SEISMIC INTERACTION BETWEEN PARTS OF A DYNAMIC SYSTEM

by

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SYNOPSIS

The motion of a discrete system which consists of two subsystems and is attached to a laterally moving rigid support is formulated in terms of the generalized coordinate properties of the modes of the two subsystems. In this approach the subsystem properties which affect the coupling between the two subsystems can be easily determined. For harmonic base accelerations, the application of the method to equipment-building interaction problems and soil-structure interaction problems is discussed. The dynamic response of a 10 storey building on a 200 ft. deep foundation is determined for a scaled El Centro earthquake record.

INTRODUCTION

The analysis of the dynamic response of two subsystems may be quite expensive when carried out in terms of the generalized coordinate properties of the modes of the coupled system, because in many practical problems the number of degrees of freedom is very large. Previous studies of soil-structure interaction problems indicate even when a coupled system analysis is practical, the generalized coordinate properties of modes of individual subsystems appear as important variables. In addition, in building-equipment interaction problems, it is computationally impracticable to consider an analysis of a coupled system, because of the large difference between the masses of the equipment and the building. Therefore, for design purposes in both equipment-building interaction problems and soil-structure interaction problems, it appears desirable to be able to formulate the analysis so that the response of the system to a seismic input is expressed in terms of the generalized coordinate properties of modes of the subsystems which can in many cases be more easily determined than for the coupled system.

THE METHOD OF ANALYSIS

We consider the motion of a dynamic system subjected to a lateral base acceleration. The generalized coordinate properties of two discrete subsystems comprising the dynamic system are shown in Table 1. The subsystem which is attached to the base is taken as subsystem 1. The mode shapes of subsystem 2 are determined assuming that it is fixed at its interface with subsystem 1.

For a given lateral base acceleration $v(t)$, the lateral acceleration $g(t)$ of the interface of two subsystems can be expressed in the form of a Volterra integral equation:

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$$g(t) = \sum_{j=1}^{N} \phi_{rj} \Gamma_j I[v(t);\Omega_j,\eta_j]$$

$$+ \sum_{j=1}^{N} \sum_{k=1}^{n} \phi_{rj}^2 \gamma_k^2 \frac{m_k^*}{M_j^*} \{F[g(t);\omega_k,\beta_k;\Omega_j,\eta_j] - I[g(t);\omega_k,\beta_k]\} \quad (1)$$

where

$$F[g(t);\omega,\beta;\Omega,\eta] = A\, I(g;\Omega,\eta) + B\, I(g;\omega,\beta) + C\, J(g;\Omega,\eta) + D\, J(g;\omega,\beta) \quad (2)$$

$$I(x;p,q) = p[\int_0^t x(\tau)e^{-pq(t-\tau)} \sin p(t-\tau)d\tau + 2q\int_0^t x(\tau)e^{-pq(t-\tau)} \cos p(t-\tau)d\tau] \quad (3)$$

$$J(x;p,q) = 2pq \int_0^t x(\tau)e^{-pq(t-\tau)} \cos p(t-\tau)d\tau \quad (4)$$

In equation (1), ϕ_{rj} corresponds to the relative lateral displacement amplitude of the interface of the two subsystems in the $j^{\underline{th}}$ mode of subsystem 1. The terms of the second sum in the right-hand side of equation (1) represent the contribution of all the possible mode pairs of the two subsystems to the coupling between the two subsystems. It is clear that the coefficients $\phi_{rj}^2\ \gamma_k^2\ m_k^*/M_j^*$ (k=1,2,...,n; j=1,2,...,N) in this sum are important factors determining the degree of coupling between the two subsystems. The symbol $I(x;p,q)$ represents the acceleration response of a single degree of freedom system with natural frequency p and fraction of critical damping q to a general support acceleration x(t) as given in equation (3). The coefficients A, B, C, D in equation (2) can be expressed in terms of the natural frequency ratio of the mode pair and the critical dampings corresponding to the members of the mode pair as:

$$\begin{aligned} A &= \{(1+2\beta m)(m^2+\varepsilon^2+1) - 2\varepsilon(\varepsilon-2\eta m) + 4\beta\eta\varepsilon(m^2+\varepsilon^2-1)\}/\Delta \\ B &= \varepsilon\{(\varepsilon-2\eta m)(m^2+\varepsilon^2+1) - 2\varepsilon(1+2\beta m) + 4\beta\eta(m^2-\varepsilon^2+1)\}/\Delta \\ C &= \{(2\beta\eta m-\beta\varepsilon+\eta)(m^2+\varepsilon^2+1) + \varepsilon m\}/(\eta\Delta) - A \\ D &= -\varepsilon\{\varepsilon(2\beta\eta m-\beta\varepsilon+\eta)(m^2+\varepsilon^2+1) + \varepsilon m\}/(\beta\Delta) - B \end{aligned} \quad (5)$$

where

$$\Delta = [m^2+(1+\varepsilon)^2][m^2+(1-\varepsilon)^2] \quad , \quad \varepsilon = \Omega/\omega \quad , \quad m = \beta-\eta\varepsilon$$

The integral equation (1) can be solved by iteration performed numerically at each time interval. Nevertheless, in both the equipment-building and the building-foundation interaction problems where the mass of subsystem 2 is negligible compared to the mass of subsystem 1 equation (1) can be approximated as:

$$g(t) = \sum_{j=1}^{N} \phi_{rj} \Gamma_j I(v;\Omega_j,\eta_j) \quad (6)$$

Then the lateral acceleration $a_i(t)$ of the $i^{\underline{th}}$ mass of subsystem 2 (equipment in equipment-building interaction problem; building in building-foundation interaction problems) becomes

$$a_i(t) = \sum_{j=1}^{N} \sum_{k=1}^{n} \phi_{rj}\, \psi_{ik}\, \Gamma_j\, \gamma_k\, F\,[v(t);\, \Omega_j, \eta_j;\, \omega_k, \beta_k] \qquad (7)$$

Here, ψ_{ik} corresponds to the relative lateral displacement amplitude of the i^{th} mass of subsystem 2 in the k^{th} mode. The terms of the sum in the right-hand side of equation (7) represent the contribution of all the possible mode pairs of the two subsystems to the acceleration response.

APPLICATIONS

A formula is developed for the steady-state part of $F[v(t);\Omega,\eta;\omega,\beta]$ when the input acceleration is $v(t) = \exp(i\theta t)$. The maximum value of the amplitude of the complex valued function $F[\exp(i\theta t);\Omega,\eta;\omega,\beta]$ is evaluated as a function of the ratio Ω/ω and the damping β and η. The results are shown in Figure 1 for the cases where $\beta=\eta=0.05$ and $\beta=0.06$, $\eta=0.04$. For design purposes, employing these curves (generated for several values of β and η) with a suitable rule to combine the maximum modal responses of mode pairs of two subsystems (super-position of the maximum responses of mode pairs or root-mean-square super-position of the maximum responses of mode pairs) the maximum acceleration response of the equipment (in equipment-building problem) or the building (in building-foundation problem) can be determined for the harmonic base accelerations from equation (7).

The dynamic structural response of a 10 storey building on a foundation layer 200 ft. in depth was examined for various foundation elastic moduli employing finite element techniques in reference [1]. The same problem will be analysed here to assess the accuracy of the method described above. The properties of the building and the foundation layers are shown in Table 2 and Table 3. The damping ratio for each mode of the building and the foundation layer is taken as 5%. The uncoupled analysis is carried out for a unit-length (1 ft.) of the building and the foundation layer employing equation (7). The N-S component of the El-Centro-California Earthquake of May 18, 1940 (the first 10 seconds) is taken as the lateral acceleration input at the base of the soil layer. The earthquake record is scaled down to give a maximum amplitude of 20% of the acceleration of gravity. The results of this study are summarized in Figure 2 which shows the variation of maximum base shear force in the building as a function of the fundamental period of the foundation. In this analysis modal pairs which are obtained from the combination of first two modes of the building and the first five modes of the soil layers are considered. The contribution from the other modal pairs does not change the base shear more than 1%. The results shown in Figure 2 are in good agreement with the results given in reference [1].

ACKNOWLEDGEMENTS

The authors express their appreciation to the National Research Council of Canada for financial support under grant NRC 1498.

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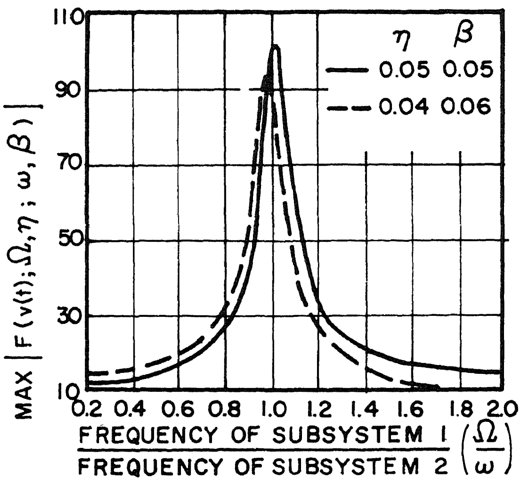


FIG.1 MAX. ACCELERATION RESPONSE FOR MODE PAIRS, HARMONIC INPUT

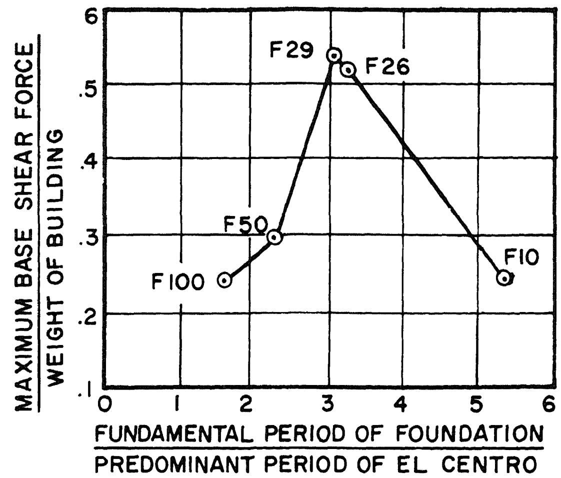


FIG.2 RESPONSE OF BUILDING-FOUNDATION LAYER TO EL CENTRO

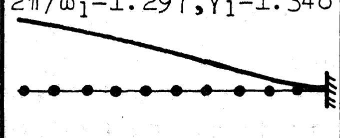
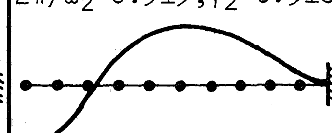
TABLE 1 SYSTEM CHARACTERISTICS

CHARACTERISTICS	SUBSYSTEM 1	SUBSYSTEM 2
Degrees of Freedom	N	n
Natural Frequencies	$\Omega_1, \Omega_2, \ldots, \Omega_N$	$\omega_1, \omega_2, \ldots, \omega_n$
Modes	$\phi_1, \phi_2, \ldots, \phi_N$	$\psi_1, \psi_2, \ldots, \psi_n$
Generalized Masses	$M_1^*, M_2^*, \ldots, M_N^*$	$m_1^*, m_2^*, \ldots, m_n^*$
Participation Factors	$\Gamma_1, \Gamma_2, \ldots, \Gamma_N$	$\gamma_1, \gamma_2, \ldots, \gamma_n$
Fraction of Critical Damping	$\eta_1, \eta_2, \ldots, \eta_N$	$\beta_1, \beta_2, \ldots, \beta_n$

TABLE 2 PROPERTIES OF THE FOUNDATION LAYERS (Subsystem 1)
unit weight = 130 lb/ft^3, Poisson's Ratio = 0.3

Cases	Young's Mod.psi	Natural Periods $2\pi/\Omega_j$					$\Gamma_j \times \phi_{rj}$				
		j=1	j=2	j=3	j=4	j=5	j=1	j=2	j=3	j=4	j=5
F10	10,000	2.38	2.30	2.22	2.18	2.08	0.06	0.00	0.74	0.45	0.09
F26	26,000	1.48	1.43	1.37	1.35	1.29					
F29	29,000	1.40	1.35	1.30	1.28	1.22					
F50	50,000	1.07	1.03	0.99	0.97	0.93					
F100	100,000	0.75	0.73	0.70	0.69	0.66					

TABLE 3 PROPERTIES OF THE BUILDING (Subsystem 2)
120 ft.high, 100 ft.x 37 ft.in plan, storey weight=200 kips

Storey Number	Total Flexural Rigidity EI (lb-in^2) per storey		$2\pi/\omega_1=1.297, \gamma_1=1.348$	$2\pi/\omega_2=0.515, \gamma_2=0.518$
	Columns	Girders		
1 to 4	24.84×10^{10}	48.60×10^{10}		
5 to 7	12.42×10^{10}	48.60×10^{10}		
8 to 10	6.21×10^{10}	37.30×10^{10}	1st Mode Shape	2nd Mode Shape