ELASTIC-PLASTIC BEHAVIOR OF STEEL BRACES UNDER REPEATED AXIAL LOADING

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INTRODUCTION
The earthquake-resistant function of bracing is well understood, and a great deal of contribution is often anticipated from braces to the strength and rigidity of steel framed structures. Therefore, the knowledge on the hysteretic behavior of braces is essential in the prediction of the elastic-plastic performance of a steel braced frame subjected to repeated loading as in an earthquake. Results of experimental [1] and theoretical [2,3] studies on the elastic-plastic behavior of steel braces under repeated loading have been published separately, and efforts are currently exerted to their access by the present authors. From these, several examples of the comparison between the theoretical and experimental results are taken and to be reported in this paper.

EXPERIMENTAL BEHAVIOR
A series of experimental studies are carried on by the authors, the details of test apparatus and some data having been reported [1]. Examples are taken here from Ref.1, together with from recent results (under preparation for publication). Specimen shown in Fig.1 is made of SS41 mild steel, having a square cross-section with 15x15mm nominal dimension. Yield stress of the material is 2.54 t/cm². The repeated axial load is applied on the specimen through the loading apparatus as shown in Fig.2, which promises the free rotation at both ends of the specimen. In the test reported in Ref.1, the maximum and minimum axial relative displacements experienced in one cycle of loading were continually changed as the number of the loading cycle increased. A typical test result is plotted in Fig.3, taking the axial load N (positive for tension) and the axial relative displacement Δ (positive for separation) for the ordinate and abscissa, respectively. On the other hand, in the recently carried test, a few cycles of the repeated loading was applied controlling the maximum and minimum axial relative displacements at fixed values until the hysteresis loop got stabilized, and then, the axial displacement amplitude was increased and another few cycles of loading were applied. Fig.4(a) and (b) show the loops of the relations of the load-axial displacement and the load-midpoint lateral deflection V, for a specimen with a slenderness ratio of about 40. Fig.5 presents the result of a specimen having a slenderness ratio of 120. The maximum axial displacement amplitude experienced in the test was about 2.5% of the effective length of the specimen.

Figs.6(a) and (b), which are the load-axial displacement and the load-midpoint lateral deflection hysteresis loops, respectively, schematically drawn for the first two cycles, serve for the description of the general behavior of a bar under the repeated axial load. Starting from Point O, the bar is shortened keeping the straightness under the compression applied with slow axial displacement rate, until it buckles at Point A. Then, the sustained compression load rapidly decreases to Point B. This process may not be properly recorded, because the loading machine cannot follow the high speed behavior of the bar. The slope of the load-deflection curve after the axial displacement is increased at Point B, becomes smaller than the slope of OA. The curve, then, once shows still smaller slope, and pro-

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ceeds to Point D, the slope being gradually steeper. The plastic elongation occurs at Point D with a small lateral deflection remaining. The direction of the change in axial displacement is again reversed at Point E and the curve passes Point F with the slope nearly equal to that of OA. The maximum compressive strength is reached at Point G, which is much lower than that at Point A. The slope of the curve from Point H is smaller than that of BCD, and the curve IJ follows approximately the same path EF. Although the maximum compressive and tensile strengths are smaller than those at Point G and I, respectively, the loop for the third cycle is similar to that for the second cycle, and the loop stabilizes after a few cycle of loading (see Figs.4 and 5).

**ANALYTICAL BEHAVIOR** According to the elastic-perfectly plastic, one-dimensional continuum type of analysis for an axially loaded bar with both ends simply supported, the relative displacement between the ends in the axial direction is composed of four components[3]. These are the components due to uniform elastic axial deformation, to change in geometry caused by lateral deflection, to plastic axial deformation at a yield hinge, and to plastic axial elongation distributed along the bar axis. Among basic assumptions are moderate bar slenderness, finite but sufficiently small deformation, a piecewise linear yield condition for the interaction of axial force and bending moment and that a compressed straight bar buckles when the compression reaches either the Euler load $N_E$ or the plastic limit $N_Q$. Quasi-static behavior of the bar has been analysed, and the relative displacement is expressed as a function of the axial load in a closed form with certain hereditary nature in the plastic components. This solution enables the history of the axial load to be determined uniquely by the history of the relative displacement. Figs.7(a) and (b) serve for the illustration of the typical analytical behavior of a bracing member under repeated axial loading. In the figures, $\Delta Q$ denotes the elastic limit displacement under uniform tension, and $M_Q$ is the plastic limit moment under pure bending (Theory I).

Since above treatment has completely neglected partial yielding in a cross section, it cannot take account of residual nature of plastic deformation strictly. In order to study this effect in a reasonably simple manner, another type of analysis (Theory II) has been made (under preparation for publication), in which it is assumed that the bending deformation is concentrated in the middle portion of the bar, the remaining portion having infinite rigidity in bending. The length of the flexural portion is determined so as to produce the exact central elastic deflection when loaded transversely at the center. By assuming the uniform distribution of curvature in this portion, plane sections remaining plane, and elastic-perfectly plastic stress-strain relationship, and by diving a cross-section into a finite number of small regions, the increments in the axial force and the bending moment at the central cross section are related with the increments in the curvature and centroidal axial strain to provide two equations. From these equations and the other equation of the equilibrium for half bars, the increments in the centroidal axial strain, axial force and bending moment are expressed analytically by the curvature increment alone. Based on the additional assumption that the axial elongation or contraction is distributed along the bar, the load-displacement relation is determined incrementally with recourse to step-by-step calculation. This is carried out.
for a square cross section (Fig. 8), and compared with experimental results in Fig. 9 as well as the one-dimensional analysis of Ref. 3. In Ref. 2, analysis is executed numerically, and is based on the same principle as in Ref. 3 except for the yield condition. Therefore, comparison is not made here with this theory.

DISCUSSIONS

The following observations are made from the experimental and theoretical results. The slope of the elastic load-displacement curve corresponding to the curve EC in Fig. 6(a) becomes smaller as the slenderness ratio increases and as the displacement amplitude increases (see Figs. 3, 4 and 5). This phenomenon is observed both in the theories and the tests. Theory I shows a point of discontinuous slope on the load-displacement curve corresponding to the path ECD in Fig. 6(a). This slope discontinuity vanishes in the results of tests and Theory II. Because of the assumed constitutive equation Theory II indicates a still larger rate of change in slope than the test results. When the tension load is applied on the buckled bar, the test results show that the tension reaches the value nearly equal to the yield load, and the lateral deflection remains. On the other hand, the both theories indicate that the yield load is attained with the lateral deflection vanishing under the loading program used in the test. The lateral deflection vanishes in an earlier stage in Theory I than Theory II. Referring to the curve EFG in Fig. 6(a) it is derived from both Theories I and II that the bar is compressed until it buckles, holding the straight shape. The actual bar action during this process is, however, similar to that of a bar with the above mentioned residual lateral deflection under compression, and the maximum compressive strength is much lower than the buckling strength. The test results show that the maximum tensile and compressive strengths of a bar recorded in each cycle of loading decrease due to the cumulated lateral deflection. The hysteresis loop stabilizes after 6 to 10 cycles of the repeated loading (see Figs. 4 and 5), while 2 cycles of loading are enough for the stabilization of the loop within a fixed displacement amplitude in Theory I, because of the phenomenon that the lateral deflection vanishes when the bar is subjected to the maximum tensile displacement. Theory II well describes but overestimates the cumulation of the lateral deflection, leading to a stabilization delay. As described above, Theory I, in which the closed form solution is obtained, gives reasonably good approximation to the real behavior of a bar without tedious numerical computation. Theory II takes care of the effect of the cumulated lateral deflection in consideration of the partial yielding. The small deflection assumption seems to hold its validity, since the maximum lateral deflection was observed in the test to be about 10% of the length of the bar. A more realistic model of the stress-strain relationship may have to be employed to improve the theoretical results.

REFERENCES

(brief notes and abstracts omitted)


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Fig. 1 Test specimen.

Fig. 2 Loading apparatus.

Fig. 3 A N-Δ relation in Ref. 1.

Fig. 4 N-Δ and N-V relations for slenderness ratio 40.

Fig. 5 N-Δ relation for slenderness ratio 120.

Fig. 6 Schematic N-Δ and N-V relations.

Fig. 7 Dimensionless load-displacement relations by Theory I.

Fig. 8 Dimensionless load-displacement relation by Theory II.

Fig. 9 Comparison between experimental and theoretical results with displacement amplitude being 0.5% and 2% of the effective length.