LONGITUDINAL WAVES PROPAGATION IN SPECIMENS OF INCOHERENT SOILS AND DISCONTINUED MATERIALS

by
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SYNOPSIS

In this work the methodologies are critically evaluated by which the propagation velocity of longitudinal waves is measured in cylindrical specimens of continued, homogeneus, isotropic and elastic mediums in connection with the band width of the transmitted signal. The experimental data have been compared to those obtained by the Pochhammer theory. The fundamental parameter for us was ϕ/λ (diameter of specimen / wavelength) as a function of the dispersion referred to the shape and not to the characteristics of the medium, hence the meaning was defined of the velocities obtained with different measurement systems. Tests were made to measure velocities in granular medium specimens with the same assumptions applied to continuous mediums. Resuls on sand specimens are then reported showing how the confining pressure, void ratio and grain size affect the wave propagation velocity, the latter being considered as a function of the discontinuities among the grains which we are trying to schematize through tests carried out on specimens obtained by putting sheets of the same material close to each other.

MEASUREMENT OF PROPAGATION VELOCITY

The propagation velocity of longitudinal waves can be experimentally determined either by resonance tests or pulse transmission tests. With the first system vibrations are originated by a continuous source of simple sinusoidal waves; and with the second system by ultrasonic transducers or shocks, also air or explosive induced. The continuous emission and the ultrasonic pulses canbe regarded as generators of narrow band signals as their frequency is controlled and known, whereas the other systems generate wide band signals, wich can be defined as noise, whose characteristics cannot be preselected. By the resonance method the phase velocity is measured and λ is immediately determined since the length of the specimen is known. By ultrasonic pulses which can be considered as wave groups, it is possible to define the group velocity and the first arrival velocity. With wide band signals the considerably different frequencies, owing to dispersion, quite distort the pulse, causing the speedier components to reach on the way the head of the pulse and viceversa. Therefore it could be said that the phenomenon of dispersion by which waves of different frequency are trasmitted at different velocities, is not depending on the specimen material but only on its cross section size, also in some real mediums. This is confirmed assuming there is only one dilatational velocity in a given medium independently from the frequency band in which it can be measured. The narrow band excitation permits to compare directly the results with those obtained by the Pochhammer's theory enunciated for infinitely long cylinders and quite true with a fair approximation (Love) also for

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long and thin cylinders of finite length. Many investigators Bancroft (1941), Tu-Brennan-Sauer (1955), Spinner-Tefft (1961), Zemanek-Rudnick (1961), Booker (1969), etc., have noticed that the Pochhammer's theory for the first mode is checked by experiments in the range of ϕ/λ less than 2, whereas no experimental check of this theory is possible for ϕ/λ ratios exceeding the aforesaid value. Field's hypothesis (1931) is that there must be a critical frequency for each cylinder beyond which the first vibrating mode disappears and the second mode appears instead. Field's approach is limited to these two modes. It should be possible that modes related to the various roots of the frequency equation may appear subsequently by which definite and overlapping fields are determined according to the various modes where it should be possible to measure the dilatational velocity. Tu, Brennan and Sauer (1955) in fact have succeeded in recording dilatational velocities over $\phi/\lambda = 5$ in long and thin rods by means of high frequency ultrasonic pulses. This means that beyond a certain limit frequency in a given specimen the phase velocity equals the group as well as the signal velocity reaching the dilatational value, thus possibly accounting for the dispersion phenomenon to disappear. Boyle and Froman (1929) obtained their dilatational values by means of a modified resonance method consisting in the measurement at high frequencies of the ultrasonic energy absorbed by a metallic disk. During their tests they recorded dilatational velocity values beyond the ratio $\phi/\lambda = 3.44$. Thus, accepting the results and the relevant hypothesis, the interpretation of the wide band pulse phenomenon is quite straightforward. Adopting the first arrival for the measurement of velocity, its determination depends on the components propagating at dilatational velocity, and therefore this value should always be reached in spite that energetically speaking it may represent only a negligible part of the signal and hence needs a very sensitive receiver in order to be detected. This hypothesis is fully confirmed both by our tests and those of other investigators Hughes-Pondrom-Mims (1949), Kolsky (1954), Miklowitz-Nisewanger (1954), etc. Each component propagates at a velocity which can be defined by the dispersion curve and has associated some energy. When speaking of energy transmission velocity it should be clear that the average is to be made of the energy quantities associated with each component against each single transmission velocity value. The propagation phenomenon is not quite clear at about $\phi/\lambda = 2$, where the curve is supposed to show a discontinuity, as already pointed out by Field; both the experimental methods by resonance and ultrasonic pulses fail here as reported by Tu, Brennan and Sauer.

EXPERIMENTAL RESULTS

By carrying out tests on specimens of incoherent granular materials, the radial strain of the specimen is affected by the container. Either thin rubber containers may be used which do not affect the specimen strain values, or containers which by their radial rigidity would limit the specimen strain. Baker and Triandafilidis (1968) say that the rigid segmented rings containers they used allowed radial strains up to 8% of the longitudinal strain. The wave propagating in these specimens should always be considered as longitudinal because it is difficult to fully reproduce in cylindrical specimens the conditions typical of the non limited medium. By transposing to the granular specimens the conclusions reached on the wave propagation in continuous material, we can see that since the high frequen cy components propagate at dilatational velocity, also in free boundary specimens the container effect for these components should be negligible. The first arrival velocity should therefore reach the dilatational value in tests with wide band pulses. Hampton and Huck (1968/70) by means of an air-shock tube and container of segmented rings have found that the pressure peak velocity is about 60% of the first arrival velocity. This would confirm the hypothesis that most of the energy propagates at longitudinal velocities. With the same type of device Stoll and Ebeido (1965) have recorded velocities about 20% higher than those obtained by resonance by Hardin and Richart (1963) on the same sand specimens. In spite they think the results reconcile within errors, we believe that the higher values obtained by shock tube should be equal to the dilatational velocity, and those obtained by resonance equal to the longitudinal velocity.

Longitudinal wave propagation tests have been carried out by us on specimens of incoherent granular material obtained by crushing a limestone rock. Various grain sizes were selected - the void ratio varying for each grain size between a maximum of about 1.05 and a minimum of 0.85. The void ratio obtained by the Bolomey 12 diagram was about 50% lower. The material was put into cylindrical containers lined with foam rubber having a diameter ranging between 20 and 30 centimetres and a height ranging between 15 and 30 centimetres. The excitation was obtained by the impact of masses falling on a rigid metallic disk resting on the specimen. The propagation velocity was measured by recording the first arrival. The device allows the confining pressures to be changed.

The results obtained have practically demonstrated that the propagation velocity is not affected by the variation of the void ratio. In fact in all the specimens with a quite homogeneous grain size the velocity values recorded were almost all equal though the void ratio was changed. The Bolomey grain size specimens tested showed - in spite of their low void ratio - a velocity slightly lower than the average values obtained with the specimens with slightly variable grain size. The variation of propagation velocity by varying the void ratio recorded by some investigators should be ascribed to boundary overpressures which inevitably occur during compaction operation either shock or vibration induced. It was noticed besides that the velocity varies with approximately the 1/4 power of the confining pressure. This result is in a good agreement with the data obtained by other investigators (Hardin-Richart). The experiences made also clear how the velocity is dependent upon the grain size according to the single confining pressure values applied. Where no confining pressure is applied the phenomenon can be plotted - according to the type of material used - into the following equation curve : $c = 160\varphi^{0.137}$, where c is the velocity and φ is the maximum grain size.

For a deeper analysis of this phenomenon prismatic specimens have been realized with definite plane discontinuities, consisting of sheets of slate - which is a natural stratified stone material - placed one close to the other, and whose contacting surfaces were machined and checked so as to obtain the same roughness degree. The velocity was determined by varying the thickness of the sheets and the contact pressure. The curves of velocity against sheet thickness were similar to those of the crushed material.

Tests were also performed on a slate cube and it was found that the propagation velocity of longitudinal waves in the slate is considerably different according to whether it was measured on the bedding plane or on the direction perpendicular to it. The velocity as measured in the first case was considerably higher and this may confirm the aforesaid hypothesis of the effect exerted by the discontinuities (stratifications) on the propagation time. This was further supported by applying loads on specimens perpendicularly to the bedding plane, thus obtaining increasing velocity values related to the load. When the load was instead applied in the direction parallel to the bedding plane, no increase in velocity appeared and this could be explained by the absence of discontinuity in the material along this direction. In the wide band pulse tests on granular specimens it is impossible to establish the wavelength. The velocity value measured in relation to the first arrival is either longitudinal or dilatational. The possibility to measure the dilatational velocity depends on improving the amplifying capacity of the receiving equipment.

The relationship between the measured propagation velocity and the elastic properties of a given material is established taking into account the conditions in which the measurement was made both of the velocities and of the elastic properties. The Joung's elastic modulus E is determined in a specimen with free radial dilatations and constant cross section stresses. The constrained modulus in granular mediums E^* - called by Barton "elongational elasticity" for continuous material - is measured with interdicted radial strain; its value is related to the Joung's modulus as follows: $E^*/E = (1 - \nu)/(1 + \nu)\ (1 - 2\nu)$, where ν is the Poisson's ratio. The correlation must be expressed by E^* in each case where dilatational velocities are recorded, and by E whenever the longitudinal velocity equals the bar velocity. Both moduli are obtained by the relationship ρc^2, where ρ is the density of the medium tested.

The velocity measurements carried out on cylindrical specimens in laboratory tests may be related to the wave propagation in situ applying the same limits and principles set out in our paper.

A more detailed study of the same subject together with exhaustive references have been developed by the authors in another paper now being printed.