A SYSTEM FOR MEASURING NORMAL MODES OF STRUCTURES

W. R. Stephenson (I), R. I. Skinner (I)

SYNOPSIS

A convenient system has been developed for measuring the natural periods, modal shapes, and modal dampings of structures. A selective measuring system gives accurate values for large buildings when excited by a portable light-weight shaking machine. The system operates successfully even in the presence of large disturbing forces, or when there are almost coincident natural periods.

1. Features of the Mode Measuring System

Central to the system for measuring normal modes is a phase sensitive detector, which measures the induced structural motion in relation to the phase of the shaker force. Fig. 1 shows a simplified diagram illustrating the main functions of the measuring system, (1). The phase sensitive detector confers a number of important features. It permits convenient narrow-band filtering, and this excludes almost all of the building responses which are not synchronous with the shaker force, including the responses to wind and other disturbances. This sharp discrimination against disturbances permits accurate measurements to be made using the small motions induced by a light-weight building shaker, which can be carried readily by two men.

(a) Period. When the phase sensitive detector measures the component of velocity V_Q, which is 90^o out of phase with the shaker force, then it indicates when the shaker period has been adjusted to a structural natural period by showing a sharp null. This response is shown by the normalized $t^2 V_Q$ curve of Fig. 2.

(b) Mode Shape. When the shape of a normal mode is measured, sharp discrimination against the responses of modes of other periods is obtained by basing measurements on those components of velocity which are in phase with the shaker force. This sharp discrimination is achieved since the in-phase response, the normalized curve $t^2 V_I$ of Fig. 2, falls much more rapidly, when the shaker period T is shifted from resonance, than does the total response vector and

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I Engineering Seismology Section, Physics and Engineering Laboratory, D.S.I.R., Private Bag, Lower Hutt, New Zealand.

hence the in-phase velocity of a nearby mode is very small. When further discrimination is required this can usually be obtained by reorientation or relocation of the light-weight shaker. Shifting the shaker is particularly easy since its only constraint is frictional force, assisted by a suitable adhesive. The mode shape is built up by measuring the ratios of the velocities at various points with respect to a fixed point, the measurement being performed conveniently and accurately using a bridge balance technique. The measuring system is adjusted to give a detector output proportional to $V_1 + G\ V_2$, and when G is adjusted by a precision potentiometer to give zero output, then $V_2/V_1 = -\ 1/G$.

When an appendage causes a complex mode response, because it has a natural period almost identical with that of the mode, that appendage can be identified readily by comparing its phase with the phase of another point on the structure, for periods above and below the mode period. This amounts to a change in null detector sign for a change in motor speed.

(c) Damping. The equivalent viscous damping of a mode can be obtained from the way in which the in-phase and 90° out-of-phase components of the velocity vary, with changes in the shaker period. It is particularly convenient to plot the data in a form which gives a straight line when the damping force is proportional to the velocity. Equation (3), below, shows that this may be achieved by plotting values of the ratio V_Q/V_I against $(t^2-1)/2t$ (where t is the ratio of the shaker period to the natural period), as shown in Fig. 4. Then the damping factor ζ is given by $\zeta = \tan\theta$, where θ is the slope angle in Fig. 4.

It should be possible to separate by measurement the contributions which the building, and the foundations, make to the overall modal damping. The input power is easily derived, and the power supplied to the foundations can be obtained from those horizontal and vertical foundation motions which are 90° out of phase with the shaker force.

(1) A. R. Morman, P.M. Glenn, "Building Mode Shape Finder", Manual No. 93, Physics and Engineering Laboratory, D.S.I.R., New Zealand.

2. Ideal Modal Responses

It is useful to compare measured modal responses with the responses which would be obtained if only one viscously damped mode were excited; then it may be shown that

$$t^2 V_I = Kt(2\zeta t)/[(t^2-1)^2 + (2\zeta t)^2] \tag{1}$$

$$t^2 V_Q = Kt(t^2-1)/[(t^2-1)^2 + (2\zeta t)^2] \tag{2}$$

$$(t^2-1)/2t = \zeta V_Q / V_I \tag{3}$$

where $t = T/T_n$; K = constant.

Measured data should conform to these relationships if

(a) There are no significant responses of other modes
(b) The damping force is proportional to velocity.

(a) Response Vector It may be shown by equations (1) and (2) that a plot of $t^2 V_Q$ against $t^2 V_I$ gives a circle, tangent at the origin, and of diameter $K/2\zeta$. When measured data is thus plotted, as in Fig. 5, and a circle is drawn, then it is easy to see the degree to which the data conforms to equations (1) and (2).

3. Measurements on a 17 Storey Building

The system has been used to measure the modes of the 17-storey shear-tower Vogel Building. The periods, shapes, and dampings were measured for the first two modes, for transverse, longitudinal, and rotational motions. The shapes of the two transverse modes are shown in Fig. 3. Normalized response curves for transverse mode 1 are given in Fig. 2, the damping factor for torsional mode 2 is obtained from Fig. 4, and the normalized response-vector for torsional mode 2 is shown in Fig. 5. From Figs 4 and 5 it is seen that conditions (a) and (b) of section 2 are approximately satisfied.

4. Conclusion

A light-weight portable system for measuring the normal modes of structures has a number of operational advantages. Tests can be made on buildings with very little disturbance to the occupants. (The shaker does not require anchor bolts). Shaking forces can be applied at locations not available to heavier machines. The convenience of the system allows it to be used for the accumulation of data on a substantial number of buildings. Particular attention should be given to those containing strong-motion recorders. After a severe earthquake the system can be used for the rapid evaluation of damaged structures.

DATA FROM VOGEL BUILDING, WELLINGTON, NEW ZEALAND.

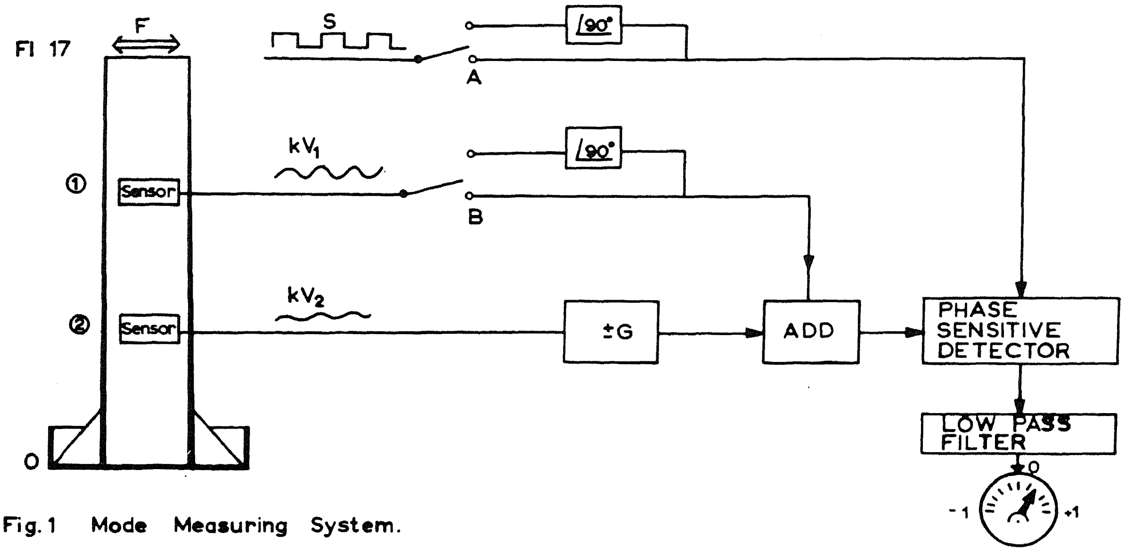

Fig. 1 Mode Measuring System.

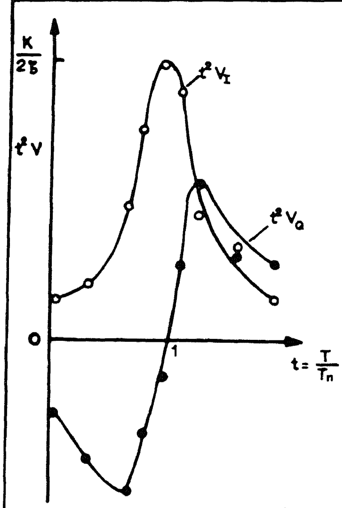

Fig. 2 Response Curves.
(Transverse Mode 1)

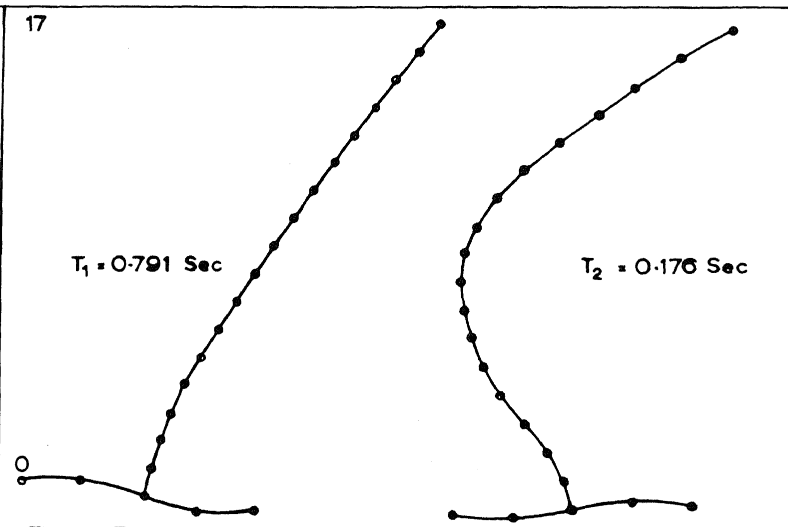

Fig. 3 Transverse Modes.

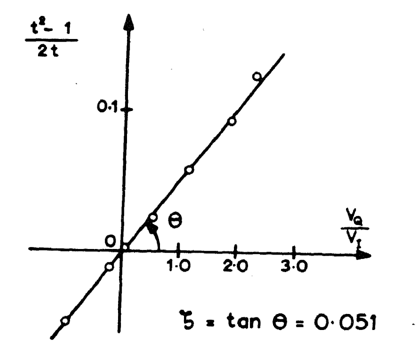

Fig. 4 Damping Factor.
(Torsional Mode 2)

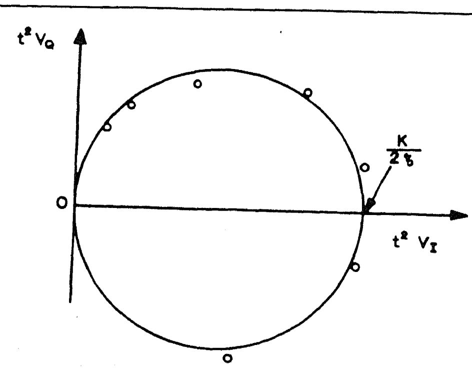

Fig. 5 Response Vector.
(Torsional Mode 2)