Consideration on Effect of Hysteresis Damping for Composite Systems

by

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SYNOPSIS

In this paper, the behaviors of damped linear composite systems, constructed by different kinds of structure having hysteretic viscous damping and structural damping properties which can not be uncoupled by classical normal modes are considered. It is shown that the dynamic properties of actual full-scale building tested by generator are nearly similar to those of hysteretic viscous damping model interacted with ground-foundation. Such a hysteretic viscous damping model may be acceptable well to analyse the damped linear composite systems interacted with ground-foundation.

INTRODUCTION

The structures with nuclear reactors, chemical plants and some kinds of buildings are usually constructed by many kinds of materials and various structural type. Furthermore, they often stand upon the ground-foundation with soft layers. Therefore, dynamic properties of these composite structures subjected to earthquake ground motion are very complicated. Especially, the behaviors of damped composite systems which are constructed by any structures having different damping properties each other have not been investigated yet. Provided that there are composite systems of which damping effect is attributed to interaction with any kinds of system having constant damping factor independent on natural frequency for all modes, then these damped coupling systems can be uncoupled by means of complex eigen value technique. Accordingly uncoupled modal damping effects of composite systems become more clear.

The objective of this paper is to introduce the hysteretic viscous damping matrix which may be diagonalized by the same transformation that uncouples the damped system and to analyse the properties of the damped coupling composite system constructed by any structures having hysteretic viscous damping matrix derived above and interacting with ground-foundation.

HYSTERETIC VISCOUS DAMPING MATRIX

Consider the following equation of motion for $(j)th$ system constructing the composite dynamic system

$$[M]_j\{\ddot{X}\}_j + [C]_j\{\dot{X}\}_j + [K]_j\{X\} = \{0\} \quad (j = 1, 2, \cdots, m) \qquad (1)$$

where $[M]_j, [C]_j$ and $[K]_j$ are mass matrix, damping matrix and stiffness matrix respectively, and $\{X\}_j$ is displacement vector.

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MODAL DAMPING PROPERTIES FOR COMPOSITE SYSTEM

The equations of motion of damped coupling n-degree-of freedom composite system having viscous damping matrix as eq.(12) is given by

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{0\} \tag{14}$$

in which $[M]$, $[C]$, and $[K]$ are mass matrix, damping matrix and stiffness matrix, respectively and $\{X\}$ is displacement vector.

Let eigenvalue and eigen vector of eq.(14) be λ and $\{\phi\}$, respectively, then the following homogeneous equation[2] is derived as

$$\begin{bmatrix} [M]^{-1}[C] + \lambda[I] & [M]^{-1}[K] \\ -[I] & \lambda[I] \end{bmatrix} \begin{Bmatrix} \lambda\{\phi\} \\ \{\phi\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \tag{15}$$

where $[I]$ is the identity matrix and $\{0\}$ is the zero vector.

Equation (15) can have a non-trivial solution only if determinant of matrix, namely characteristic equation for eigen value λ is zero. Solving this characteristic equation yields the following complex and its conjugate eigen value, respectively:

$${}_s\lambda = {}_s\alpha + i\,{}_s\beta \;,\; {}_s\bar{\lambda} = {}_s\alpha - i\,{}_s\beta \quad (s = 1, 2, \cdots, n) \tag{16}$$

Therefore, the effect on damping properties of damped composite system may be appreciated by the following modal damping ratio ${}_s\zeta$, so-called fraction of critical damping for each natural mode:

$${}_s\zeta = {}_s\alpha / {}_s\beta \quad (s = 1, 2, \cdots, n) \tag{17}$$

On the other hand, the equation of motion of damped coupling composite system having complex stiffness matrix is given by

$$[M]\{\ddot{X}\} + [[K]_R + i[K]_I]\{X\} = \{0\} \tag{18}$$

As mentioned previously, let eigen value and eigen vector of eq.(18) be $\omega^2 = -\lambda$ and $\{\phi\} = \{\phi\}_R + i\{\phi\}_I$, respectively, the following homogeneous equation is derived as

$$\begin{bmatrix} [M]^{-1}[K]_R - \lambda[I] & -[M]^{-1}[K]_I \\ [M]^{-1}[K]_I & [M]^{-1}[K]_R - \lambda[I] \end{bmatrix} \begin{Bmatrix} \{\phi\}_R \\ \{\phi\}_I \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \tag{19}$$

As above, solving the characteristic equation derived from eq.(19) yields the same eigen values as eq.(16). Modal natural circular frequencies ${}_s\omega$ and modal damping ratio ${}_s\zeta$ may be expressed by the following way:

$${}_s\omega = \sqrt{({}_sR + {}_s\alpha)/2} \;,\; {}_s\zeta = \sqrt{({}_sR - {}_s\alpha)/({}_sR + {}_s\alpha)} \;,\; {}_sR = \sqrt{{}_s\alpha^2 + {}_s\beta^2} \tag{20}$$

Therefore, it will be expected that we can investigate reasonably the effects of any systems having hysteretic viscous and structural damping properties on damped coupling composite system.

APPLICATION TO SOME COMPOSITE SYSTEMS

System Connected in Parallel

Consider the effect of damped coupling composite system connecting two systems in parallel as shown in Fig.-1. System (1) and system (2) of its composite system possess constant modal damping factor $\zeta_1 = 3\%$ and $\zeta_2 = 6\%$, respectively. Also the fundamental datum such as masses and spring constants are shown in Fig.-2.

that when natural modes corresponding to any systems of structure-foundation dynamic system are predominant explicitly, damping properties corresponding to their systems appear remarkably. Especially according as the natural mode corresponding to sway or rocking of ground-foundation is predominant in lower modes, modal damping ratio of structure-foundation dynamic system is inclined to increase in its magnitude.

Example of Analysis of Actual Building

Modal damping properties consisting in an actual building tested by K. Kinoshita[3] show decreasing tendency in accordance with increase in its modal natural frequency. The values of modal damping ratio are ${}_1\zeta = 3.2\%$, ${}_2\zeta = 3.0\%$, ${}_3\zeta = 2.2\%$ and ${}_4\zeta = 2.0\%$ for each normal mode, respectively. Damping mechanism of actual building is complicated, then we assume that the effects of such a damping property depend on the interaction with sway and rocking of foundation, and flexure of structure. Mass m_i and spring constant k_i calculated for each story are shown in Table 2. And both constant damping factor for all modes of flexure of structure $\zeta = 2.5\%$ and damping factor for 1st mode of flexure of structure ${}_1\zeta = 0.9\%$ such as for higher modes ${}_s\zeta = {}_1\zeta \cdot {}_s\omega / {}_1\omega$ proportional to their natural frequency ${}_s\omega$ are assumed considering the results of vibrational test. Furthermore damping factor for sway and rocking of foundation, $\zeta_0 = 10\%$ and $\zeta_\theta = 40\%$ are assumed for convenience.

Three types of damping model are considered in the following discussion: Namely, first type is hysteretic viscous damping model which possesses hysteretic viscous damping for flexure and viscous damping for sway and rocking of foundation, second type is frequency proportional damping model which possesses frequency proportional viscous damping for flexure and viscous damping for sway and rocking of foundation and third type is structural damping model which possesses structural damping for flexure and dissipation damping for sway and rocking of ground-foundation.

The modal damping ratio for each damping model are shown in Fig.-5. From the results indicated in Fig.-5, it can be seen that hysteretic viscous damping model represents fairly well the damping properties of building obtained from vibrational test. Therefore, for composite dynamic system accompanied with sway and rocking of ground-foundation, such a hysteretic viscous damping model may be adequately effectual one. But we have to study on damping mechanism of damped composite systems interacted with ground-foundation.

SUMMARY AND CONCLUSION

We directly derived hysteretic viscous damping matrix of damped dynamic model of which modal damping ratio is constant to all natural modes by using orthogonalized mode shape matrix of undamped system from Caughey series. Considering the composite dynamic system, for instance, the structures which are interconnected with ground-foundation, as having hysteretic viscous damping property, we investigated the damping properties of damped coupling composite system by means of complex eigen value problem. As the results of the previous consideration, it will be expected that damping properties of some sorts or composite structure interacted with ground-foundation can be explained well by analysing damped composite systems which possess the hysteretic viscous damping for each system.

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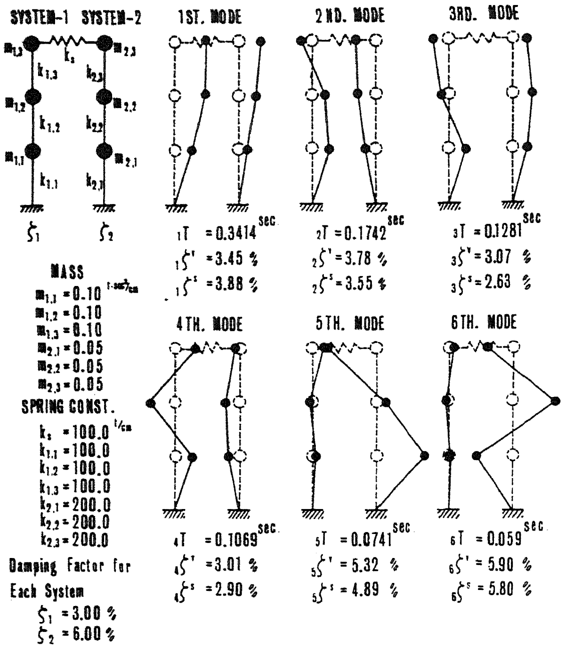

NOTE $_s\zeta^v$: Modal Fraction of Critical Damping for Hysteretic Viscous Damping (s = 1, 2, ..., 6)
$_s\zeta^s$: Modal Fraction of Critical Damping for Structural Damping (s = 1, 2, ..., 6)

FIG.1 Results of Analysis for System Connected in Parallel

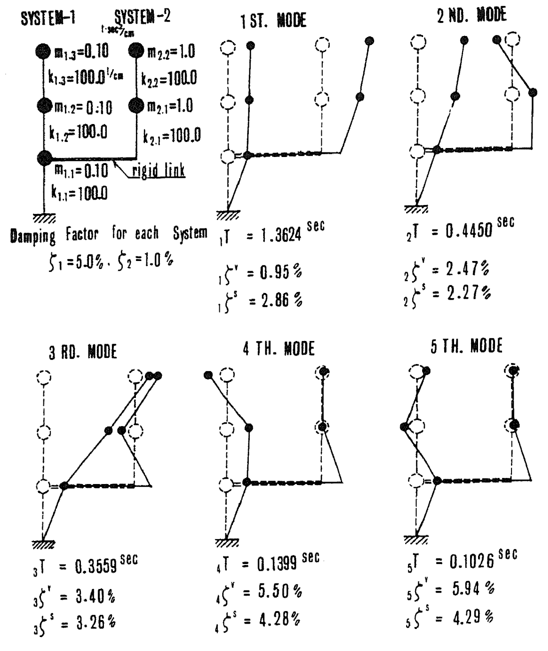

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FIG.2(a) Results of Analysis for Branched System (Fixed Model)

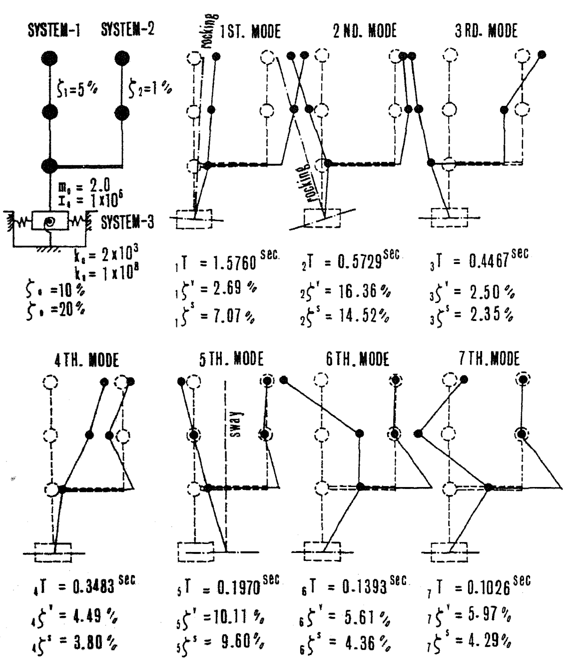

FIG.2(b) Results of Analysis for Branched System Accompanied with Sway and Rocking of Ground-Foundation

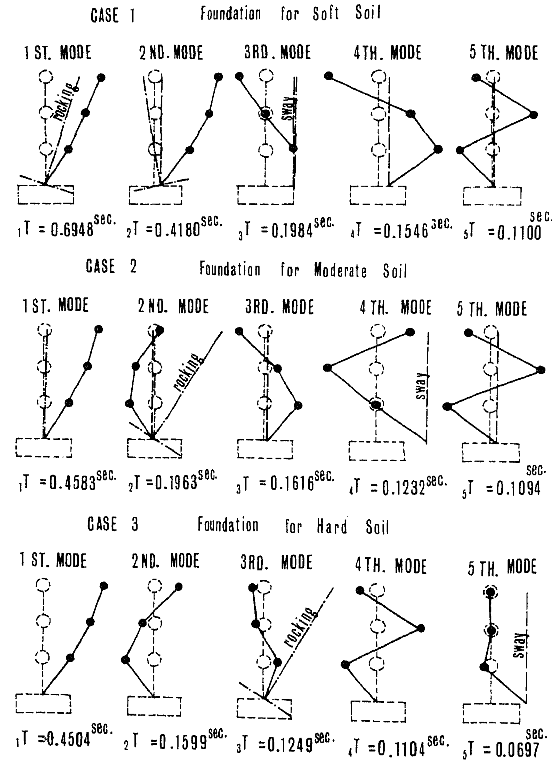

FIG.3 Modal Shapes for Systems Accompanied with Sway and Rocking of Ground-Foundation