EARTHQUAKE BEHAVIOR OF ARCH DAM-RESERVOIR SYSTEMS

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SYNOPSIS

An interaction model to determine the earthquake behavior of a coupled arch dam-reservoir system is presented. The modal properties of the dam and the complex frequency responses of the hydrodynamic pressures acting on the dam are used to define the interaction model. Responses of several dam-reservoir systems to Taft earthquake are obtained by the Fast Fourier Transform method. These results are compared with approximate solutions which neglect interaction and water compressibility. Earthquake interaction between the arch dam and reservoir is found to be important and should therefore be considered in the aseismic design of arch dams.

INTRODUCTION

Destructive earthquakes have caused in the past extensive damage of property and loss of life. Structures should therefore be designed to safely withstand earthquakes in regions of active seismicity. Aseismic design of dams is especially important due to the catastrophic consequences of failure of dams. In conventional structures, such as buildings, damage due to an earthquake is usually confined to the structure and its occupants, if any. On the other hand, failure of a dam causes extensive destruction of property and loss of life on the downstream side due to the sudden release of a large quantity of water in addition to the damage to the dam itself. Therefore, dams should be adequately designed to function safely during strong earthquakes.

When a dam-reservoir system is subjected to an earthquake, hydrodynamic pressures in excess of the usual hydrostatic pressures are set up on the dam due to the vibration of the dam and reservoir. These dynamic water pressures and the deformation of the dam interact with each other. Therefore, any satisfactory method of analysis should treat the system as a coupled dynamic interaction problem. The earthquake behavior of concrete arch dam-reservoir systems is the subject of this investigation.

The present design practice for considering hydrodynamic pressures on arch dams(5,7,8) is based on earlier investigations of gravity dams(16). Hydrodynamic pressures acting on gravity dams have been studied in considerable detail(1,10,16). More recently, Chopra(2,3) has investigated the earthquake interaction between concrete gravity dams and reservoirs. The use of gravity dam solutions for the aseismic design of arch dams is

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questionable due to the obvious differences between the geometries of the two types of dams. Specifically, the motion of banks of the arch dam contribute significantly to the hydrodynamic pressures(9,11). Therefore, the arch dam-reservoir system should be treated as a three-dimensional problem.

Assuming water to be incompressible, earthquake hydrodynamic pressures acting on rigid arch dams have been studied(18,19). Recent studies dealing with hydrodynamic pressures on rigid arch dams during earthquakes have demonstrated that significant errors are involved in solutions based on the assumption of incompressible water(9,12). Therefore, water compressibility should be considered in evaluating earthquake hydrodynamic pressures acting on dams. Earthquake responses of flexible arch dams based on the assumptions that the interaction between the dam and reservoir and water compressibility can be neglected have been reported in the literature(6,13,15). While these assumptions greatly simplify the complexity of the problem, significant errors may be involved in the solutions. The present investigation treats the earthquake behavior of the arch dam-reservoir system as a coupled interaction problem with flexible dam and compressible water.

INTERACTION MODEL

Arch Dam-Reservoir System

The arch dam is assumed to have a constant upstream radius and radially extended banks with vertical water faces as shown in Fig. 1. The material of the dam is considered to be linearly elastic. The foundation and banks of the dam, which are usually of hard rock, are assumed to be rigid. Only the horizontal component of earthquake ground motion acting in the direction of the course of the river is considered in this work.

Equation of Motion

Assuming the displacements to be small, the deformation of the dam can be approximated as

$$\Psi(\theta,z,t) = Y(t)\ \phi(\theta,z) \tag{1}$$

where $Y(t)$ and $\phi(\theta,z)$ are respectively the generalized coordinate and mode shape for the fundamental mode of vibration of the dam. Therefore, total acceleration of the dam is given by

$$\ddot{u}(\theta,z,t) = \ddot{u}_g(t) + \ddot{\Psi}(\theta,z,t) \tag{2}$$

where $\ddot{u}_g(t)$ = earthquake acceleration. The acceleration given by Eq. 2 is the excitation for the reservoir.

The equation of motion for the dam may be written as

$$\ddot{Y}(t) + 2\xi_d\omega_d\ \dot{Y}(t) + \omega_d^2\ Y(t) = \frac{\overline{P}(t)}{M^*} \tag{3}$$

$$\text{where } M^* = \int_{-\theta_d}^{\theta_d} \int_0^{H_d} m(\theta,z)\, \phi^2(\theta,z)\, R\, d\theta\, dz \tag{4}$$

$$\text{and } \overline{p}(t) = -\ddot{u}_g(t) \int_{-\theta_d}^{\theta_d} \int_0^{H_d} m(\theta,z)\, \phi(\theta,z)\, R\, d\theta\, dz - \int_{-\theta_d}^{\theta_d} \int_0^{H} p(\theta,z,t)\, \phi(\theta,z)\, R_d\, d\theta\, dz \tag{5}$$

In Eqs. 4 and 5, $m(\theta,z)$ represents the distributed mass of the dam, R is the radius to the center of the dam, ω_d and ξ_d are respectively the natural frequency and damping ratio of the dam in the first mode of vibration, H_d, R_d and θ_d are respectively the height, upstream radius and half arch central angle of the dam and H is the depth of the reservoir. In Eq. 5, $p(\theta,z,t)$ is the hydrodynamic pressure acting on the dam.

Response to Earthquake Excitation

The response of the dam due to earthquake can be conveniently obtained by the complex frequency response method and the Fourier integral(4) as

$$\Psi(\theta,z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_\Psi(\theta,z,\omega)\, \ddot{U}_g(\omega)\, e^{i\omega t}\, d\omega \tag{6}$$

where H_Ψ = complex frequency response of Ψ and $\ddot{U}_g(\omega)$ = Fourier transform of $\ddot{u}_g(t)$, given by

$$\ddot{U}_g(\omega) = \int_{-\infty}^{\infty} \ddot{u}_g(t)\, e^{-i\omega t}\, dt \tag{7}$$

Complex Frequency Responses

Let $H_{p_o}(\theta,z,\omega)$ and $H_{p_1}(\theta,z,\omega)$ be the complex frequency responses of hydrodynamic pressures acting on the dam due to the excitations $\ddot{u}_g(t) = e^{i\omega t}$ and $\phi(\theta,z)e^{i\omega t}$ respectively. Expressions for H_{p_o} and H_{p_1} can be derived by solving the equations of motion for water subject to appropriate boundary conditions(9). Substituting from Eqs. 1 and 2, Eq. 3 may be written as

$$[M^* + C^R(\omega)]\, \ddot{Y}(t) + [2\xi_d\omega_d M^* - \omega\, C^I(\omega)]\, \dot{Y}(t) + \omega_d^2\, M^*\, Y(t) = -\,[A + B(\omega)]\, e^{i\omega t} \tag{8}$$

$$\text{in which } A = \int_{-\theta_d}^{\theta_d} \int_0^{H_d} m(\theta,z)\, \phi(\theta,z)\, R\, d\theta\, dz \tag{9}$$

$$B(\omega) = \int_{-\theta_d}^{\theta_d} \int_0^{H} H_{p_o}(\theta,z,\omega)\, \phi(\theta,z)\, R_d\, d\theta\, dz \tag{10}$$

and $C^R(\omega)$ and $C^I(\omega)$ are respectively the real and imaginary parts of the complex coefficient $C(\omega)$, given by

$$C(\omega) = \int_{-\theta_d}^{\theta_d} \int_0^H H_{p_1}(\theta,z,\omega)\, \phi(\theta,z)\, R_d\, d\theta\, dz \tag{11}$$

Eq. 8 governs the generalized coordinate response of the dam due to the excitation of the coupled dam-reservoir system by the earthquake acceleration $\ddot{u}_g(t) = e^{i\omega t}$.

Case 1. Empty Reservoir

When there is no water in the reservoir, the equation of motion for the dam is obtained from Eq. 8 as

$$\ddot{Y}(t) + 2\xi_d\omega_d\, \dot{Y}(t) + \omega_d^2\, Y(t) = -\frac{A}{M^*}\, e^{i\omega t} \tag{12}$$

It is seen from Eqs. 8 and 12 that the presence of the reservoir modifies the properties of the dam. These changes represent the dynamic interaction between the flexible dam and the reservoir.

Case 2. Uncoupled Dam-Reservoir System

If the interaction effects between the dam and reservoir are assumed to be negligible, the excitation forces are due to the inertia of the dam and the hydrodynamic pressure obtained on the assumption that the dam is rigid. The equation of motion for the dam is written as

$$\ddot{Y}(t) + 2\xi_d\omega_d\, \dot{Y}(t) + \omega_d^2\, Y(t) = -\frac{[A + B(\omega)]}{M^*}\, e^{i\omega t} \tag{13}$$

Case 3. Uncoupled Dam-Reservoir System: Water Compressibility Neglected.

If water is assumed to be incompressible, the complex frequency response of the hydrodynamic pressure can be obtained from that due to compressible water by setting the excitation frequency $\omega = o$. The equation of motion for the dam is then obtained from Eq. 13 as

$$\ddot{Y}(t) + 2\xi_d\omega_d\, \dot{Y}(t) + \omega_d^2\, Y(t) = -\frac{[A + B(o)]}{M^*}\, e^{i\omega t} \tag{14}$$

A study of the governing equations of motion given by Eqs. 8, 12, 13 and 14 reveals the effects of the various assumptions that are usually made in studies dealing with the earthquake responses of arch dams. It is known that under steady state conditions the acceleration, velocity and displacement of the dam are related as

$$\ddot{Y}(t) = i\omega\, \dot{Y}(t) = -\omega^2 Y(t) \tag{15}$$

Complex frequency responses of the generalized coordinates of the dam can be easily obtained by substituting from Eq. 15 into the equations of motion given by Eqs. 8, 12, 13 and 14. The complex frequency response of the deformation of the dam may be written from Eq. 1 as

$$H_\psi\,(\theta,z,\omega) = H_Y(\omega)\;\phi(\theta,z) \tag{16}$$

where $H_Y(\omega)$ = complex frequency response of $Y(t)$. The complex frequency response of the hydrodynamic pressure acting on the dam due to the excitation given by Eq. 2 with $\ddot{u}_g(t) = e^{i\omega t}$ may be written as

$$H_p\,(\theta,z,\omega) = H_{p_o}\,(\theta,z,\omega) - \omega^2\,H_Y(\omega)\,H_{p_1}\,(\theta,z,\omega) \tag{17}$$

Response to earthquake excitation may be obtained from the complex frequency response function and the Fourier transform of the earthquake as indicated in Eqs. 6 and 7. The integrals appearing in Eqs. 6 and 7 are evaluated by the Fast Fourier Transform method[14].

NUMERICAL RESULTS

The parameters controlling the geometry of an arch dam-reservoir system are the height, radius, arch central angle and thickness of dam and depth of reservoir. Properties of six dam-reservoir systems considered in this study are listed in Table 1. The thickness of the dam is assumed to vary linearly with height as shown in Fig. 1. The arch central angle is taken to be 90^o for convenience of deriving simple closed-form mathematical solutions for the complex frequency responses of the hydrodynamic pressures. Expressions for these pressures involved in Eqs. 10 and 11 have been presented previously[9]. The material of the dam is assumed to be homogeneous, isotropic and linearly elastic with modulus of elasticity of 5×10^6 psi, Poisson's ratio of 0.17 and unit weight of 150 pcf. These values represent properties of mass concrete commonly used for arch dams. The N69^{o}W component of Taft Earthquake, California, July 21, 1952 shown in Fig. 3 is chosen for the excitation of the dam-reservoir systems.

The dams selected may be considered as thin arch dams on the basis of their thickness to height ratios. Using quadrilateral shell finite elements to discretize the dam, the natural frequencies and mode shapes of the dam are obtained by solving the standard eigenvalue problem in the computer[9,17]. The first symmetric mode of vibration of a 300-ft. arch dam is shown in Fig. 2. A value of 5 percent of critical damping is chosen for the material damping in the first mode of vibration of the dam.

The complex frequency response of the generalized acceleration of a 300-ft. dam-reservoir system obtained from Eq. 8 is shown in Fig. 4. The first natural frequency of the coupled system is seen to be the excitation frequency at which the response curve attains its first maximum. Natural frequencies of all the dam-reservoir systems considered are given in Table 1. It is apparent that the natural frequency of a coupled-system is lower than that of the reservoir alone due to the interaction between the dam and reservoir. It is convenient to consider the total hydrodynamic force acting on a vertical section of the dam. This force is obtained by integrating the hydrodynamic pressure over the depth of the reservoir. Absolute maximum values of earthquake hydrodynamic forces normalized with respect to the hydrostatic force are listed in Table 2. It is seen that the hydrodynamic forces of the coupled system are significantly different from those of the uncoupled system. Time histories of crest displacements due to Taft earthquake are shown in Figs. 5 and 6 for a 300-ft. dam-reservoir system. Maximum values of crest displacements for all the cases considered in this study are given in Table 3. It is apparent that the solution based on uncoupled system with incompressible water differs considerably from that of the coupled system. The errors are generally on either side - safe or unsafe. The same types of errors - safe or unsafe - are involved in the solution for the uncoupled system with compressible water. It should be noted that the response of a system to a particular earthquake depends on the system properties and the characteristics of the earthquake. System properties are modified according to the assumption made in the simpler analyses. Therefore, there seems to be no correlation between the various solutions based on different assumptions. Maximum values of the crest accelerations are listed in Table 4. It is seen that these values represent large magnifications with respect to the peak ground acceleration.

CONCLUSIONS

A method to analyze the effects of interaction between an arch dam and the reservoir during earthquakes is presented. An interaction model is defined using the modal properties of the dam and the complex frequency responses of the hydrodynamic pressures. Responses of several dam-reservoir systems to Taft earthquake are obtained by the Fast Fourier Transform method. The numerical results presented demonstrate that (1) the response of the dam is significantly increased due to the presence of the reservoir, (2) analyses which neglect interaction between dam and reservoir may not be satisfactory due to their generally unreliable results, and (3) water compressibility significantly influences the behavior of the dam-reservoir system. Finally, it may be stated that the earthquake behavior of an arch dam-reservoir system should be evaluated by properly taking into account the flexibility of the dam and compressibility of water.

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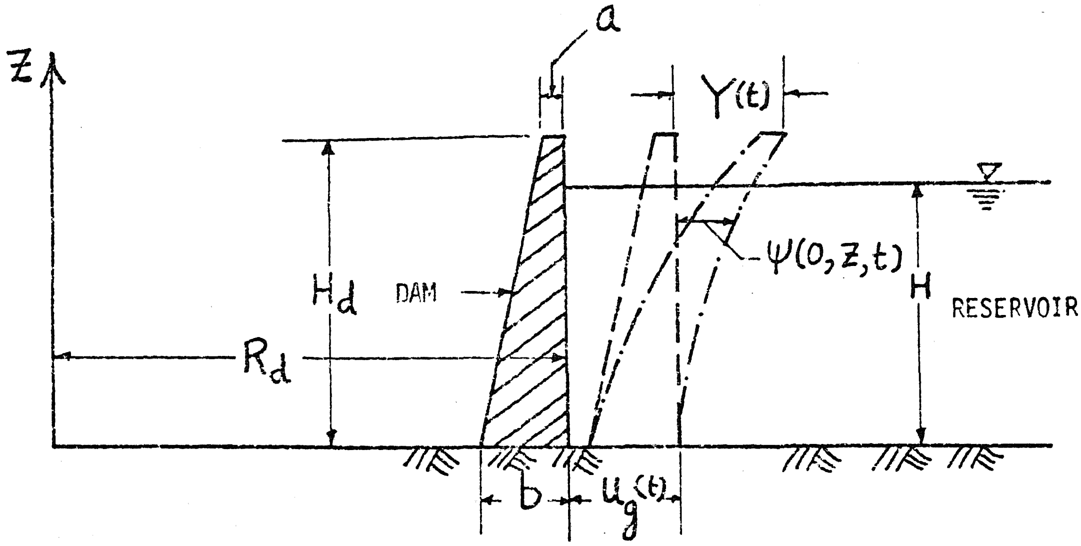
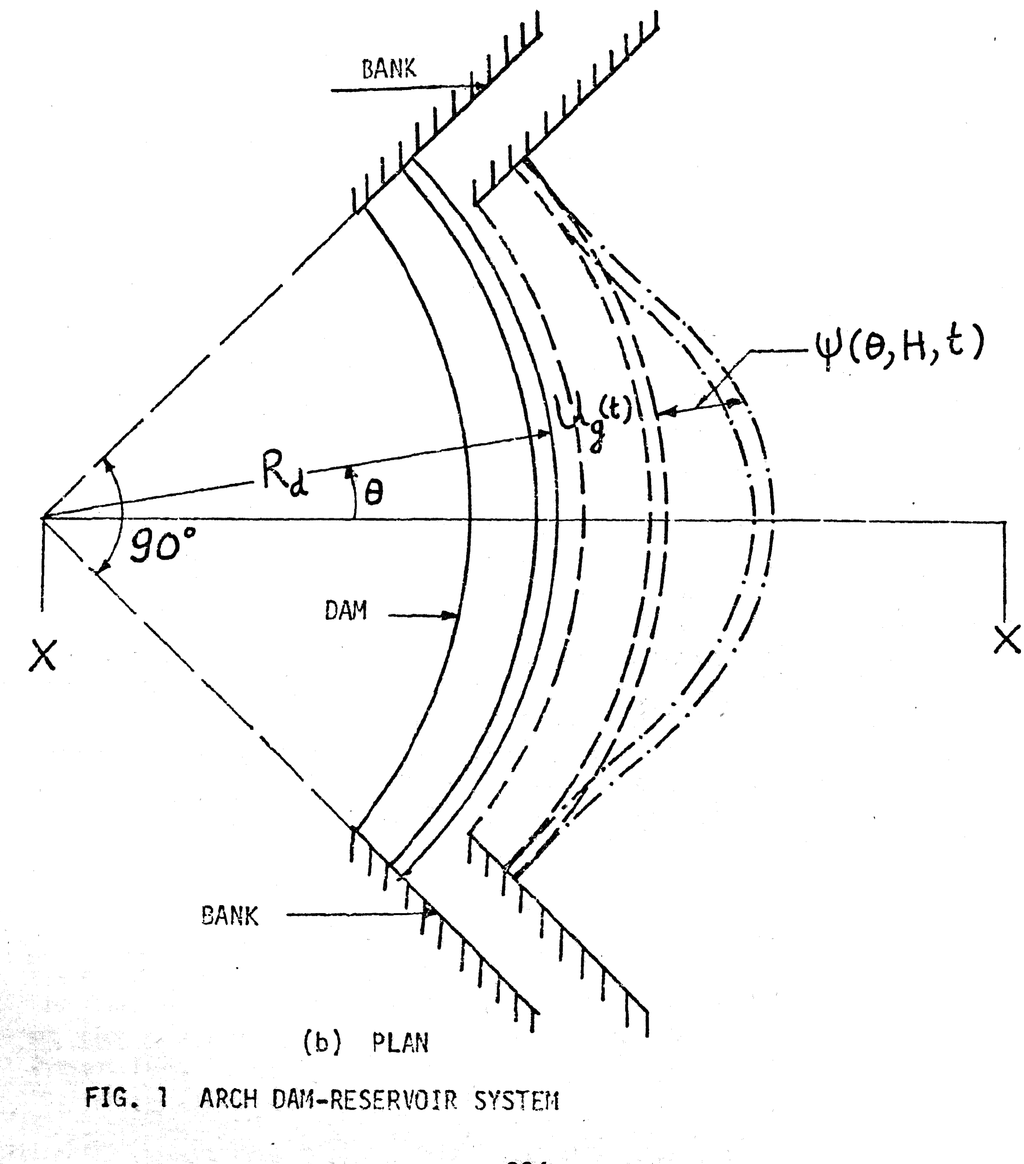


(a) SECTION ON X-X



(b) PLAN

FIG. 1 ARCH DAM-RESERVOIR SYSTEM

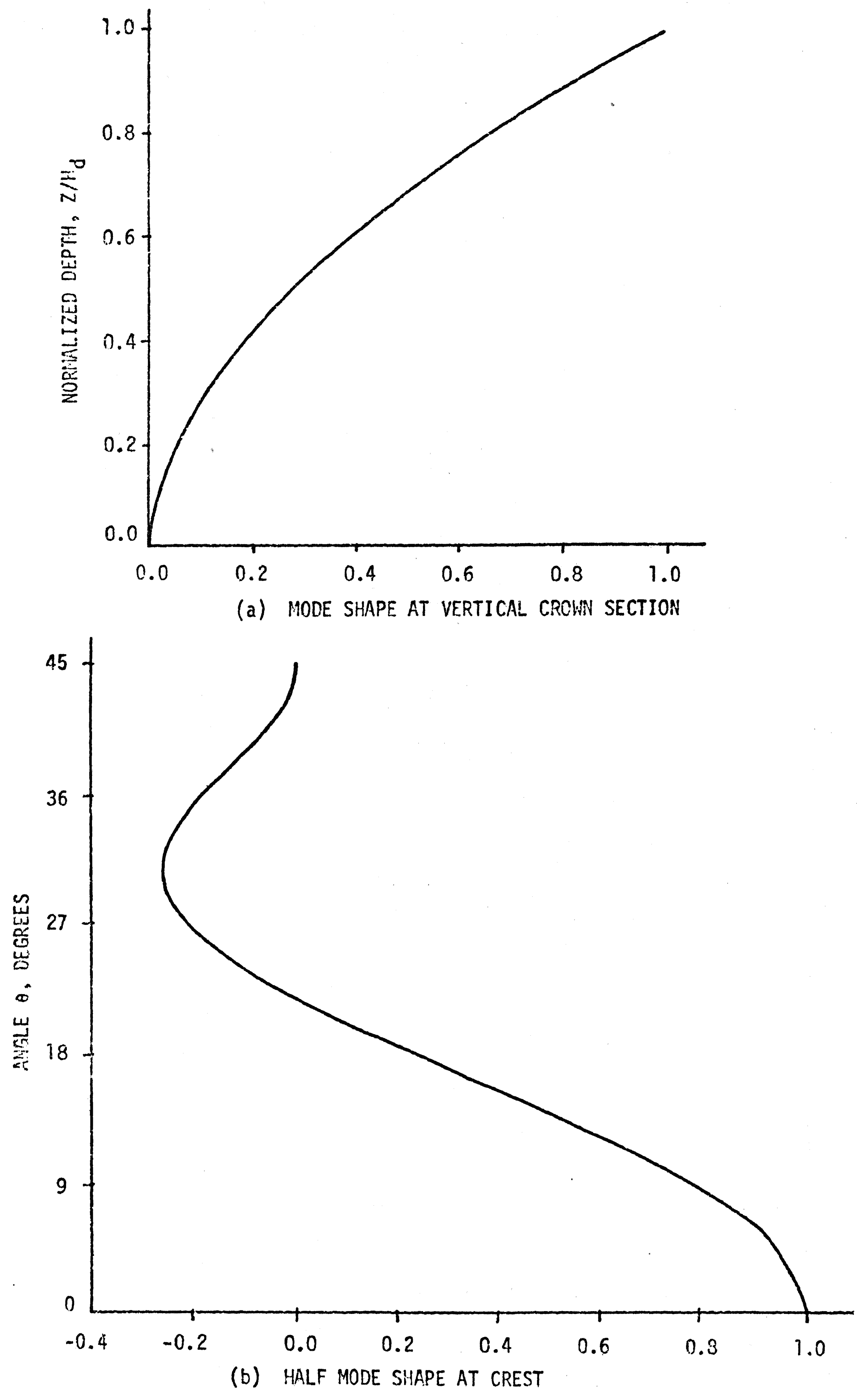


FIG. 2 FIRST SYMMETRICAL MODE SHAPE OF ARCH DAM, H_d = 300 FT. AND R_d/H_d = 1.5

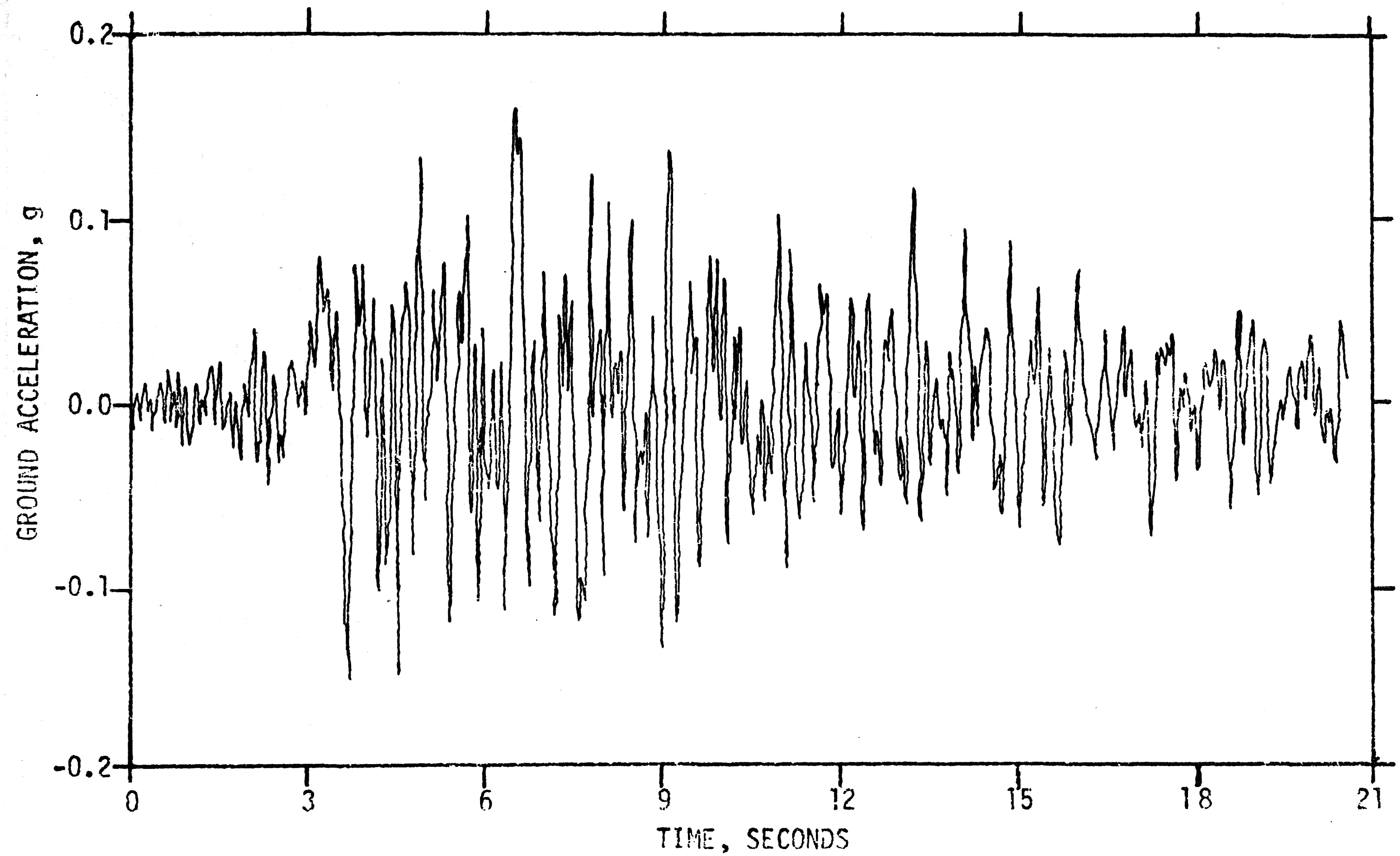


FIG. 3 ACCELERATION RECORD - TAFT, CALIFORNIA EARTHQUAKE, JULY 21, 1952 (N69°W COMPONENT)

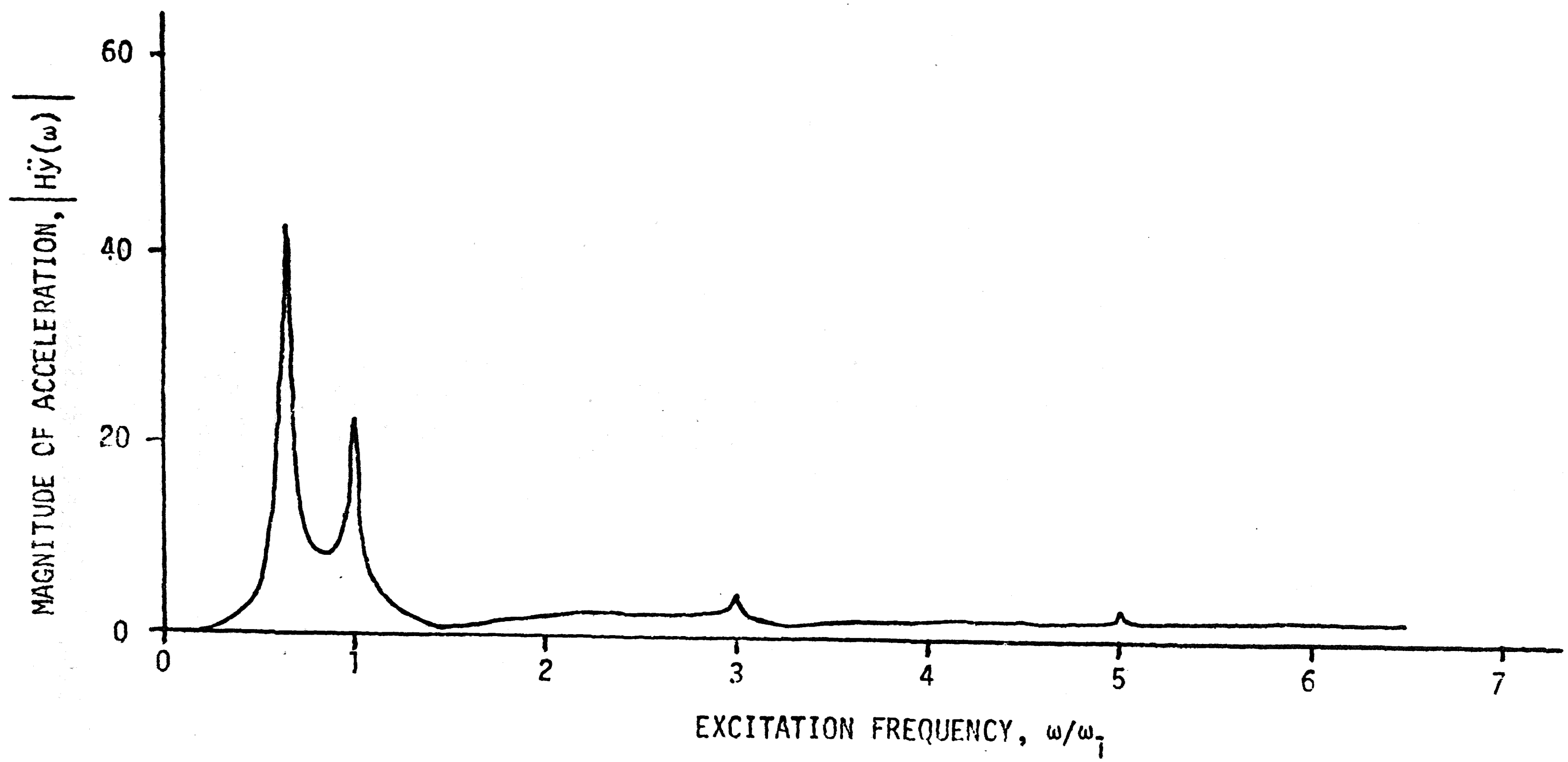


FIG. 4 COMPLEX FREQUENCY RESPONSE OF GENERALIZED ACCELERATION OF ARCH DAM, $H = H_d$ = 300 FT. AND R_d/H_d = 1.5; COUPLED SYSTEM

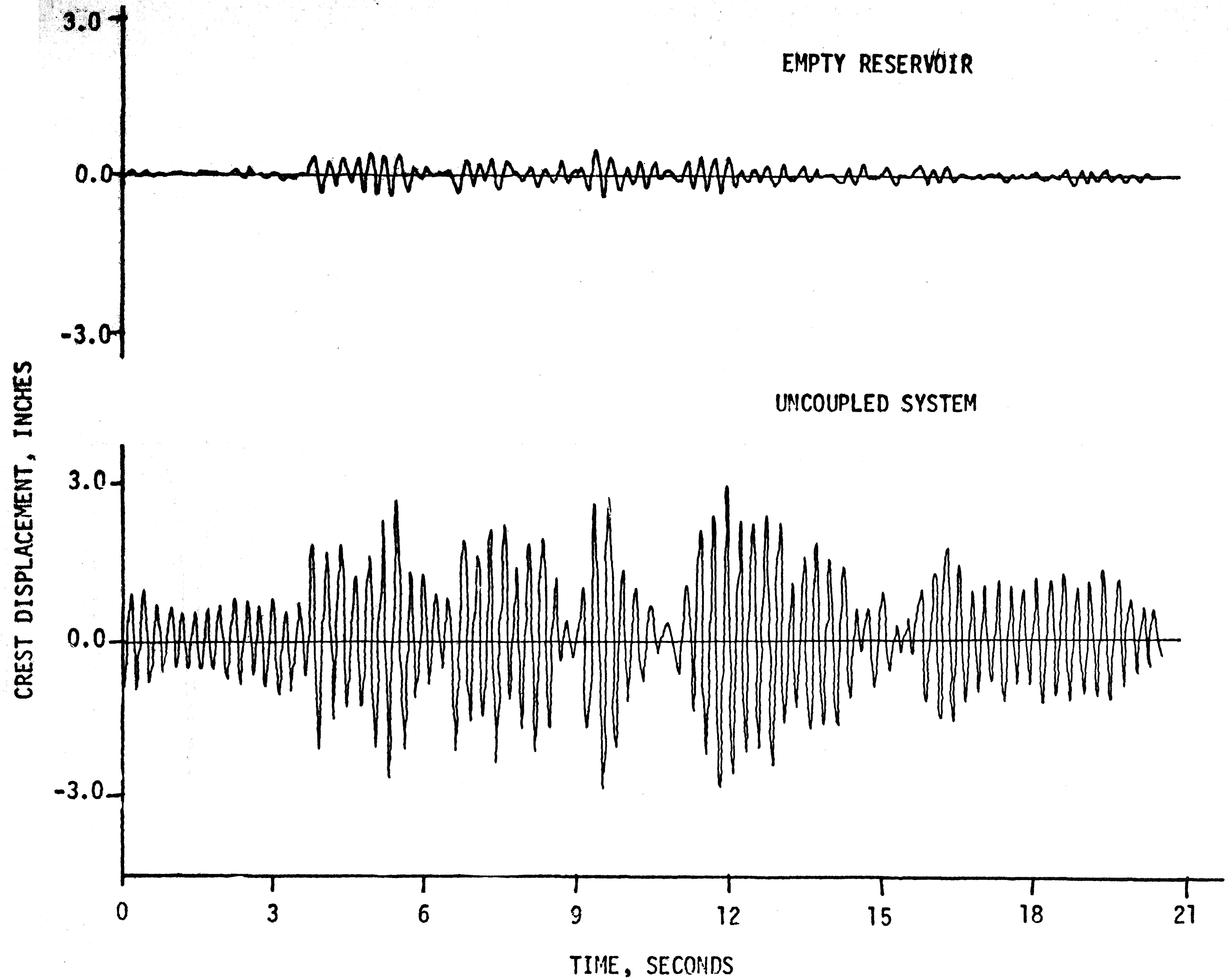


FIG. 5 DISPLACEMENT RESPONSE OF CREST OF ARCH DAM, $H = H_d = 300$ FT. AND $R_d/H_d = 1.5$; TAFT EARTHQUAKE

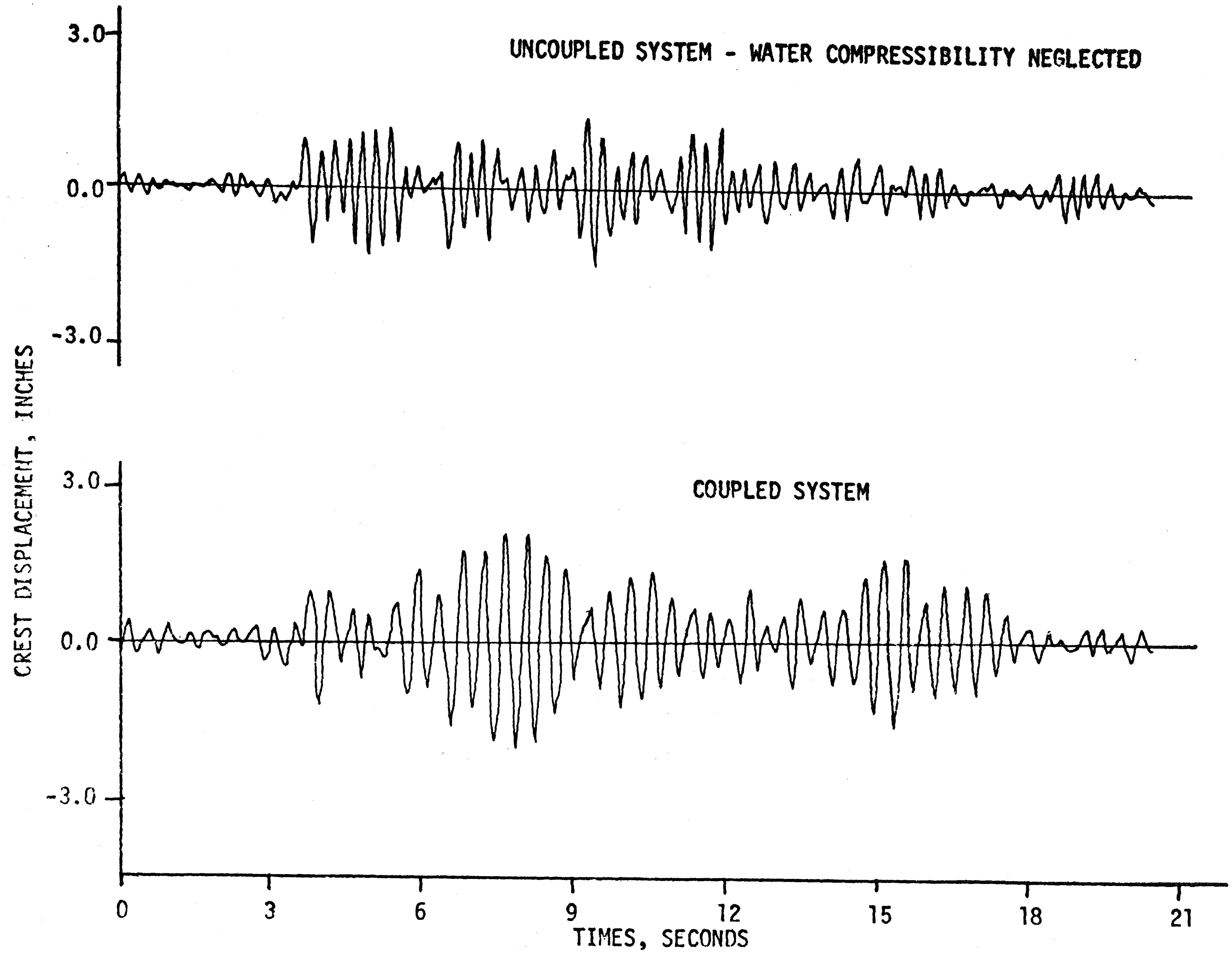


FIG. 6 DISPLACEMENT RESPONSE OF CREST OF ARCH DAM, $H = H_d$ = 300 FT. AND R_d/H_d = 1.5; TAFT EARTHQUAKE

TABLE 1. PROPERTIES OF DAM-RESERVOIR SYSTEMS

$H = H_d$ (Ft.)	R_d/H_d	a/H_d	b/H_d	First Natural Frequency (rad./sec.)		
				Dam Only, ω_d	Reservoir Only, ω_1	Coupled System, ω_c
300	1.0	0.033	0.133	31.02	24.72	20.88
300	1.5	0.042	0.167	22.58	24.72	15.66
300	2.0	0.050	0.200	19.58	24.72	13.82
500	1.0	0.030	0.150	19.14	14.83	13.20
500	1.5	0.035	0.200	14.53	14.83	10.13
500	2.0	0.040	0.240	12.77	14.83	8.91

TABLE 2. ABSOLUTE MAXIMUM VALUES OF FORCES (HYDRODYNAMIC/HYDROSTATIC)

TAFT 1952 EARTHQUAKE

$H = H_d$ (Ft.)	R_d/H_d	Force at Crown Section ($\theta = 0^o$)		Force at Mid-Section ($\theta = 22\ 1/2^o$)		Force at Abutment ($\theta = 45^o$)	
		Coupled System	Uncoupled System	Coupled System	Uncoupled System	Coupled System	Uncoupled System
300	1.0	0.388	0.282	0.289	0.295	0.327	0.350
300	1.5	0.357	0.274	0.263	0.282	0.327	0.347
300	2.0	0.303	0.254	0.206	0.270	0.334	0.369
500	1.0	0.258	0.341	0.236	0.355	0.294	0.388
500	1.5	0.278	0.316	0.286	0.341	0.358	0.383
500	2.0	0.228	0.312	0.335	0.323	0.353	0.376

TABLE 3. ABSOLUTE MAXIMUM VALUES OF CREST DISPLACEMENTS

$H = H_d$ (Ft.)	R_d/H_d	Static Displacement due to Hydro-Static Pressure (in.)	Displacement (Dynamic/Static) Taft 1952 Earthquake			
			Empty Reservoir	Uncoupled System		Coupled System
				Compressible Water	Incompressible Water	
300	1.0	0.495	0.705	3.385	2.535	3.640
300	1.5	0.991	0.463	2.998	1.419	2.140
300	2.0	1.390	0.724	2.825	1.932	2.527
500	1.0	1.411	0.740	3.540	2.619	3.357
500	1.5	2.475	0.910	6.125	2.678	2.045
500	2.0	3.565	0.520	2.660	1.370	1.293

TABLE 4. ABSOLUTE MAXIMUM VALUES OF CREST ACCELERATIONS

TAFT 1952 EARTHQUAKE

$H = H_d$ (Ft.)	R_d/H_d	Peak Ground Acceleration (g)	Maximum Crest Acceleration / Peak Ground Acceleration
300	1.0	0.16	14.30
300	1.5	0.16	10.50
300	2.0	0.16	12.92
500	1.0	0.16	15.39
500	1.5	0.16	10.81
500	2.0	0.16	8.21