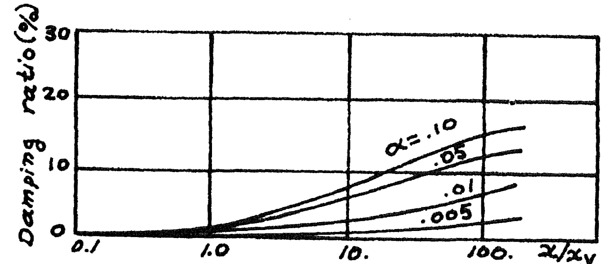


Fig.6: Damping ratio vs x/x_y for Ramberg-Osgood. (r=2)

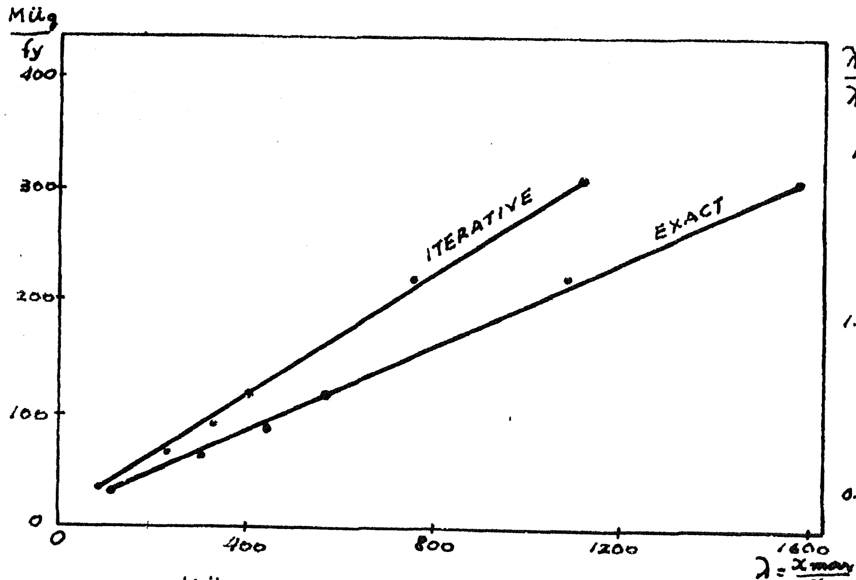


Fig. 7: $M\ddot{u}g/f_y$ vs corresponding λ for a SDOF system with $T_{init} = 0.50$ sec.

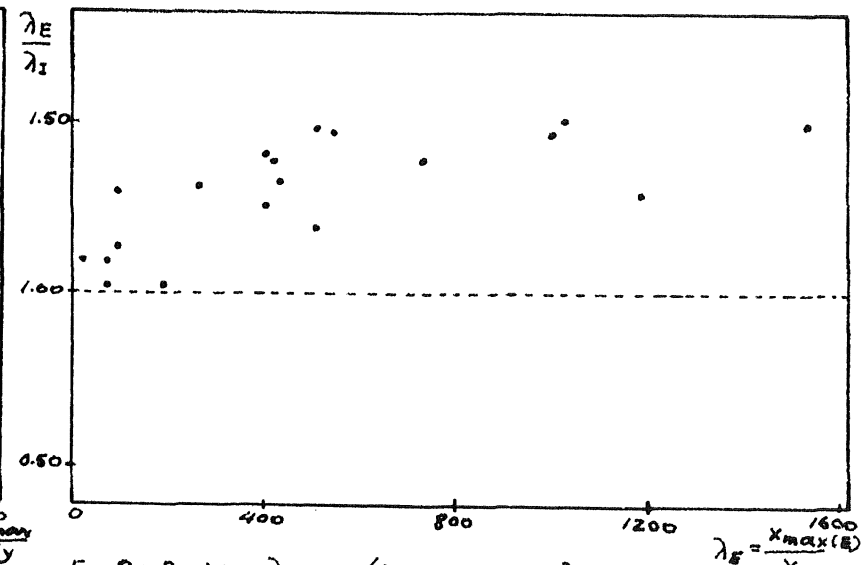


Fig. 8: Ratio $\lambda_{EXACT}/\lambda_{ITER}$ vs λ_{EXACT}

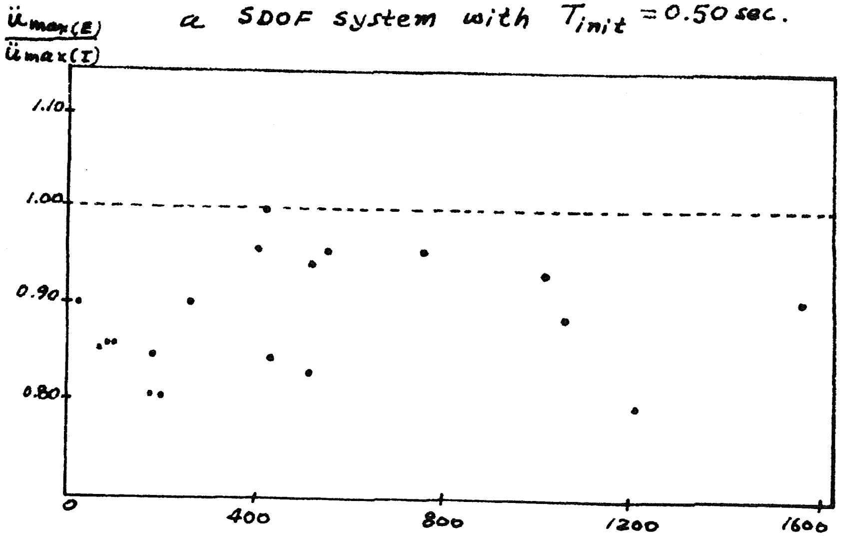


Fig. 9: $\ddot{u}_{max(EXACT)}/\ddot{u}_{max(ITERAT)}$ vs λ_{EXACT}

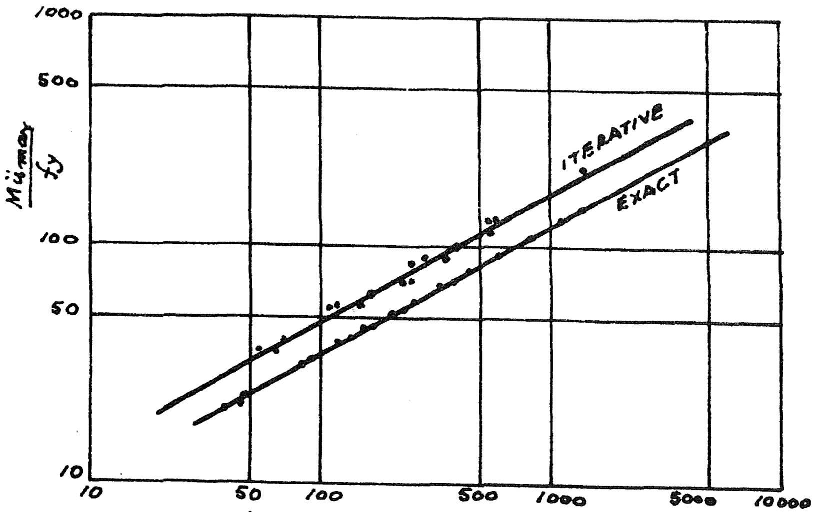


Fig. 10: $M\ddot{u}_{max}/f_y$ vs Corresponding λ

Fig. 11

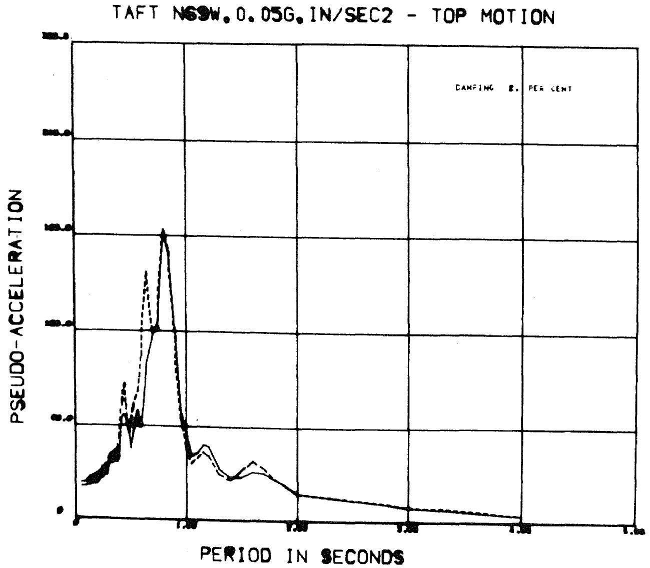
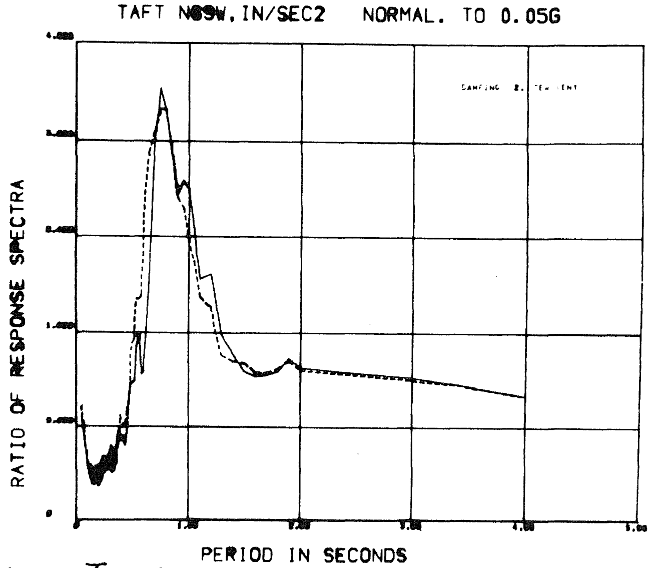


Fig. 12



SDOF system $T_{init}=0.50$ sec

Fig. 13

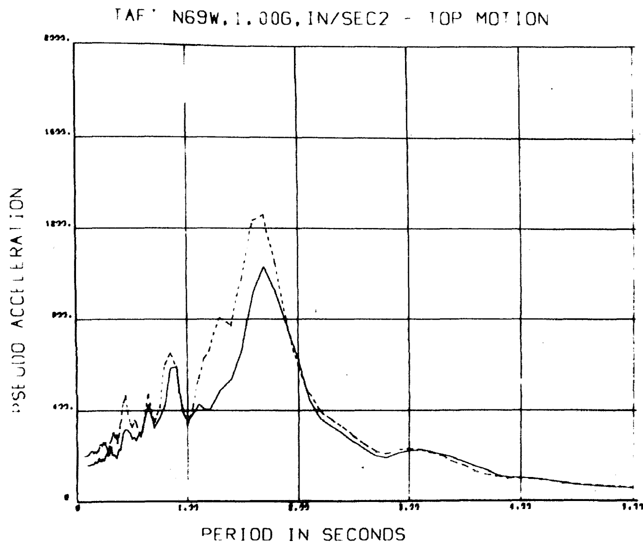


Fig. 14.

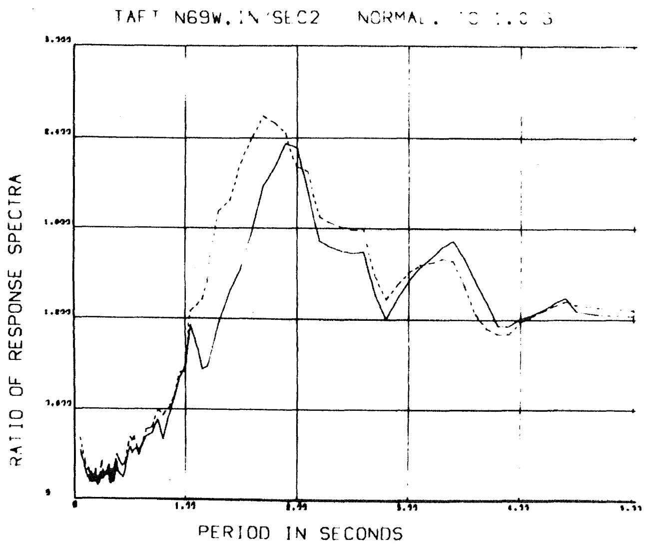


Fig.15

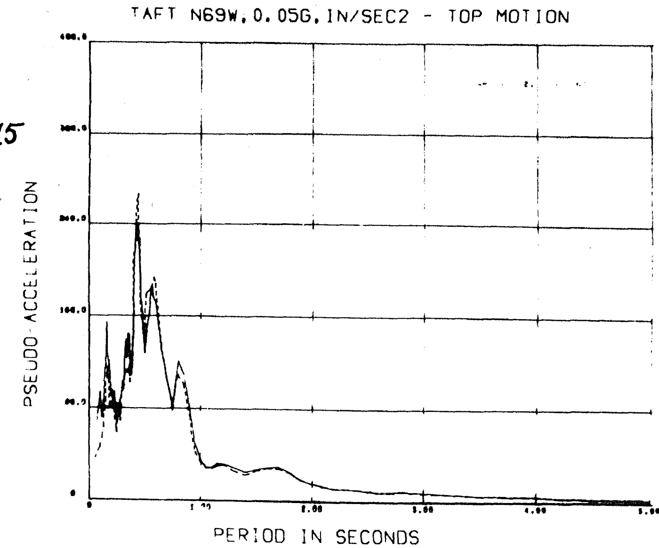
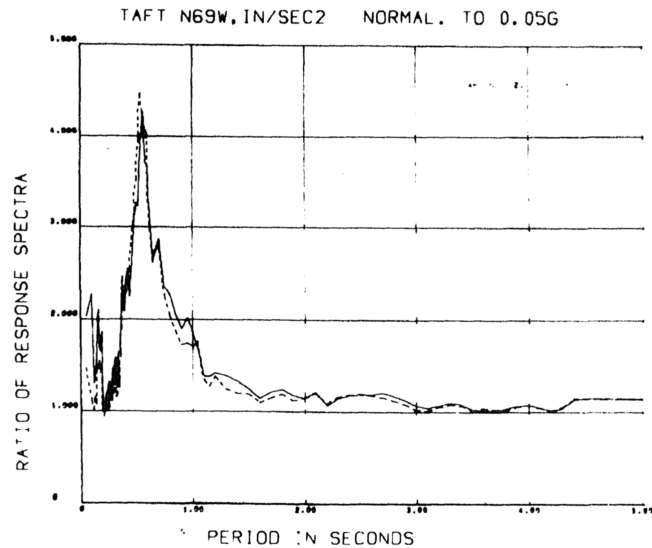


Fig. 16



Uniform profile

Fig.17

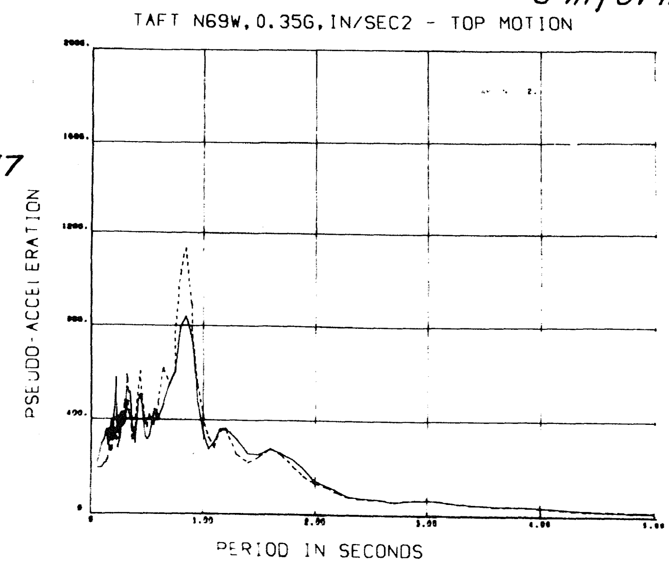


Fig.18

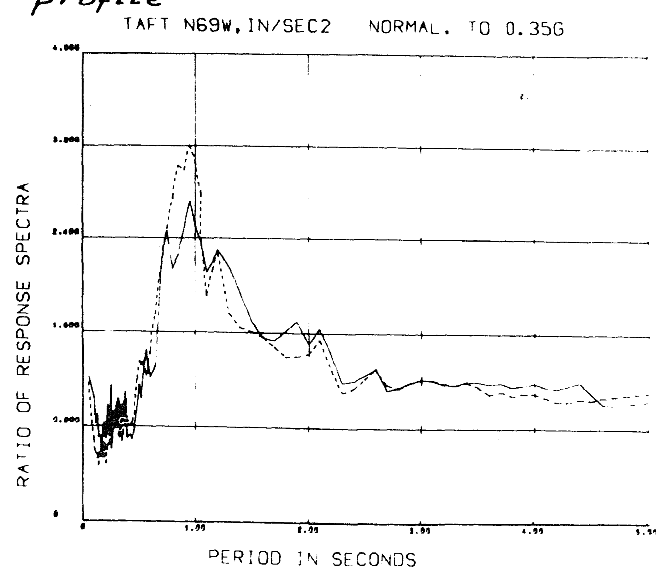


Fig. 19

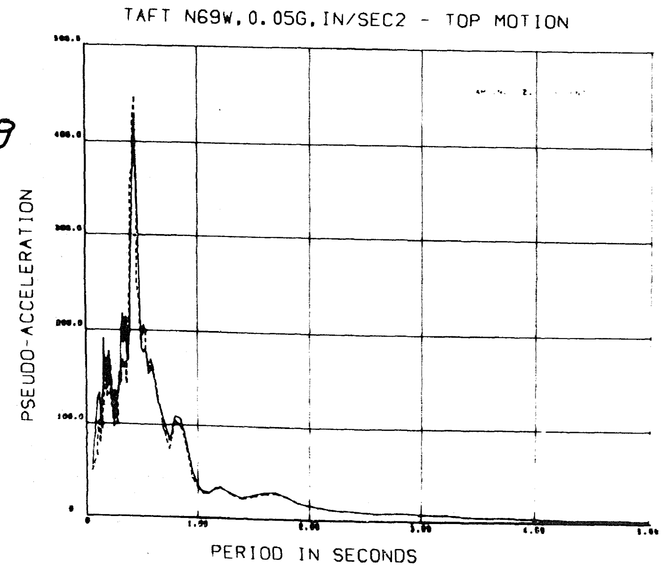
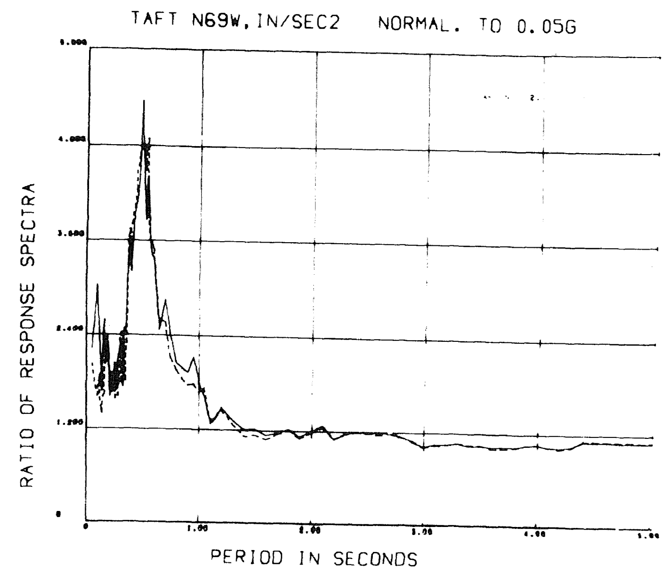


Fig. 20



Variable profile

Fig. 21

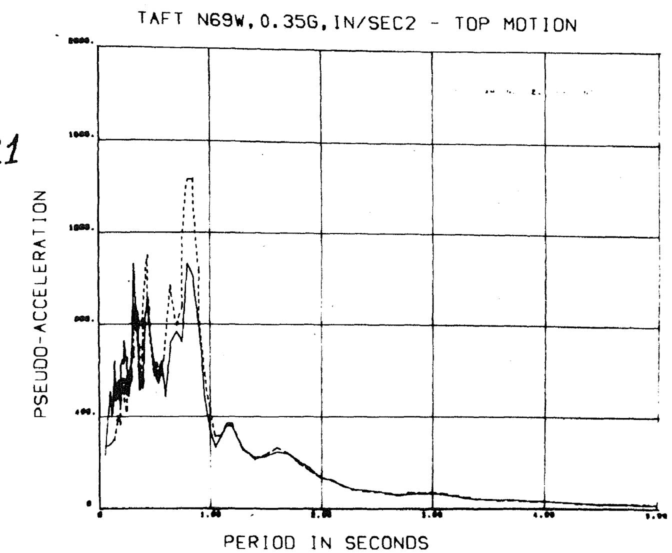


Fig. 22