DYNAMIC RESPONSE OF ARCHES UNDER SEISMIC FORCES

by

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SYNOPSIS

Elastic response of arches having various shapes and sizes has been studied under the action of two severe ground motions. A wide range of their geometrical and vibrational parameters has been considered and normalised moment, thrust and shear force distribution curves have been obtained. Using these curves, and other data for time periods and modal response factors, a simplified dynamic procedure is suggested for practical design of arches.

INTRODUCTION

The behaviour of an arch structure under seismic acceleration is different than other structures like buildings and dams, which have been extensively studied, because of its shape and typical mass and stiffness distribution. To get an insight into the inplane elastic response of arches to earthquake excitations, their various parameters, viz., rise-span ratio, shape and moment of inertia variation (5), slenderness ratio, mass distribution and fundamental time period, have been considered in this paper and dynamic response computed for two recorded ground motions, having different predominant frequency contents. The earthquake motion has been considered as horizontal component alone or horizontal component together with vertical motion of 50% intensity in phase with horizontal motion. Internal moments, thrusts and shear forces have been computed at springings, crown, quarter points and other sections by timewise superposition of modal values and normalised distributions plotted along the span (3,4).

The various parameters of the arch are listed in Table 1 alongwith their practical range of values. Response was computed to study the effect of each parameter separately.

RESULTS

The significant conclusions arrived at from the dynamic response study are: a) The rise-span ratio and the fundamental time period of the arch turn out as the most significant parameters; b) Shallow arches attract less earthquake forces; c) Uniform seismic coefficient method (2) leads to unconservative distribution of forces; and d) It is possible to arrive at a simplified dynamic design approach using averaged normalised distribution of forces and tabulated time period and modal response coefficients as described below.

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SIMPLIFIED MODAL RESPONSE ANALYSIS FOR PRACTICAL DESIGN

a) Compute the periods in first three antisymmetrical and first three symmetrical modes using curves given in Fig. 2 (1,4).

b) Using the appropriate acceleration response spectra for the site, obtain the seismic coefficients $\alpha_r = S_{ar}/g$ for the rth period and appropriate damping. These seismic coefficients will be taken in horizontal and vertical directions (α_{rH} and α_{rV}) for the anti-symmetrical and symmetrical modes respectively.

c) Compute the values of moment, thrust and shear force at the springing point in the rth mode by using the seismic coefficients and modal response coefficients tabulated in Tables 2 to 4.

d) Using the force values of the springing, compute the values at other points of the arch using the normalised design curves given in Fig. 3.

CONCLUSION

The parametric study of dynamic response of arches in the elastic range gives a distribution of seismic forces in the arch which is more severe than obtained by uniform seismic coefficient method. Data is presented in the form of curves and tables which would enable dynamic modal analysis of arches for practical design.

REFERENCES

1. Arya, A.S. and Thakkar, S.K., "Computation of Natural frequencies and modes for hinged and fixed circular arches", Journal of Institution of Engineers (India), Vol. 50, July, 1970, pp. 285-289.

2. Indian Standard Institution, IS: 1893 - 1970, "Criteria of Earthquake resistant Design of Structures", New Delhi.

3. Thakkar, S.K. and Arya, A.S., "Earthquake response of circular arches", 4th symposium on Earthquake Engineering, University of Roorkee, Roorkee, Nov. 14-16, 1970.

4. Thakkar, S.K., "Response of Arches under Earthquake Excitation", Ph.D. Thesis, University of Roorkee, Roorkee, July, 1972.

5. Whitney, C.S., "Design of Symmetrical Concrete Arches", Transactions, ASCE, Vol. 88, pp. 931-1029, 1925.

TABLE 1

RANGE OF ARCH PARAMETERS INVESTIGATED

No.	Parameter	Range
1.	Rise-span ratio, f/L	0.15, 0.20, 0.25
2.	Ratio of span to radius of gyration at crown, L/k	200, 300, 400
3.	Shapes	i) Uniform circular, ii) Non Uniform circular, iii) Parabolic, iv) Whitney (1925).
	Shape Factor S for Whitney Arches (Fig.1)	0.15, 0.20, 0.25
4.	Form factor, F, for Non-uniform arches (Fig.1)	0.15, 0.25, 0.40
5.	Fundamental Period, T	0.5 , 1.0 , 2.0 , 3.0 Sec.
6.	Mass distribution	i) Uniform along curved length, ii) Uniform along horizontal span iii) filled spandrel arch type
7.	Damping	i) Uniform in all modes, ii) 5% in fundamental mode increasing to 20% in highest mode.
8.	Ground motion	Koyna 1967 Longitudinal and El-Centro 1940 N-S, i) Horizontal alone, ii) Horizontal + vertical equal to half horizontal

TABLE 3

MODAL RESPONSE COEFFICIENTS FOR PARABOLIC ARCHES

Arch Parameter f/L	F	Mode	m_r			c_r			n_r		
			Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
.15	.25	A	.01711	.0034	.00121	.10327	.03700	.01950	.01789	.00802	.00433
		S	.00162	.00187	.00312	.01601	.03339	.03957	.03112	.71172	.03189
.25	.25	A	.02931	.00484	.00173	.10792	.04749	.02490	.04091	.01012	.00550
		S	.00282	.00378	.00485	.02243	.05553	.03910	.01513	.17906	.34700
.20	.15	A	.02353	.00467	.00167	.12687	.04477	.00236	.02425	.00927	.00501
		S	.00136	.00157	.00441	.01116	.05637	.04765	.01207	.53214	.06125
.20	.25	A	.02317	.00419	.00146	.12277	.04352	.02205	.02902	.00985	.00527
		S	.00125	.00208	.00422	.01096	.06157	.04838	.01184	.53469	.08422
.20	.40	A	.02251	.00376	.00132	.13743	.04195	.02095	.03412	.01036	.05540
		S	.00098	.00260	.00624	.00920	.06654	.05550	.01014	.51752	.1230

f = Rise of arch, L = Span of arch, F = Form factor (Fig.1),

S = Shape factor (Fig.1) , A = Antisymmetric, S = Symmetric

TABLE 2

MODAL RESPONSE COEFFICIENTS FOR CIRCULAR ARCHES

Arch Parameter f/L	F	Mode	m_r Mode 1	m_r Mode 2	m_r Mode 3	q_r Mode 1	q_r Mode 2	q_r Mode 3	n_r Mode 1	n_r Mode 2	n_r Mode 3
Uniform Circular Arches											
.20	1.38	A	.02158	.00357	.00118	.15099	.04737	.02237	.04154	.02049	.01106
		S	.00245	.00138	.00130	.02908	.02554	.05761	.04046	.05028	.46331
.15	1.20	A	.01562	.00277	.00093	.11764	.03779	.01827	.02732	.01370	.00749
		S	.00199	.00237	.00135	.02486	.05012	.01299	.04704	.18761	.62217
.25	1.668	A	.02809	.00430	.00138	.19515	.05492	.02528	.05360	.02629	.01404
		S	.00339	.00126	.00146	.03798	.02158	.03761	.04248	.03069	.11220
Non Uniform Circular Arches											
.15	0.15	A	.01772	.00421	.00157	.10036	.04246	.02289	.01127	.00968	.00617
		S	.00096	.00408	.00497	.00897	.08716	.04184	.01465	.4766	.25990
.25	0.15	A	.02809	.00430	.00138	.19515	.05492	.02529	.05359	.02629	.01404
		S	.00339	.00126	.00146	.03798	.02158	.03761	.04245	.03069	.11216
.20	0.15	A	.02443	.00566	.00209	.13086	.05419	.02890	.01450	.01415	.00913
		S	.00073	.00310	.00171	.00631	.04479	.02252	.00703	.09649	.46649
.20	0.25	A	.02424	.00513	.00183	.13801	.05326	.02753	.01948	.01564	.00966
		S	.0010	.00254	.00085	.00924	.03889	.03212	.01059	.08077	.49606
.25	0.40	A	.02372	.00466	.00162	.14398	.05193	.02614	.02045	.01702	.01012
		S	.00136	.00211	.00012	.01344	.03418	.04091	.01606	.06916	.51330

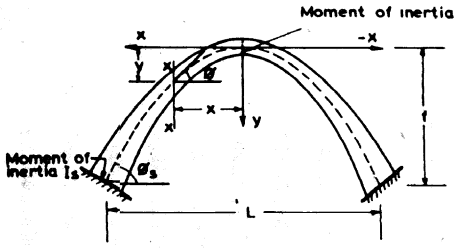
TABLE 4

MODAL RESPONSE COEFFICIENTS FOR WHITNEY ARCHES

Arch Parameter f/L	F	S	Mode	m_r Mode 1	m_r Mode 2	m_r Mode 3	q_r Mode 1	q_r Mode 2	q_r Mode 3	n_r Mode 1	n_r Mode 2	n_r Mode 3
.15	.25	.15	A	.01969	.00638	.00227	.11532	.07027	.03613	.00290	.02755	.01883
			S	.01285	.00382	.00068	.15647	.07600	.03556	.24592	.18072	.32259
.25	.25	.15	A	.03528	.00849	.00265	.17904	.08071	.03615	.00973	.03597	.01621
			S	.00966	.00193	.00067	.08465	.02482	.01165	.09016	.02836	.08500
.20	.25	.15	A	.02738	.00770	.00251	.14994	.07908	.03715	.00188	.03405	.01866
			S	.01110	.00284	.00148	.11356	.04354	.05654	.14268	.07014	.27411
.20	.40	.15	A	.02724	.00705	.00221	.15923	.07787	.03509	.00809	.03766	.01914
			S	.01092	.00246	.00135	.11904	.03977	.05251	.15471	.06382	.24210
.20	.25	.20	A	.02523	.00581	.0020	.14240	.06013	.02989	.01612	.02125	.01188
			S	.00389	.00257	.00034	.03728	.03936	.03870	.04755	.07644	.45313

For 'A' Mode: $M_{rA} = m_{rA}\alpha_{rH}WL$; $Q_{rA} = q_{rA}\alpha_{rH}W$; $N_{rA} = n_{rA}\alpha_{rH}W$

For 'S' Mode: $M_{rS} = m_{rS}\alpha_{rV}WL$; $Q_{rS} = q_{rS}\alpha_{rV}W$; $N_{rS} = n_{rS}\alpha_{rV}W$



EQUATION OF SHAPE OF ARCH RIB

$$y = \frac{f}{g_1 - 1}\left(\cosh\frac{2x}{L}\,k - 1\right)$$

$$g_1 = \frac{1}{2}\left(1/S - 2\right)^2 - 1$$

Shape factor, $S = \dfrac{y \text{ AT } x = L/4}{f}$

$$k = \cosh^{-1} g_1$$

EQUATION FOR FORM OF ARCH RIB

Form factor, $F = \dfrac{I_c}{I_s \cos\phi_s}$

$$\frac{I_x}{I_c} = \frac{1}{\left[1 - (1-F)(2x/L)\right]\cos\phi}$$

FIG. 1 _SHAPE AND FORM OF WHITNEY ARCH

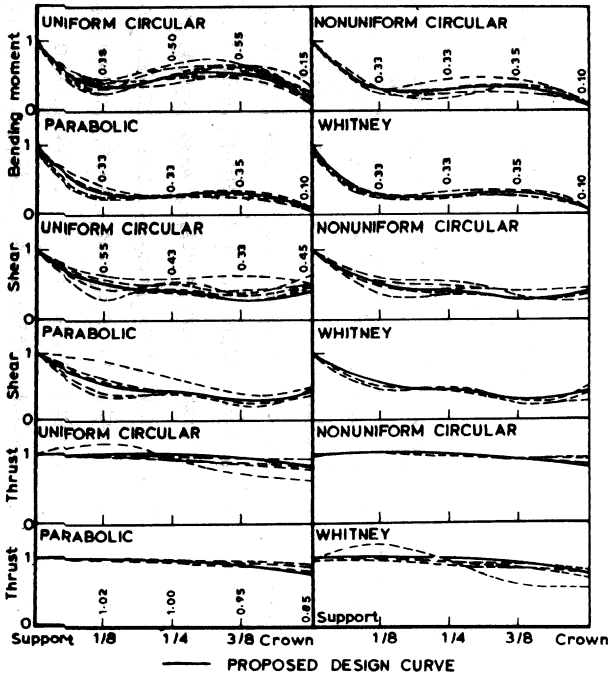
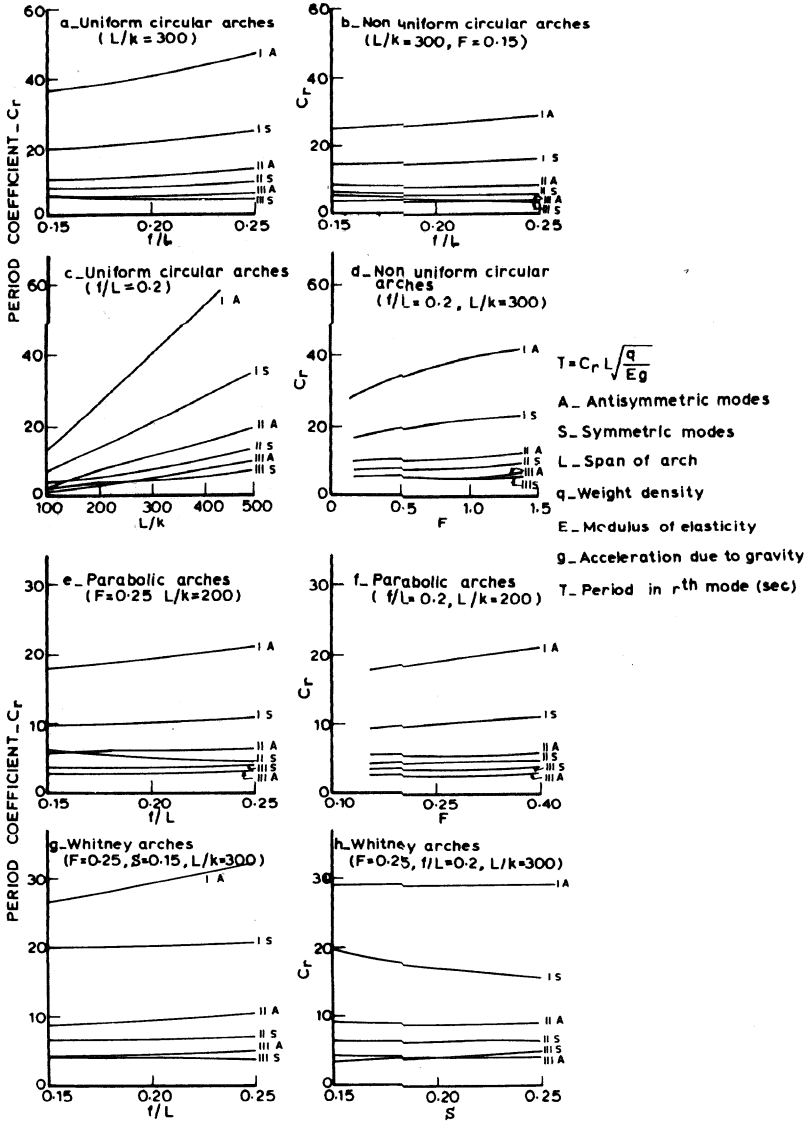


FIG. 3 _NORMALIZED DISTRIBUTION OF FORCES ALONG THE SPAN OF ARCH



$$T = C_r L\sqrt{\frac{q}{Eg}}$$

A_ Antisymmetric modes

S_ Symmetric modes

L_ Span of arch

q_ Weight density

E_ Modulus of elasticity

g_ Acceleration due to gravity

T_ Period in r^{th} mode (sec)

FIG. 2 _PERIOD CHARACTERISTICS OF ARCHES