A NEW MEASURING METHOD OF VIBRATION USING CORRELATION TECHNIQUE

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A new measuring method named "MIK System", using a correlator, is used to measure the vibration response of structures caused by a vibrator.

I. Introduction

The vibration test of many large-scale structures such as dams, nuclear power plants, high rise buildings, etc. has become very important for earthquake resistant design. The highlight of the MIK System is to make the measurement of these structures possible, even if the response caused by a vibrator is small and regardless of the noise mixed in. Because this system can exclude the disturbing noise from the measured vibration, real response and phase angle of the structure can be obtained accurately.

All measuring apparatus of this system are installed in a large automobile named "MIK Measuring Van" having speedy mobility to anywhere in Japan. (Refer to Fig. 1 and Fig. 2)

II. Fundamental Principle

The most important part of the MIK System is a real time digital correlator, shown in Fig. 2, which can calculate the cross correlation function between the synchronizing sinusoidal signal and the measured vibration signal.

Expressed below is the exciting force $F(t)$, the synchronizing sinusoidal signal from the vibrator $x(t)$, the response of the structure $y(t)$, the accompanied noise $n(t)$ and the measured vibration $z(t)$: (Refer to Fig. 3)

$$ F(t) = P \cdot \cos \omega_0 t \quad (1) $$
$$ x(t) = \cos \omega_0 t \quad (2) $$
$$ y(t) = A \cdot \cos (\omega_0 t - \theta) \quad (3) $$
$$ z(t) = y(t) + n(t) \quad (4) $$

Where,

$T_0 = \frac{2\pi}{\omega_0}$

$T_0$ : period of the vibrator  
$\omega_0$ : circular frequency of the vibrator

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\[ P = M \cdot r \cdot \omega_0^2 \]

- \( P \): maximum exciting force
- \( M \): effective eccentric mass
- \( r \): eccentric radius

- \( A \): response amplitude
- \( \theta \): phase angle of the response relative to the vibrator

Then, the cross correlation function \( \Phi_{xz}(\tau) \) between \( x(t) \) and \( z(t) \) is expressed in the following form:

\[
\Phi_{xz}(\tau) = \frac{1}{T} \int_{0}^{T} x(t) \cdot z(t+\tau) dt
\]

\[
= \frac{1}{T} \int_{0}^{T} x(t) \cdot y(t+\tau) dt + \frac{1}{T} \int_{0}^{T} x(t) \cdot n(t+\tau) dt
\]

\[
= \Phi_{xy}(\tau) + \Phi_{xn}(\tau)
\]

(5)

Where, \( T \): observation time
\( T_d \): shifted time

Thus, the number of waves \( m \) during \( T \) is \( T/T_0 \). In case where \( m \) is large enough, the second noise term \( \Phi_{xn}(\tau) \) generally approaches zero and Eq.(5) becomes as follows: (Refer to Fig. 10)

\[
\Phi_{xz}(\tau) = \Phi_{xy}(\tau)
\]

\[
= \frac{1}{2} A \cdot \cos(\omega_0 \tau - \theta)
\]

(6)

Fig. 4 shows an example of \( \Phi_{xz}(\tau) \) curve. Where, the vertical axis is \( \Phi_{xz} \) value and the horizontal axis is \( \tau \) value. Response amplitude \( A \) can be obtained from the peak value of \( \Phi_{xz} \), while time lag \( T_d \) is determined by the \( \tau \) value of the first peak and then \( T_d \) can be transferred to \( \theta \) by \( \theta = \omega_0 T_d \).

III. MK System

Fig. 5 shows the block diagram of the correlator, whose function is described as follows:

1. \( x(t) \) and \( z(t) \)

\( x(t) \) converted into the sinusoidal signal by the differential transducer and the cam of the vibrator is kept so as to have the maximum voltage of 1 volt and enters into the A/D CONVERTER, while \( z(t) \) converted into the voltage by the transducer enters into another A/D CONVERTER.

2. Adjustment

There are five special data in the first part of \( \Phi_{xz}(\tau) \) curve in Fig. 4 and they are in order \( x^2, z, -x, z^2 \) and \( \bar{z} \), to be used for
data control for the correlator. $\bar{x}$, $-\bar{x}$ and $\bar{z}$; voltage drifts, must be adjusted to zero voltage by BIAS, while $\bar{x}^2$ and $z^2$, mean squares of voltages, should be controlled for calculation of $\Phi_{xz}(\tau)$ by GAIN.

(3) A/D CONVERTER

Because the correlator is a digital computer, both $x(t)$ and $z(t)$ must be digitized by the A/D CONVERTER. The scanning time intervals of $x$-sampling ($\Delta t$) and $z$-sampling ($k \cdot \Delta t$) are selected considering the interval of shifted time $\Delta t$ and vibration frequency.

(4) Cross correlation function

$\Phi_{xz}(\tau)$ is calculated by the MULTIPLIER and stored in MEMORY 3. In this case, AVERAGE value or INTEGRATE value can be selected. As the result, $\Phi_{xz}(\tau)$ appears on the monitor in a real time and is recorded on the oscillograph.

(5) SPECTRUM ANALYZER

SPECTRUM ANALYZER is also installed in the van. A cross spectrum or power spectrum can be obtained in as short a time as 0.2 sec.

IV. Application Example

The Fukushima Nuclear Power Plant No. 1 (boiling water type reactor) of Tokyo Electric Power Company shown in Fig. 6 was tested by the MK system in 1969.

In order to explain the vibration characteristics of the nuclear power plant from a theoretical point of view, the test results were simulated by computer analysis using the vibration model shown in Fig. 7. The analysis was performed based on the assumption that each structural viscous damping.

In this case the equation is expressed as follows:

$$[M]\ddot{\{x\}} + [C]{\dot{x}} + [K]\{x\} = \{P\}$$

Where,

$[M]$ : mass matrix
$[C]$ : damping matrix ($[C] = [r_sK_s + r_dK_d + r_stK_st]$)
$[K]$ : stiffness matrix
$\{P\}$ : exciting force vector
$\{\ddot{x}\}, \{\dot{x}\}, \{x\}$ : acceleration, velocity and displacement vectors, respectively

The following damping coefficients were used:

<table>
<thead>
<tr>
<th>Material</th>
<th>$r_s$</th>
<th>$r_d$</th>
<th>$r_st$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil</td>
<td>0.0414</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete</td>
<td>0.00064</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>0.00032</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 8 shows the resonance curve of the horizontal movement at the fifth floor. The first peak corresponds to the first resonance mode of the buildings, as shown in Fig. 9. On the other hand, the second and the third peak correspond to the resonance mode of the ST (Steel Truss) and the RPV (Reactor Pressure Vessel), respectively. The irregularity seen in the test resonance curve may be based on the resonance or vibration absorption by the vibration elements which are not considered in the vibration model.

Fig. 9 illustrates the vibration mode at the above mentioned first resonance peak. Though both the building and the foundation seem to vibrate in the same direction, a time lag of the foundation relative to the fifth floor is found to be 0.04sec in test (0.03sec in analysis). This may be based on the great difference of the damping coefficients between the building and the soil.

V. Summary

It is possible to obtain the true response of structures in a real time, because all noise can be excluded by the MIK System. Furthermore, as all apparatus are installed in the van, the measurement can be easely performed anywhere in Japan.

The applications of the MIK System from 1969 through 1972 are as follows:

<table>
<thead>
<tr>
<th>Category</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear Power Plant</td>
<td>2</td>
</tr>
<tr>
<td>High-rise building</td>
<td>6</td>
</tr>
<tr>
<td>Tower and Stack</td>
<td>5</td>
</tr>
<tr>
<td>Bridge</td>
<td>1</td>
</tr>
<tr>
<td>Sea berth</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>15 cases</td>
</tr>
</tbody>
</table>

The MIK System has applications of not only the measurement described in this paper but also wider uses such as the measurement of shear wave velocity and spectrum.

In the future, we intend to continue accumulating data on establishing reasonable earthquake resistant designs, by measuring vibration of many structures by making use of the MIK System.

We would like to acknowledge the continuing guidance of Dr. Yasushi Ishii of Institute of Space and Aeronautical Science, University of Tokyo.

REFERENCE

FIG. 1 MIK MEASURING VAN

FIG. 2 MEASURING ROOM
FIG. 3 FORCED VIBRATION TEST

FIG. 4 CROSS CORRELATION FUNCTION
FIG. 5 BLOCK DIAGRAM OF CORRELATOR
FIG. 6 FUKUSHIMA NUCLEAR POWER PLANT NO.1

\[ F_1, F_2, F_3: \text{FOUNDATION} \]
\[ W_1, W_2, W_3: \text{WALL} \]

\[ ST: \text{STEEL TRUSS} \]
\[ Sw: \text{SHIELD WALL} \]
\[ RPV: \text{REACTOR PRESSURE VESSEL} \]
\[ GSW: \text{Y-SHIELD WALL} \]
\[ PCV: \text{PRIMARY CONTAINMENT VESSEL} \]
\[ \odot: \text{RESONANCE CURVES ARE SHOWN IN FIG.8.} \]

FIG. 7 OUTLINE OF PLANT AND VIBRATION MODEL
FIG. 8 RESONANCE CURVES AT FIFTH FLOOR

FIG. 9 VIBRATION MODE SHAPES AT FIRST PEAK
If there is any stationary wave having the same frequency as that of the vibrator, eq. (5) and eq. (6) are not applicable.