A NEW MEASURING METHOD OF VIBRATION USING CORRELATION TECHNIQUE

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A new measuring method named "MIK System", using a correlator, is used to measure the vibration response of structures caused by a vibrator.

I. Introduction

The vibration test of many large-scale structures such as dams, nuclear power plants, high rise buildings, etc. has become very important for earthquake resistant design. The highlight of the MIK System is to make the measurement of these structures possible, even if the response caused by a vibrator is small and regardless of the noise mixed in. Because this system can exclude the disturbing noise from the measured vibration, real response and phase angle of the structure can be obtained accurately.

All measuring apparatus of this system are installed in a large automobile named "MIK Measuring Van" having speedly mobility to anywhere in Japan. (Refer to Fig. 1 and Fig. 2)

II. Fundamental Principle

The most important part of the MIK System is a real time digital correlator, shown in Fig. 2, which can calculate the cross correlation function between the synchronizing sinusoidal signal and the measured vibration signal.

Expressed below is the exciting force $F(t)$, the synchronizing sinusoidal signal from the vibrator $x(t)$, the response of the structure $y(t)$, the accompanied noise $n(t)$ and the measured vibration $z(t)$: (Refer to Fig. 3)

$$F(t) = P\cdot\cos\omega_0 t \qquad (1)$$

$$x(t) = \cos\omega_0 t \qquad (2)$$

$$y(t) = A\cdot\cos(\omega_0 t - \theta) \qquad (3)$$

$$z(t) = y(t) + n(t) \qquad (4)$$

Where, $T_0 = 2\pi/\omega_0$

T_0 : period of the vibrator
ω_0 : circular frequency of the vibrator

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$$P = M \cdot r \cdot \omega_0{}^2$$

P : maximum exciting force
M : effective eccentric mass
r : eccentric radius

A : response amplitude

θ : phase angle of the response relative to the vibrator

Then, the cross correlation function $\Phi xz(\tau)$ between $x(t)$ and $z(t)$ is expressed in the following form:

$$\begin{aligned} \Phi xz(\tau) &= \frac{1}{T}\int_0^T x(t)\cdot z(t+\tau)\,dt \\ &= \frac{1}{T}\int_0^T x(t)\cdot y(t+\tau)\,dt + \frac{1}{T}\int_0^T x(t)\cdot n(t+\tau)\,dt \\ &= \Phi xy(\tau) + \Phi xn(\tau) \end{aligned} \tag{5}$$

Where, T : observation time
τ : shifted time

Thus, the number of waves m during T is T/T_0. In case where m is large enough, the second noise term $\Phi xn(\tau)$ generally approaches zero and Eq.(5) becomes as follows: (Refer to Fig. 10)

$$\begin{aligned} \Phi xz(\tau) &= \Phi xy(\tau) \\ &= \frac{1}{2} A\cdot \cos(\omega_0\tau - \theta) \end{aligned} \tag{6}$$

Fig. 4 shows an example of $\Phi xz(\tau)$ curve. Where, the vertical axis is Φxz value and the horizontal axis is τ value. Response amplitude A can be obtained from the peak value of Φxz, while time lag τ_d is determined by the τ value of the first peak and then τ_d can be transferred to θ by $\theta = \omega_0\tau_d$.

III. MIK System

Fig. 5 shows the block diagram of the correlator, whose function is described as follows:

(1) $x(t)$ and $z(t)$

$x(t)$ converted into the sinusoidal signal by the differential transducer and the cam of the vibrator is kept so as to have the maximum voltage of 1 volt and enters into the A/D CONVERTER, while $z(t)$ converted into the voltage by the transducer enters into another A/D CONVERTER.

(2) Adjustment

There are five special data in the first part of $\Phi xz(\tau)$ curve in Fig. 4 and they are in order $\overline{x^2}$, $\overline{x}$, $-\overline{x}$, $\overline{z^2}$ and $\overline{z}$, to be used for data control for the correlator. $\bar{x}$, $-\bar{x}$ and $\bar{z}$; voltage drifts, must be adjusted to zero voltage by BIAS, while x^2 and z^2, mean squares of voltages, should be controled for calculation of $\Phi xz(\tau)$ by GAIN.

(3) A/D CONVERTER

Because the correlator is a digital computer, both $x(t)$ and $z(t)$ must be digitized by the A/D CONVERTER. The scanning time intervals of x-sampling ($\Delta\tau$) and z-sampling ($k \cdot \Delta\tau$) are selected considering the interval of shifted time $\Delta\tau$ and vibration frequency.

(4) Cross correlation function

$\Phi xz(\tau)$ is calculated by the MULTIPLIER and stored in MEMORY 3. In this case, AVERAGE value or INTEGRATE value can be selected. As the result, $\Phi xz(\tau)$ appears on the monitor in a real time and is recorded on the oscillograph.

(5) SPECTRUM ANALYZER

SPECTRUM ANALYZER is also installed in the van. A cross spectrum or power spectrum can be obtained in as short a time as 0.2 sec.

IV. Application Example

The Fukushima Nuclear Power Plant No. 1[1)] (boiling water type reactor) of Tokyo Electric Power Company shown in Fig. 6 was tested by the MIK system in 1969.

In order to explain the vibration characteristics of the nuclear power plant from a theoretical point of view, the test results were simulated by computer analysis using the vibration model shown in Fig. 7. The analysis was performed based on the assumption that each structural element had the different internal viscous damping.

In this case the equation is expressed as follows:

$$[M]\{x''\} + [C]\{x'\} + [K]\{x\} = \{P\}$$

Where,
- $[M]$: mass matrix
- $[C]$: damping matrix ($[C] = [r_s K_s + r_c K_c + r_{st} K_{st}]$)
- $[K]$: stiffness matrix
- $\{P\}$: exciting force vector
- $\{x''\}$, $\{x'\}$, $\{x\}$: acceleration, velocity and displacement vectors, respectively

The following damping coefficients were used:

Soil	r_s = 0.0414	sec
Concrete	r_c = 0.00064	sec
Steel	r_{st} = 0.00032	sec

Fig. 8 shows the resonance curve of the horizontal movement at the fifth floor. The first peak corresponds to the first resonance mode of the buildings, as shown in Fig. 9. On the other hand, the second and the third peak correspond to the resonance mode of the ST (Steel Truss) and the RPV (Reactor Pressure Vessel), respectively. The irregularity seen in the test resonance curve may be based on the resonance or vibration absorption by the vibration elements which are not considered in the vibration model.

Fig. 9 illustrates the vibration mode at the above mentioned first resonance peak. Though both the building and the foundation seem to vibrate in the same direction, a time lag of the foundation relative to the fifth floor is found to be 0.04sec in test (0.03sec in analysis). This may be based on the great difference of the damping coefficients between the building and the soil.

V. Summary

It is possible to obtain the true response of structures in a real time, because all noise can be excluded by the MIK System. Furthermore, as all apparatus are installed in the van, the measurement can be easely performed anywhere in Japan.

The applications of the MIK System from 1969 through 1972 are as follows:

Nuclear Power Plant	:	*2 cases*
High-rise building	:	*6*
Tower and Stack	:	*5*
Bridge	:	*1*
Sea berth	:	*1*
Total	:	*15 cases*

The MIK System has applications of not only the measurement described in this paper but also wider uses such as the measurement of shear wave velocity and spectrum.

In the future, we intend to continue accumulating data on establishing reasonable earthquake resistant designs, by measuring vibration of many structures by making use of the MIK System.

We would like to acknowledge the continuing guidance of Dr. Yasushi Ishii of Institute of Space and Aeronautical Science, University of Tokyo.

REFERENCE

1) K. Muto and K. Omatsuzawa; "Earthquake Response Analysis for a BWR Nuclear Power Plant Using Recorded Data," an invited paper at the First International Conference on Structural Mechanics in Reactor Technology, Berlin, 20-24, September 1971, published on Nuclear Engineering and Design, Vol. 20, No.2; North Holland Publishing Company, Amsterdam.

FIG. 1 MIK MEASURING VAN

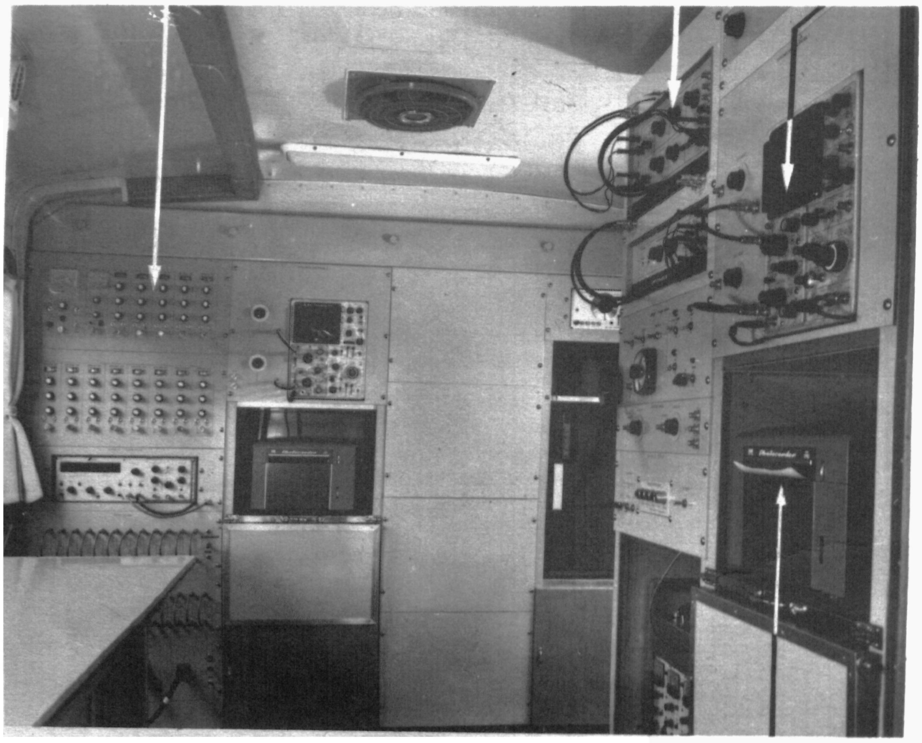


FIG. 2 MEASURING ROOM

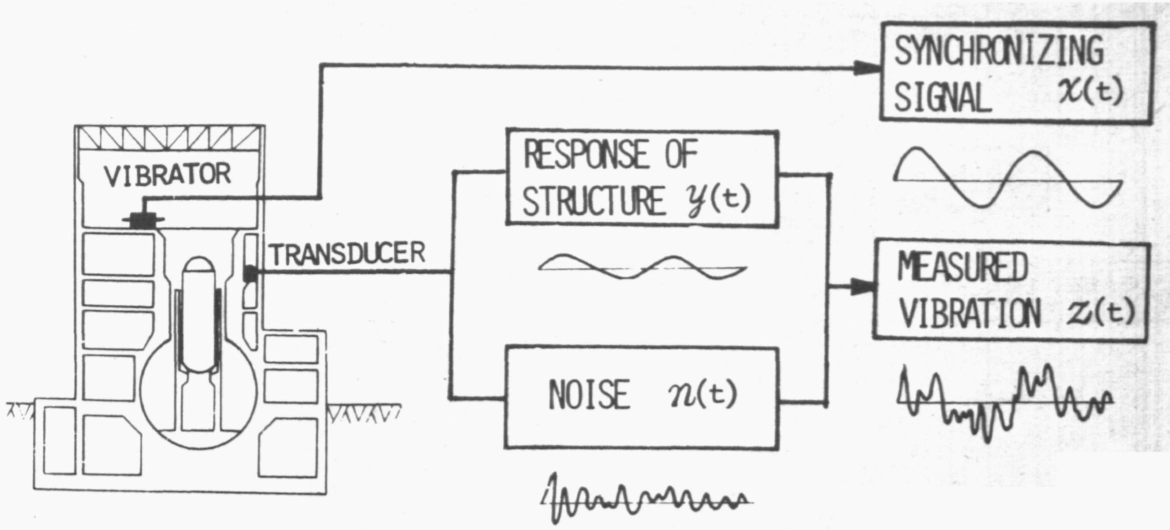


FIG. 3 FORCED VIBRATION TEST

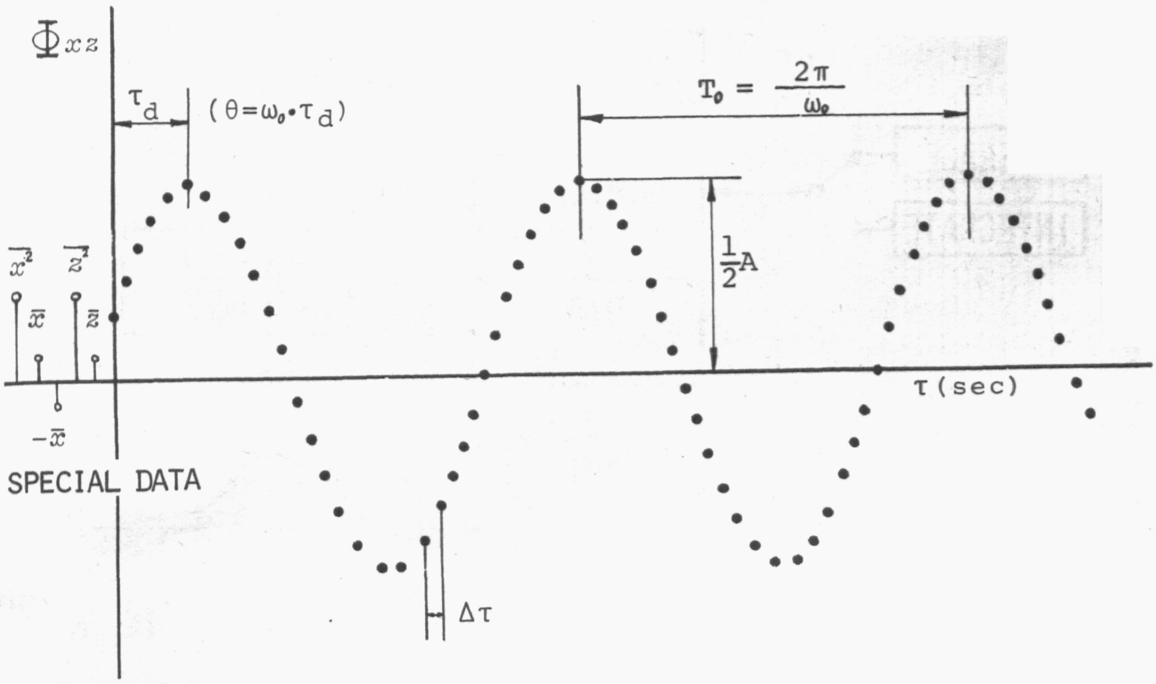


FIG. 4 CROSS CORRELATION FUNCTION

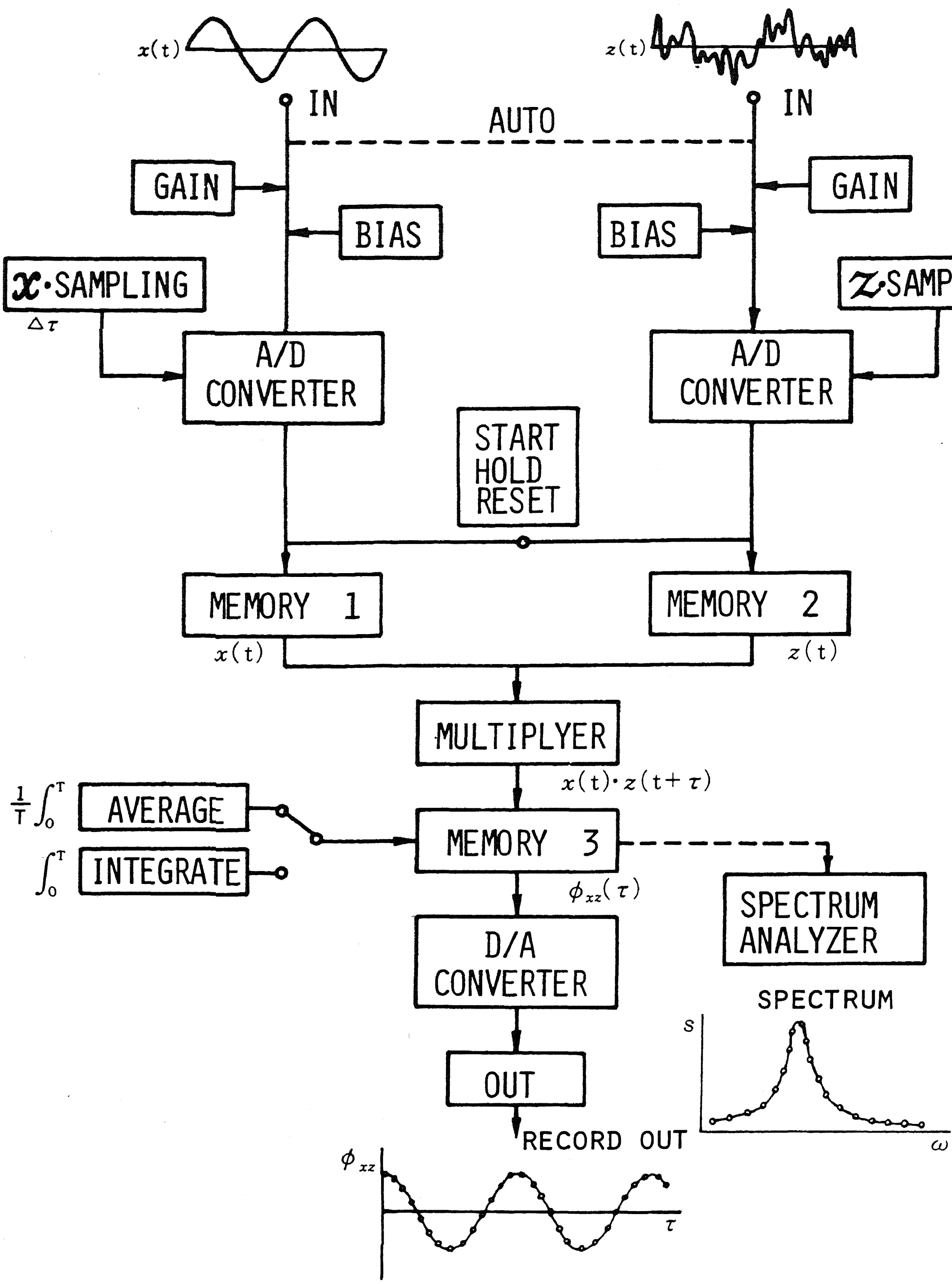


FIG. 5 BLOCK DIAGRAM OF CORRELATOR

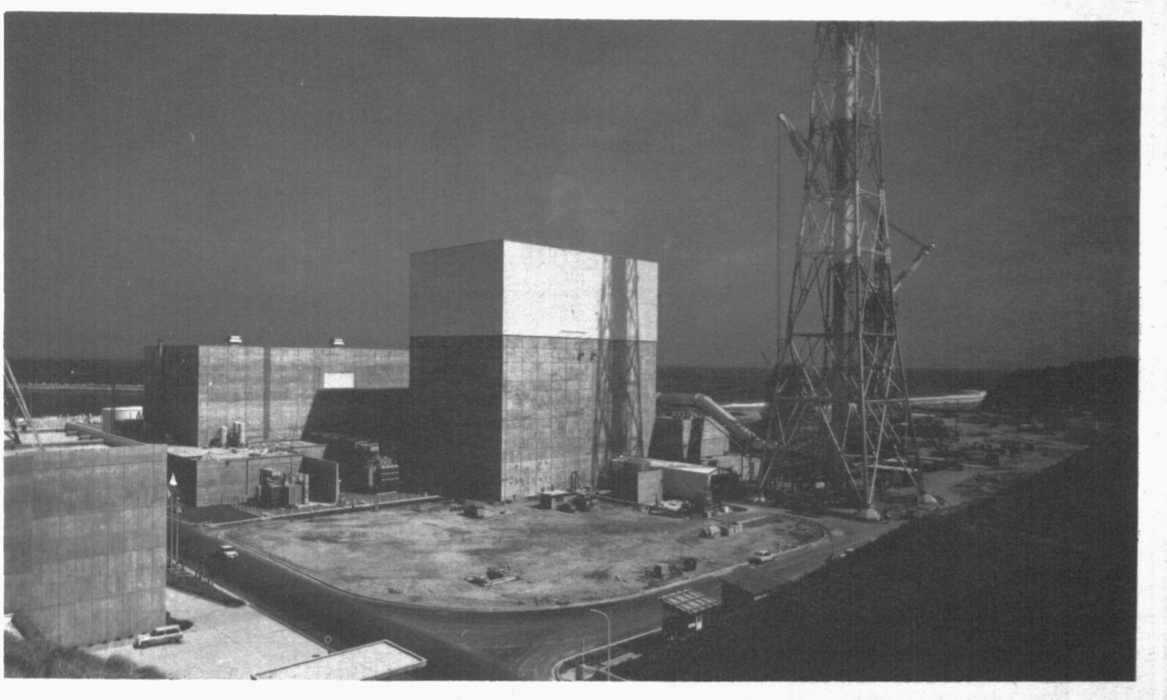


FIG. 6 FUKUSHIMA NUCLEAR POWER PLANT NO.1

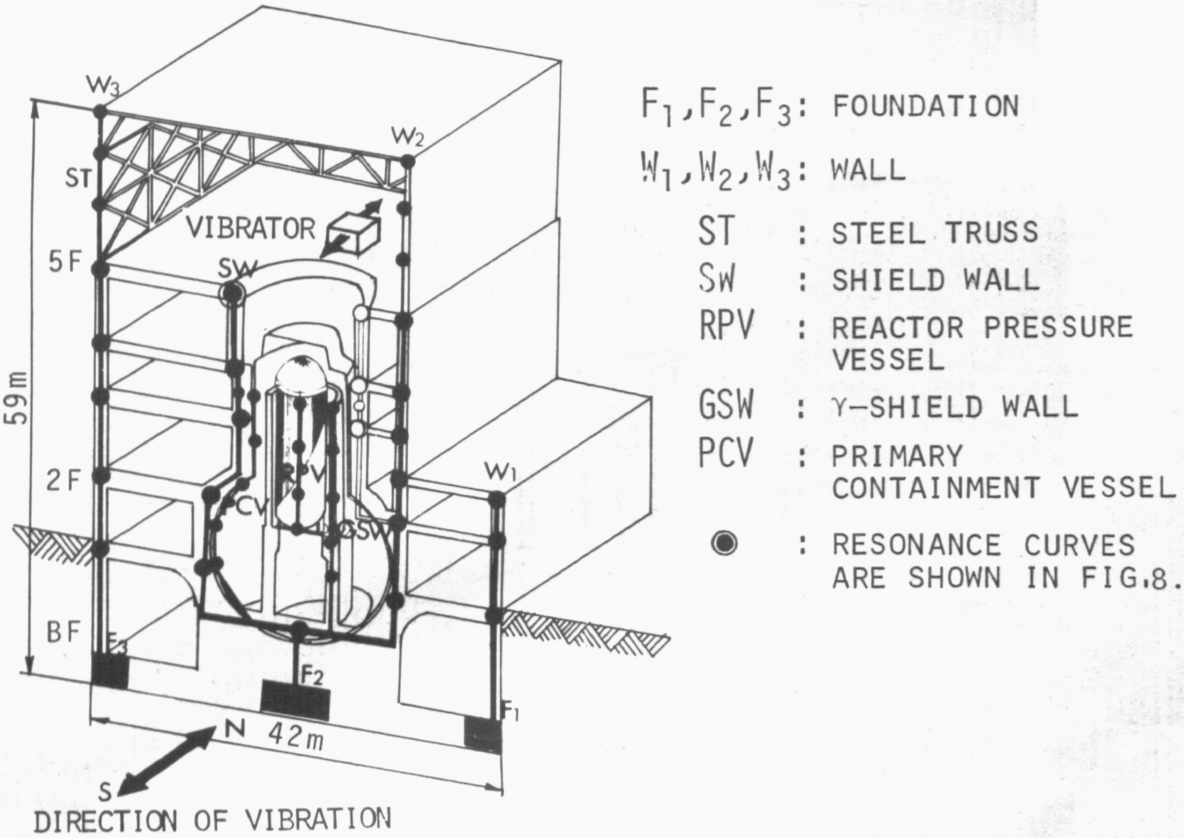


FIG. 7 OUTLINE OF PLANT AND VIBRATION MODEL

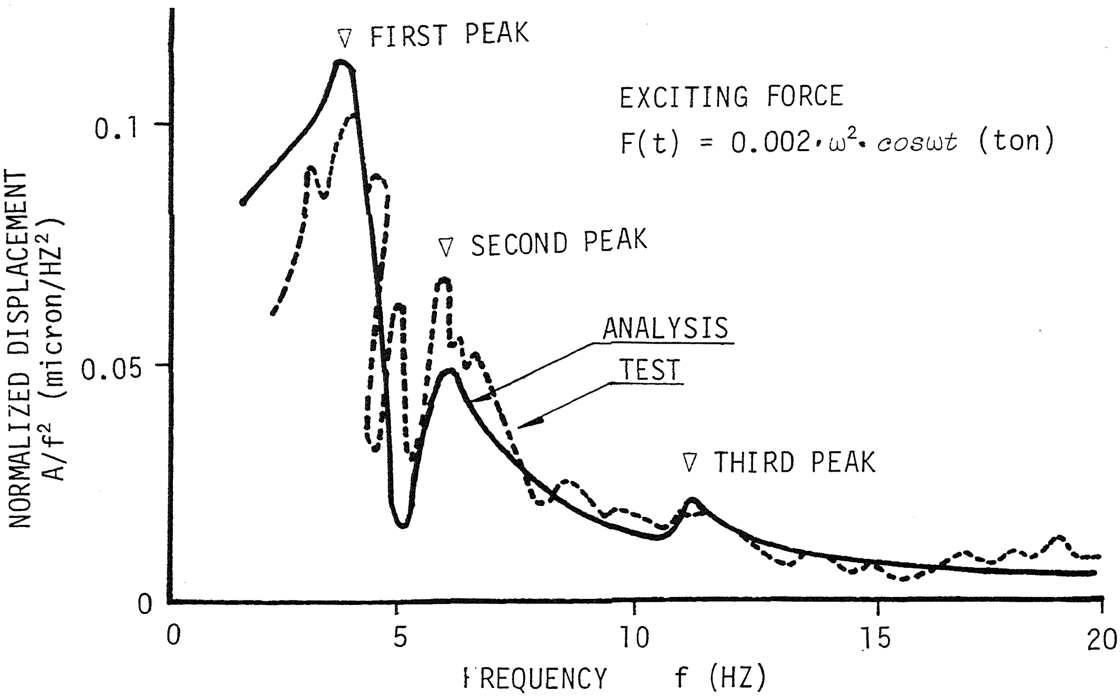


FIG. 8 RESONANCE CURVES AT FIFTH FLOOR

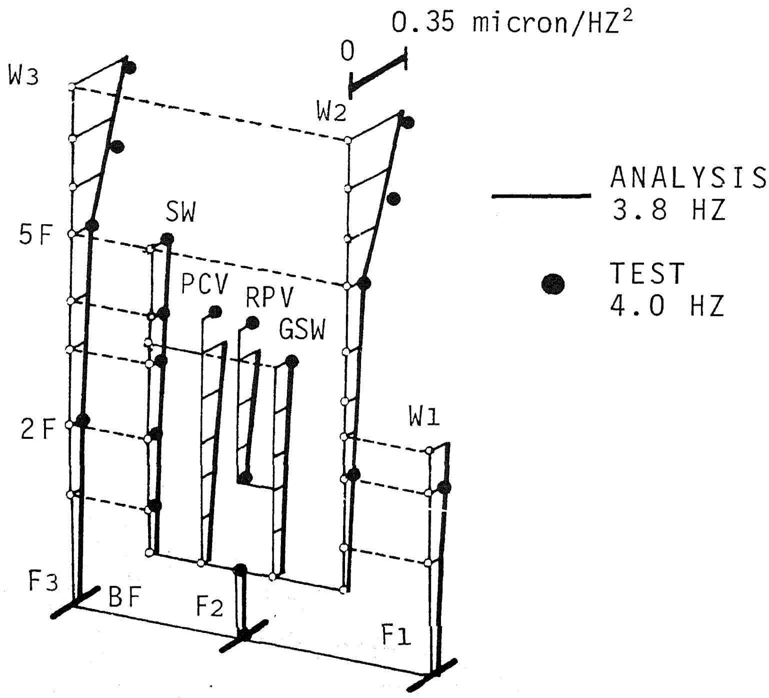
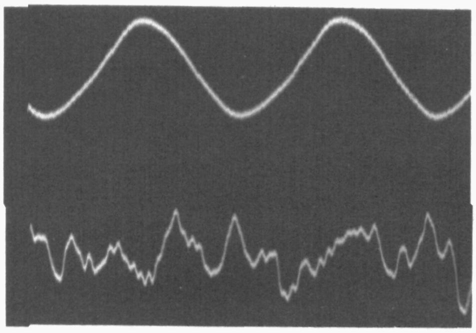
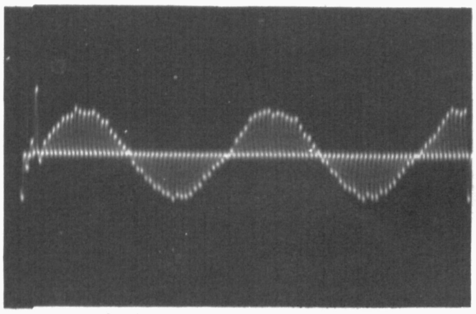


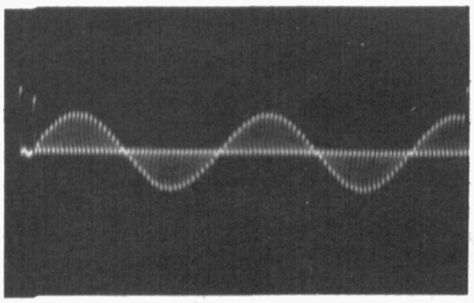
FIG. 9 VIBRATION MODE SHAPES AT FIRST PEAK



(1) upper: vibrator signal $x(t)$
lower: measured vibration
$z(t) = y(t) + n(t)$

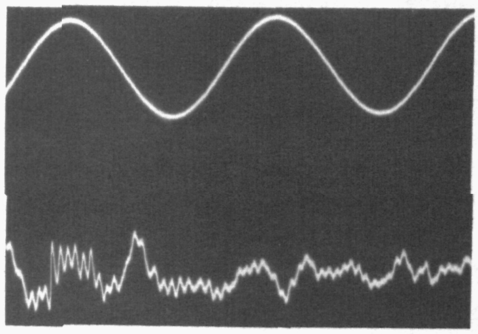


(2) $\Phi xz(\tau)$ is still unstable
after integrating 5 waves.

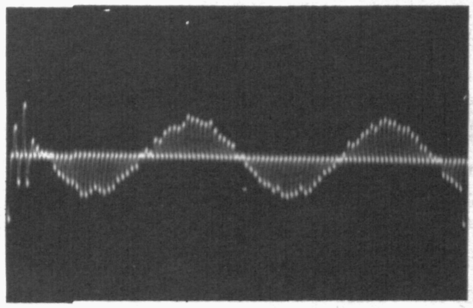


(3) $\Phi xz(\tau)$ becomes clear
after integrating 140 waves.

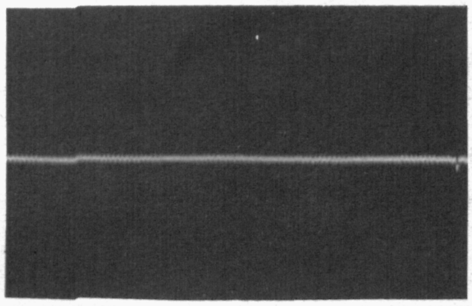
$\Phi xz(\tau)$ BY USE OF VIBRATOR



(1) upper: sinusoidal signal $\overline{x}(t)$
from function generator
lower: measured noise
$z(t) = n(t)$ $(y(t) = 0)$



(2) $\Phi \overline{x}z(\tau)$ is still existing
after integrating 5 waves.



(3) $\Phi \overline{x}z(\tau)$ becomes nearly zero
after integrating 140 waves.

$\Phi \overline{x}z(\tau)$ BY NO USE OF VIBRATOR

If there is any stationary wave having the same frequency as that of the vibrator, eq.(5) and eq.(6) are not applicable.

FIG. 10 EXAMPLE SHOWING CHARACTERISTICS OF CORRELATOR
FOLLOW UP ILLUSTRATION OF EQ.(5) AND EQ.(6)