ON THE RESPONSE CALCULATION IN THE TIME DOMAIN OF THE GROUND CONSIDERED THE VISCOSITY AND THE ENERGY DISSIPATION TO THE BEDROCK LAYER

by

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INTRODUCTION

The calculation of the frequency response by using the equation of the wave motion and the calculation of the response in the time domain of multi-layered ground not including the viscosity have been studied.[1)~8)] But it has not been sufficiently studied to calculate the response in the time domain of the system considered the viscosity and the energy dissipation to the bedrock layer. It is possible to calculate the response in the time domain of the visco-elastic ground considered the energy dissipation to the bedrock layer by a translation to the equations of the equivalent lumped mass system from that of the one-dimensional shear wave motion system. The translation is carried out by equating amplification and phase properties respectively in two systems. It is useful to apply the method of least squares to determine the parameters M, C and K of the equivalent lumped mass system.

Using this lumped mass system, it is possible to handle the non-linear problems in soil structure and the connection between soil and upper building structure.

IDEALIZATION OF THE GROUND

Consider the idealized models of the multi-layered ground as shown in Fig. 1. The left and the right hand side models in Fig. 1 are those for the wave motion system and for the lumped mass system, respectively. The effect of energy dissipation to the bedrock layer is considered by using the incident wave F_N instead of the boundary surface motion U_N as input. U_N is affected by the property of upper soil structure. Hence U_N is not pure input wave and the incident wave F_N should be considered as input.

Equations of wave motion of the system shown in the left hand side in Fig. 1 are given by

$$\rho_k \frac{\partial^2 U_k}{\partial t^2} = \left(\mu_k + \mu_k' \frac{\partial}{\partial t}\right) \frac{\partial^2 U_k}{\partial z_k^2} \qquad (k = 1, 2, \cdots, N) \tag{1}$$

where
- ρ_k ------------- mass density of the k-th layer
- H_k ------------- layer thickness
- μ_k ------------- Lame's elastic constant
- μ_k' ------------- viscosity coefficient
- F_k ------------- incident wave
- G_k ------------- reflected wave
- U_k $(=F_k+G_k)$ ------ displacement

and boundary conditions at $z_k=-H_k$, $z_{k+1}=0$ are

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$$U_k = U_{k+1} \qquad (2)$$

$$(\mu_k + \mu_k' \frac{\partial}{\partial t}) \frac{\partial U_k}{\partial z_k} = (\mu_{k+1} + \mu_{k+1}' \frac{\partial}{\partial t}) \frac{\partial U_{k+1}}{\partial z_{k+1}} \qquad (3)$$

and at $z_1 = 0$ is

$$(\mu_1 + \mu_1' \frac{\partial}{\partial t}) \frac{\partial U_1}{\partial z_1} = 0 \qquad (4)$$

On the other hand, equations of motion of the lumped mass system shown in the right hand side in Fig. 1 are given by

$$M_k \ddot{U}_k + C_k(\dot{U}_k + \dot{U}_{k+1}) - C_{k-1}(\dot{U}_{k-1} - \dot{U}_k) + K_k(U_k - U_{k+1}) - K_{k-1}(U_{k-1} - U_k) = 0 \quad (k = 1, 2, \cdots, N) \qquad (5)$$

where $C_k = K_k = 0$ when k equals to zero and

M_k -------- the k-th mass
C_k -------- the k-th damping coefficient
K_k --------- the k-th spring constant
$U_{N+1} = F_N$ ---- incident wave

EQUIVALENT TRANSLATION TO THE LUMPED MASS SYSTEM

It is possible to obtain amplification and phase properties of wave motion system by solving Eqs. (1)~(4). Now consider the translation to equivalent lumped mass system from the wave motion system by using the method of the least squares. Parameters C_k and K_k of the equivalent lumped mass system are determined by equating the amplification and the phase properties respectively in two systems. Parameters M_k are fixed by Eq.(6) when variable parameters C_k and K_k are determined by the method of the least squares.

$$\left. \begin{array}{ll} M_k = \dfrac{\rho_{k-1} H_{k-1} + \rho_k H_k}{2} & (k = 1, 2, \cdots, N-1) \\ M_N = 0 & \end{array} \right\} \qquad (6)$$

The N-th mass, spring and dash-pot of the lumped mass system shown in Fig. 1 give the effect of the energy dissipation to the bedrock layer. Namely, the story displacement, $U_N - U_{N+1} = U_N - F_N$, corresponds to the dissipation wave, G_N, to the bedrock layer.

EXAMPLE

Nine-layered ground which parameters are given in Table 1 is analyzed by this translation method.

The values of M, C, K and h in Table 1 are the fittest values obtained by applying the method. Damping constants, h, in this Table are obtained by the relation

$$h_k = \frac{C_k}{2K_k} \omega^1 \qquad (7)$$

where ω^1 is foundamental circular frequency of the system.

Figs. 2 and 3 show the comparison of amplification and phase properties of both systems, respectively.

CONCLUSION

It is proved by the example that parameters of equivalent lumped mass system are determined with good accuracy by applying the translation method. Using this equivalent lumped mass system, the calculation of the response in the time domain of multi-layered ground considered the viscosity and the energy dissipation to the bedrock layer can be simply carried out.

REFERENCES

1) K. Kanai, "Relation between the Nature of Surface Layer and the Amplitudes of Earthquake Motions", Bulletin of Earthquake Research Institute, vol. 30, 1952.
2) K. Kanai, "same as above -2-", Bulletin of E.R.I., vol. 31, 1953.
3) K. Kanai and S. Yoshizawa, "same as above -3-", Bulletin of E.R.I., vol. 31, 1953.
4) K. Kanai and S. Yoshizawa, "same as above -4-", Bulletin of E.R.I., vol. 34, 1956.
5) N. A. Haskell, "The dispersion of Surface Waves on Multi-layered Media", Bulletin of Seismological Society of America, vol. 43, 1953.
6) S. Yoshizawa, T. Tanaka and K. Kanai, "Some Features of Strong Underground Earthquake Motions Computed from Observed Surface Records", Bulletin of E.R.I., vol. 46, 1968.
7) R. V. Whitman, "Equivalent Lumped System for Structure Founded upon Stratum of Soil", Proc. of the 4th WCEE, 1970.
8) H. Kobayashi and Y. Kagami, "Amplifications due to Reflection Theory of Shear Wave in Multi-layered Ground", Translation of the Architectural Institute of Japan, No. 173, July, 1970.

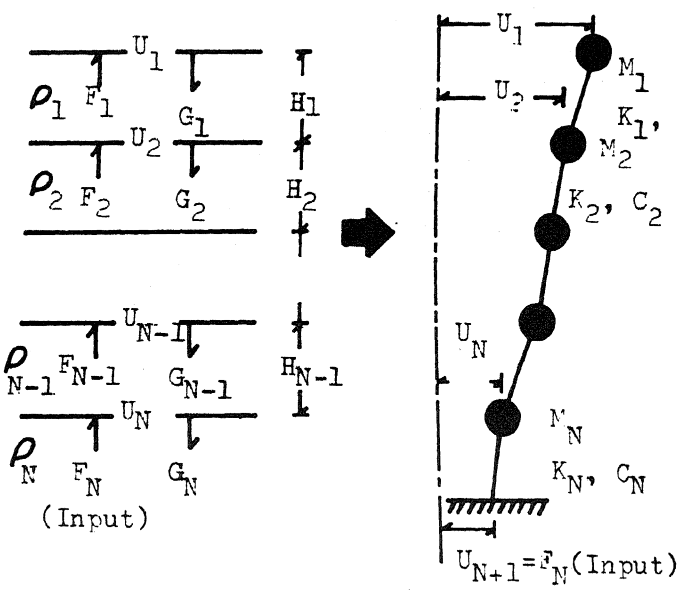
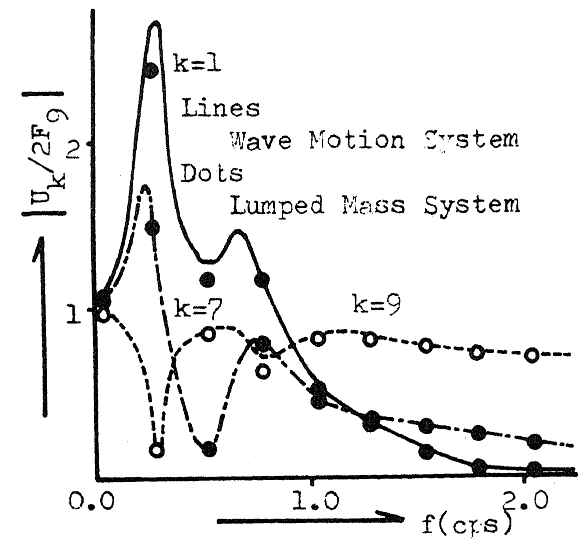


Fig. 1 Idealized Models of the Ground



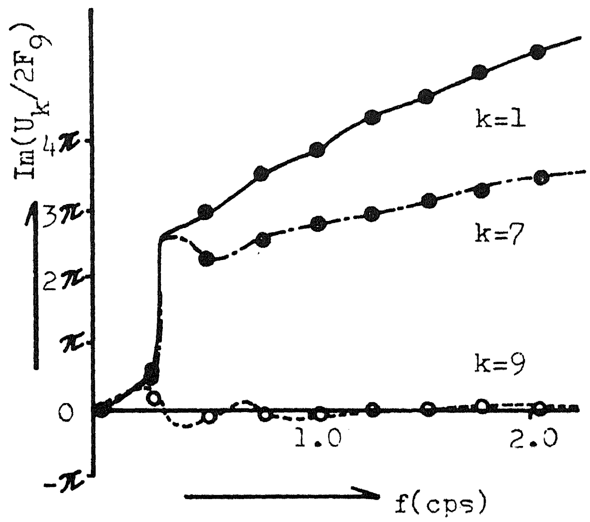


Fig. 2 Amplification $|U_k/2F_9|$

Fig. 3 Phase Lag $Im(U_k/2F_9)$

Table 1 Example

Layer	H (M)	ρ (T/M^3)	μ (T/M^2)	μ'/μ	M	C	K	h
1	16.5	1.67	4352	0.05	1.00 2.00	37.6 37.6	756 756	0.045 0.045
2	12.5	1.52	2232		2.38	12.7	262	0.044
3	5.5	1.67	5508		2.05	72.7	1429	0.046
4	1.0	1.67	5508		0.79	400.0	7842	0.046
5	25.5	1.76	9522		1.76 3.27	53.8 54.1	1070 1070	0.046 0.046
6	18.0	1.67	3332		2.73 2.18	26.9 27.4	531 531	0.046 0.047
7	46.0	1.86	26011		2.34 2.49 2.49 2.49 2.49	217.1 212.0 209.2 207.7 206.8	4034 4041 4046 4049 4050	0.049 0.048 0.047 0.047 0.046
8	55.0	1.67	6800		2.58 2.67 2.67 2.67 2.67	44.9 45.5 46.0 45.7 45.3	899 900 904 913 920	0.045 0.046 0.046 0.046 0.045
9	-	1.96	72000		0.00	141.1	91	1.408

T^1 = 3.4524 sec.

Input Data ←——→ Parameters obtained