SUBSTITUTE DAMPING RATIOS FOR REINFORCED CONCRETE FRAMES IN THE POST-YIELD RANGE

by

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ABSTRACT. By equating the energy supplied to a test frame during a base disturbance to that which would have been dissipated had a linear dashpot been imparted the same velocity response as the original frame substitute damping ratios have been obtained. It is shown that these ratios are relatively independent of the maximum deformation. Comparison is made between the experimental displacement and acceleration maxima and those obtained from linear response spectra using substitute damping ratios and frequencies related to the maximum displacements. Implications on design are given.

NOTATION.

c = dashpot constant

k = stiffness; slope of the line joining the point on the force-displacement curve where maximum displacement occurs with origin

m = mass

t = time

T = total duration of the base disturbance

$x(t)$ = relative displacement; $\dot{x} = dx/dt$, $\ddot{x} = d^2x/dt^2$

$\ddot{y}(t)$ = base acceleration

β_s = substitute damping ratio

λ = "characteristic" circular frequency, (Eq. 3)

INTRODUCTION. The design engineer is frequently beset with the problem of ascertaining the amount of damping which should be included in the dynamic analysis of structures. Often, linear analyses prove to be more expedient so that damping becomes a matter of grave concern. To answer in part the needs of the designer in a viable way, the equivalent coefficient of viscous friction has been proposed (2), (3). In this paper, a different approach to determine experimentally substitute damping ratios for reinforced concrete frames is described.

THE TEST PROGRAM. Technological advances have made possible the operation of systems which allow the testing of structures or structural subcomponents under a variety of base motions (5). The University of Illinois Earthquake simulator was used for the experiments described herein. Two different-size series of frames, shown in Fig. 1, were tested. Each frame was bolted on the platform of the Simulator and then shaken either with sinusoidal or with simulated earthquake motions (1). Recordings were made on magnetic tapes of the base and the response (absolute) accelerations and the relative displacement of the top girder. A given frame was subjected to successive base disturbances. The dynamic response of the frames has analytically been predicted with good accuracy (1).

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SUBSTITUTE DAMPING RATIOS. Resonance tests have been conducted on structures to determine damping, e.g. (4). To determine the damping ratio for a reinforced concrete frame stressed past the yield limit by this approach would, however, be untenable as there is no unique and stable stiffness, or frequency, at which resonance occurs. To obtain a measure of the energy dissipated and then to express it in the familiar linear terms, the following approach was adopted:

It can be shown that the total energy supplied to a one-degree-of-freedom oscillator subjected to a base acceleration of $\ddot{y}(t)$ for a total duration of T seconds is given by

$$\text{Energy Input} = -m \int_0^T \ddot{y}(t)\ \dot{x}(t)\ dt \qquad (1)$$

Equation (1) holds irrespective of the spring properties of the oscillator. The input energy is then dissipated by hysteresis, damping (linearly viscous or otherwise), heat, etc. If it is supposed that all of the input energy is dissipated within a hypothetical dashpot of constant c which is imparted the same relative velocity, $\dot{x}(t)$, as the oscillator, this constant can be found from the relation

$$c = -m \int_0^T \ddot{y}(t)\ \dot{x}(t)\ dt \Big/ \int_0^T \left[\ \dot{x}(t)\ \right]^2 dt \qquad (2)$$

In the test program, all quantities on the right hand side of Eq. 2 were either known a priori, or measured, or could be derived from measured quantities. To relate c to the properties of the system for which it was derived, a "characteristic" circular frquency, λ, for a given test frame and for a given test run, was defined as

$$\lambda = \sqrt{\left|\ \ddot{x}(t) + \ddot{y}(t)\ \right|_{max} \Big/ \left|\ x(t)\ \right|_{max}} \qquad (3)$$

where the square root sign extends over the entire expression. It can be shown that neglecting the "damping" force, λ can be found by taking the square root of the ratio of the stiffness obtained by joining the maximum displacement on the force-displacement curve for a frame with the origin and its mass. In Fig. 2, the qualitative force-displacement curve for a reinforced concrete frame stressed to ultimate is shown along with two different "stiffnesses". Replacing c in Eq. 2 by $2\ \beta_s\ m\ \lambda$, the substitute damping ratio, β_s, was then obtained for each frame for each run. In Fig. 3, the variation of β_s with the ductility ratio, i.e., the ratio of the maximum displacement to the yield displacement, is shown. For small values of ductility, i.e., small amounts of incurred damage, the substitute damping ratios are also small. For ductility ratios of 2 to 4, β_s generally tends to remain below 0.12, and even for ductility ratios of 6 or 7 it is below 0.15. The qualitative similarity between the average dashed line passed through the experimental points and those given by Hudson (2) should be noted.

TEST RESULTS VS RESPONSE SPECTRUM VALUES. To provide an indirect check of the data reduction and to obtain a vehicle for using substitute damping ratios for preliminary design, the test results were reevaluated "backwards" by using the response spectrum technique with the experimentally determined values of frequency (Eq. 3), and the damping ratio obtained from Eq. 2.

Using the numerical data for the response spectrum for each base acceleration, the spectral displacement and acceleration corresponding to each test frame and for each run were compared with the experimentally measured maximum displacement and absolute acceleration. This comparison is summarized in Fig. 4. In view of the usual jagged form common to response spectra and the wide differences between values found for a given frequency but for different damping ratios, the agreement is very good. It should be noted that the mean value for the ratio of spectral to measured acceleration for all data points is 0.99, and that for displacements, 0.98.

DESIGN IMPLICATIONS. Although caution should be exercised not to draw too universal conclusions from the results presented, it may be stated that the substitute damping ratio, a measure of the energy dissipation over the entire duration of the base disturbance (as opposed to the equivalent damping ratio which is based on the energy dissipated per cycle in steady-state response) is a slowly varying function of the ductility. The close agreement between spectral and experimental maxima (Fig. 4) suggests that linear response spectrum curves may be used to estimate the deformation of similar reinforced concrete frames. Given the "design" spectrum, the maximum displacement may be estimated, and with the dimensions and material properties known, the force-displacement curve for the frame can be calculated. Joining the estimated maximum displacement on this curve with the origin (such as k_1 in Fig. 2), a stiffness is obtained. λ is given by the square root of the ratio of this stiffness to the mass of the structure. With the ductility ratio known from the estimated displacement, the damping ratio to be used in design may be obtained from Fig. 3. Then, armed with a frequency and a damping ratio, the design spectrum may be entered to find the spectral displacement. If this value and the original estimate agree, design should be based on the corresponding ductility; otherwise the spectral displacement should be taken as the new estimate and the previous steps covered again (1).

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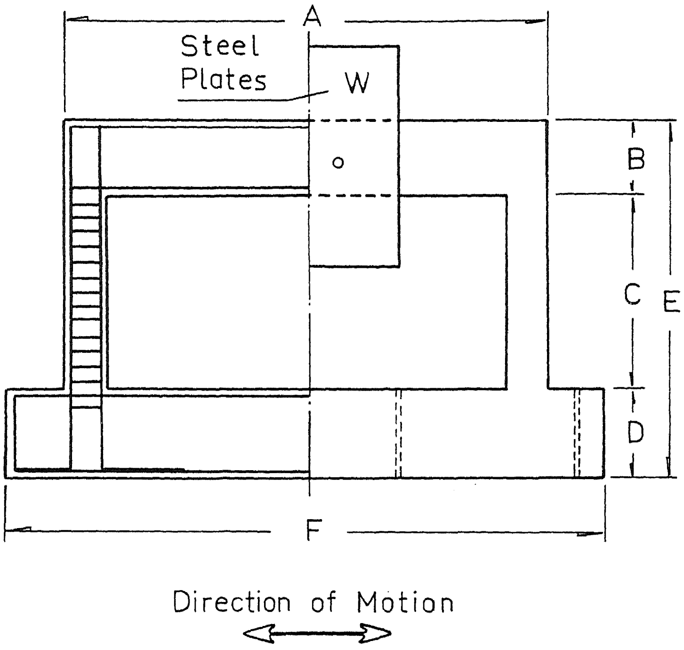

Series	A in.	B in.	C in.	D in.	E in.	F in.	f'_c psi	f_y ksi	W lb
1	32	5	13	6	24	40	5700	40	1400
2	64	10	26	12	48	80	5400	49	4000

FIG. 1. TEST FRAMES

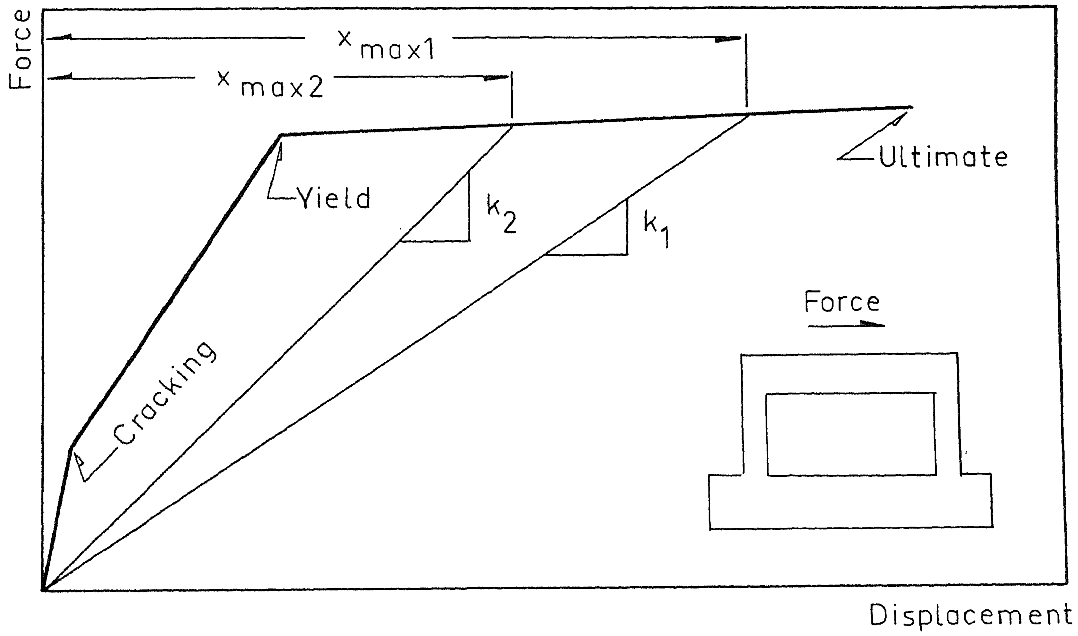

FIG. 2. FORCE-DISPLACEMENT CURVE FOR A FRAME

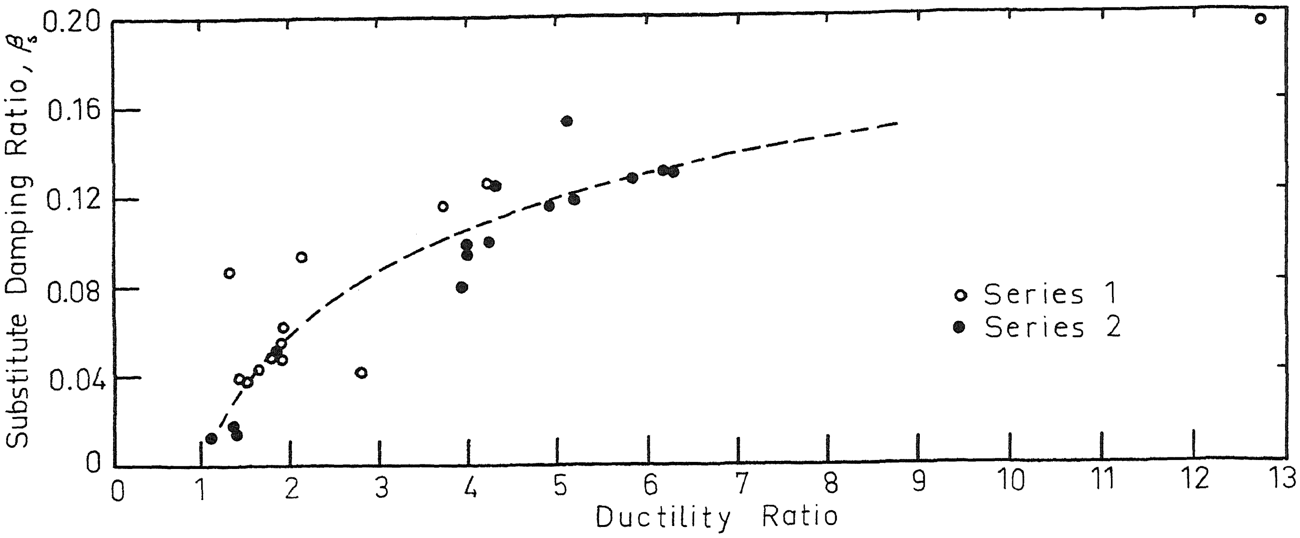

FIG. 3. VARIATION OF SUBSTITUTE DAMPING WITH THE DUCTILITY RATIO

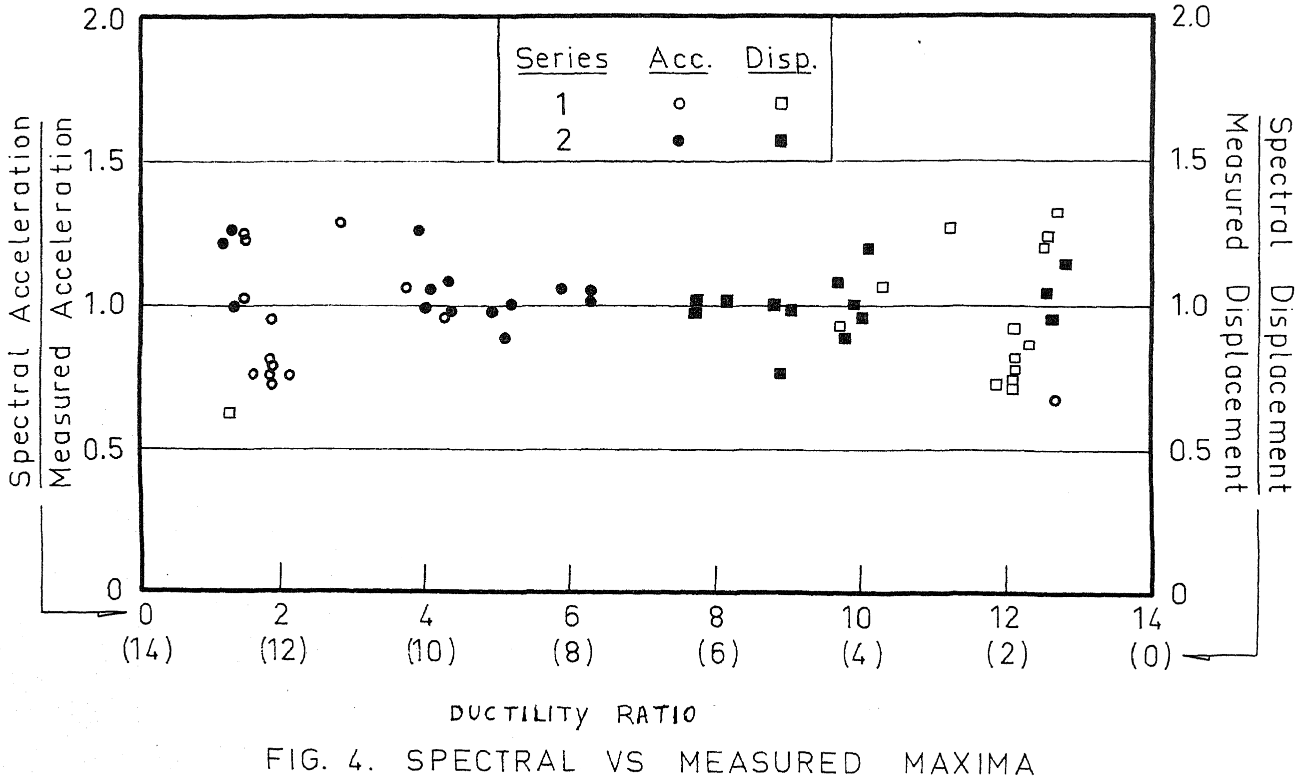

FIG. 4. SPECTRAL VS MEASURED MAXIMA