

# AN EMPIRICAL EVALUATION OF INELASTIC BEHAVIOR OF STRUCTURAL ELEMENTS IN REINFORCED CONCRETE FRAMES SUBJECTED TO LATERAL FORCES

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## SYNOPSIS

Test results of reinforced concrete framing elements, i.e., beams, columns, shear walls and beam-column connections were reviewed to derive equations for skeleton curves of restoring force characteristics. Based on some discussions of influences of geometry and materials on test results, empirical equations of strength and inelastic stiffness of the elements were proposed by statistical method.

## INTRODUCTION

A practical method to evaluate inelastic seismic deflection of reinforced concrete frames based on the test results of elements was proposed<sup>1)</sup> and was further developed<sup>2),3)</sup>. For this method as well as general earth quake resistant design of frames, existing data of the elements should be systematically and statistically reviewed to predict more realistic restoring force characteristics.

For the first step of the study, the authors investigated skeleton curves of restoring force characteristics which can generally be idealized into tri-linear model. 22 papers were reviewed<sup>4)~25)</sup> and test data of more than 250 rectangular beams and columns, 120 framed shear walls without opening and 40 beam-column connections were studied with respect to strength and inelastic stiffness which define the tri-linear relation in Figs. 1 to 3. Influences of geometry and materials on the behavior were discussed with regression analysis and set of empirical or approximate equations were proposed. Reliability and application of the equations were also discussed.

It was observed that yield strength and stiffness at yielding of the elements could be estimated with fair accuracy by the proposed equations. On the other hand, measured cracking strength had wide scatter to the proposed equations. However error in estimating cracking strength is not very critical in frame analysis because the idealized tri-linear relation is not very sensitive to the cracking strength.

The authors acknowledge gratefully the guidance of Prof. H. Umemura and H. Aoyama of University of Tokyo, and to authors of original papers for their permission to use their data in this study.

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## INELASTIC BEHAVIOR OF BEAMS AND COLUMNS FAILING IN BENDING

The tri-linear relation of beams and columns in Fig. 1 is defined by cracking moment  $M_c$ , yield moment  $M_y$  and stiffness reduction at yielding  $\alpha_y$ . They are estimated by following set of equations.

According to elastic beam theory, cracking moment  $M_c$  is expressed by

$$M_c = (c\sigma_t + N/A) Z \quad \dots\dots 1)$$

where  $Z$  and  $A$ , transformed uncracked section modulus and area, and  $N$ , axial compression on the section.  $c\sigma_t$  which normally corresponds to modulus of rupture of concrete is found as follows from eq. 1) and measured  $M_c$

$$c\sigma_t = 1.8 \sqrt{F_c} \quad (\text{kg/cm}^2) \quad \dots\dots 2)$$

where  $F_c$  is compressive strength of concrete.

Instead of analyzing yield moment  $M_y$  rationally, the authors attempted to estimate  $M_y$  with much simpler approach. Assuming tension and compression steel yield, we can obtain following equation,

$$M_y = (g_1 p_t \sigma_y / F_c + 0.5 \eta_o (1 - \eta_o)) F_c b D^2 \quad \dots\dots 3)$$

where  $g_1$  is the distance between tension and compression steel divided by depth of section,  $p_t$  and  $\sigma_y$ , ratio and yield strength of tensile steel and  $\eta_o = N / (b D F_c)$ . However eq. 3) is applicable even if compressive steel does not yield and test results show satisfactory agreement, i.e., 90% of data has within 20% of error.

Stiffness reduction at yielding  $\alpha_y$  is influenced by tensile steel  $np_t$  ( $n$ : modular ratio), span to depth ratio  $a/D$ , axial force  $\eta_o$  and effective depth ratio  $d/D$ . An empirical equation is obtained as follows,

$$\alpha_y = (0.043 + 1.64 np_t + 0.043 a/D + 0.33 \eta_o) (d/D)^2 \quad \dots\dots 4)$$

and test results show good agreement, i.e., 90% of data has within 30% of error. This equation may be used for members with  $p_t = 0.4 \sim 2.8\%$ ,  $a/D = 2 \sim 5$  and  $\eta_o = 0 \sim 0.55$ .

## INELASTIC BEHAVIOR OF FRAMED SHEAR WALLS WITHOUT OPENING

Cracking shear stress  $\tau_c$ , yield shear stress  $\tau_y$  and stiffness reduction at yielding  $\beta_y$  define the tri-linear relation of shear walls in Fig. 2. Here shear stress is expressed by average, i.e.,  $\tau = Q/A_w$ ;  $Q$  is shear force and  $A_w$  is section area of wall.

Effect of axial force on  $\tau_c$  is not considered here because it is usually transmitted to columns and wall panel relatively free of axial force. It appears that  $\tau_c$  is influenced by  $F_c$  and stress due to bending, and is expressed by empirical equation in terms of  $F_c$  and  $A_g/A_w$ ,

$$\tau_c = (4.3 A_g / A_w + 0.05) F_c \quad (\text{kg/cm}^2) \quad \dots\dots 5)$$

where  $A_g$  is total area of steel in a column.

The authors attempted to estimate yield stress  $\tau_y$  introducing "truss effect" of concrete in wall panel. Assuming that concrete diagonal member would crush at yielding,  $\tau_y$  can be estimated as the sum of strength of truss and wall reinforcement if the effective width of diagonal member  $B_e$  is known

$$\tau_y = B_e \cdot F_c / \sqrt{\ell^2 + h^2} + p_w \cdot w \sigma_y \cdot h / \ell \quad \dots\dots 6)$$

According to test results, first term of eq. 6) can be estimated by following equation,

$$B_e \cdot F_c / \sqrt{\ell^2 + h^2} = \left\{ A_g \cdot \sigma_y / A_w + p_w \cdot w \sigma_y (1 - h^2 / \ell^2) / 2 + N / (2A_w) \right\} \ell / h$$

$$\leq 5.6t \cdot F_c / \sqrt{\ell^2 + h^2} \quad \dots\dots 7)$$

where  $\ell, h, t$ , are length, height and thickness of wall respectively and  $p_w$  and  $w \sigma_y$  are steel ratio and yield strength of wall reinforcement. Eqs 6) and 7) show good agreement, i.e., 80% of test data has within 20% of error.

Stiffness reduction at yielding  $\beta_y$  is expressed by following equation in terms of  $p_w w \sigma_y$  and  $F_c$ ,

$$\beta_y = 0.46 \cdot p_w \cdot w \sigma_y / F_c + 0.14 \quad \dots\dots 8)$$

#### INELASTIC BEHAVIOR OF BEAM-COLUMN CONNECTIONS

Cracking shear stress  $\tau_c$ , yield shear stress  $\tau_y$  and yield shear strain  $\gamma_y$  define the tri-linear relation in Fig. 3. Here shear stress is expressed as follows,

$$\tau = (1 - u - v) \cdot Q \cdot h / (t u \ell v h) \quad \dots\dots 9)$$

where  $Q, h$  and  $\ell$  are column shear force, story height and span and  $t, u \ell$  and  $v h$  are thickness, width and height of connection panel respectively.

A rational expression for cracking shear stress  $\tau_c$  is

$$\tau_c = \sqrt{{}_c \sigma_t^2 + {}_c \sigma_t \cdot \sigma_n} \quad (\text{kg/cm}^2) \quad \dots\dots 10)$$

where  $\sigma_n$  is normal stress due to column load (compressive positive).  ${}_c \sigma_t$  which normally corresponds to tensile strength of concrete is found as follows from eq. 10) and measured  $\tau_c$

$${}_c \sigma_t = 0.205 \cdot F_c - 0.0004 \cdot F_c^2 \quad (F_c < 420 \text{ kg/cm}^2) \quad \dots\dots 11)$$

It appeared in the test results that yield shear stress  $\tau_y$  is little affected by axial stress  $\sigma_n$ . Assuming that  $\tau_y$  is expressed by equation 12),

$$\tau_y = \tau_{\text{conc}} + 2.7 \sqrt{p_w \cdot w \sigma_y} \quad \dots\dots 12)$$

where  $p_w$  and  $w \sigma_y$  are steel ratio and yield strength of shear reinforcement, the relation between  $\tau_{\text{conc}}$  and  $F_c$  is expressed by following empirical equation.

$$\tau_{\text{conc}} = 0.51 \cdot F_c - 0.001 \cdot F_c^2 \quad (F_c < 420 \text{ kg/cm}^2) \quad \dots\dots 13)$$

All of 31 test results which failed at connections show good agreement within 20% of error.

Yield shear strain  $\gamma_y$  is generally greater than 0.4%, and

$$\gamma_y = 0.5 \% \quad \dots\dots 14)$$

will result in the best representation of tri-linear relation in Fig. 3.

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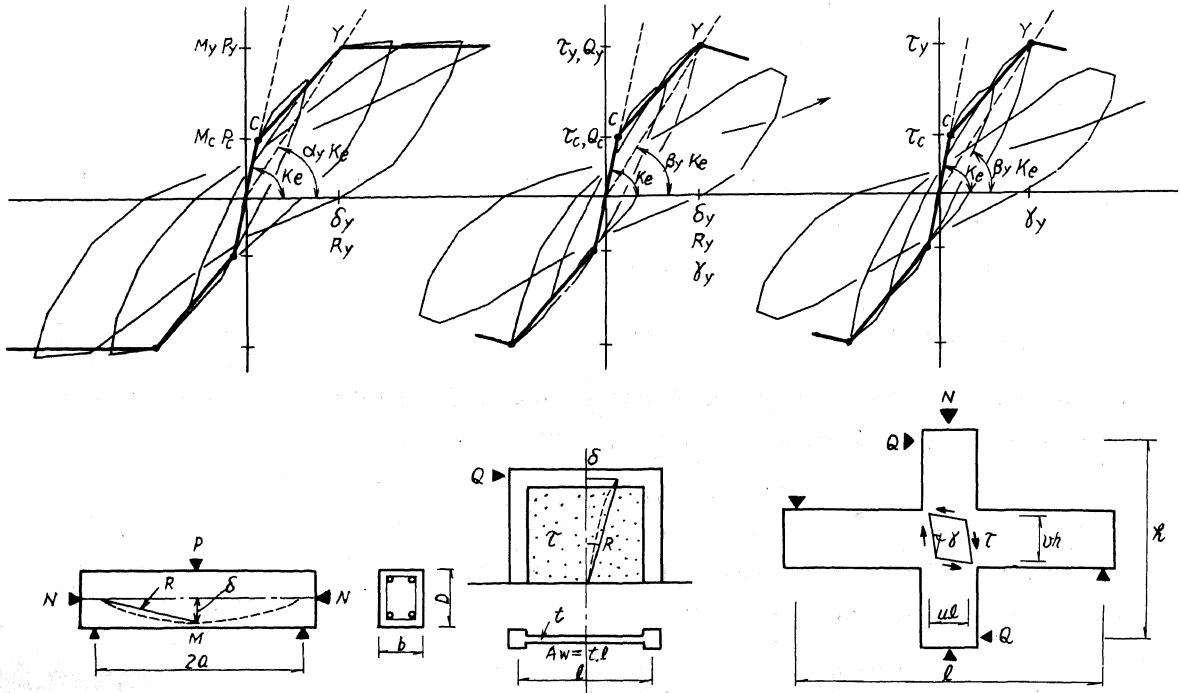


Fig. 1. Beams and Columns

Fig. 2. Shear Walls

Fig. 3. Beam-Column Connections