SEISMIC ANALYSIS OF LIQUID SODIUM STORAGE TANKS

by

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SYNOPSIS

The reactor building of the Fast Flux Test Facility (FFTF) houses several tanks which are used for the storage of liquid sodium at 1200°F. To minimize the effects of the severe thermal environments, the tanks are supported by a pendulous support system. Base slide pins are used to constrain the horizontal motion of the tanks during a potential seismic disturbance. A spectral analysis of these tanks is presented, including fluid sloshing effects, to determine peak hanger loads as well as base pin forces developed by the seismic disturbance.

INTRODUCTION

In the design of equipment and structures housed within primary structures of FFTF, the seismic design criteria is specified in terms of support point floor response spectra. In particular, support point spectra for the liquid sodium storage tanks are available at both the ceiling and floor attachment points. The support configuration for these tanks (Fig. 1) consists of four pin-ended hanger members attached to the ceiling and slide pins to restrict horizontal motions of the tanks caused by seismic disturbances. The objective of this support configuration was to minimize the effects of thermal expansion.

FLUID SLOSHING MODEL

When the tanks are subjected to seismic induced motions, the fluid in the tank will tend to undergo sloshing motions. As a result, a simple rigid mass assumption for the fluid is inadequate unless either the sloshing frequencies of the fluid are sufficiently high compared to the frequency range of the input seismic motion, or the mass of fluid associated with the sloshing is sufficiently small so that the resulting inertial forces acting on the vessel are negligible.

An equivalent lumped mass model for the fluid is used herein based on the general results presented by Abramson[1]. For sloshing normal to the tank axis, the following

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assumptions are made: (1) the tank is represented as a cylinder with flat plate ends and the effects of the ellipsoidal heads are neglected; (2) the flexibility of the tank walls are neglected; and (3) the liquid in the tank is assumed to be inviscid and incompressible. On the basis of these assumptions, both analytic solutions and experimental data are available for frequencies of fluid sloshing as well as the total forces and moments applied to the tank by the sloshing motions. A lumped mass model for the fluid was generated consisting of a rigid fluid mass component and a series of fluid mass elements connected to the tanks by means of equivalent springs. By comparing the frequencies, forces and moments developed by this lumped mass model with the available analytic solution, the parameters of the lumped mass model can be determined and are shown in Fig. 2.

For sloshing in the longitudinal direction, an analytic solution for a circular cylinder is not available. However, the lumped parameter representation of the sloshing model is available[1] for a tank of rectangular cross section. The rectangular tank would be expected to give more severe sloshing problems than the cylindrical tank. The parameters of the lumped mass model are also presented in Fig. 2. For one of the tanks analysed (12 ft. diameter, 20 ft. long) the longitudinal slosh frequencies for the first mode were determined as:

Depth of Fluid (h/2a)	% Full by Volume	w_1 (rad/sec)	$\frac{m_o}{M_f}$	$\frac{m_1}{M_f}$
0.50	50.0	0.557	.313	.687
0.75	80.5	0.768	.418	.582
0.84	90.0	0.833	.454	.547

As may be noted, for all levels of fluid, a significant percentage of the fluid is involved in the sloshing mass.

RIGID BODY SEISMIC ANALYSIS

The seismic analysis of the tank was first conducted assuming no sloshing of the fluid, with the only flexibility in the system being that of the vertical hangers. Preliminary analysis using a finite element shell analysis indicated that shell bending frequencies were significantly higher than the frequency range of the seismic input. The potential modes of motion for both tranlational and longitudinal seismic input motions are shown in Fig. 3. The equations of motion for the two modes are presented in Fig. 4. Since seismic inputs were specified in terms of support point shock spectra and not motion-time histories, shock spectral analyses were used to obtain upper bounds on the peak hanger loadings as well as base lateral forces, for both the operating basis earthquake (OBE) and design basis earthquake (DBE). In designing the hangers, the required area of the four hangers was determined so as to yield a translatory rocking frequency of 10 cps. With these hanger areas, the frequencies (and associated stresses) were determined for both the vertical and longitudinal modes.

EFFECT OF SLOSHING MASS

The equations of motion for the translatory and longitudinal modes were again derived assuming only one sloshing fluid mass together with one rigidly connected fluid mass. The vertical equation of motion is the same as previously found for the no sloshing analyses, and since it is uncoupled from the two rotational equations, leads to the same peak vertical stresses. The equations of motion for the rotational modes are inertially coupled so that a modal analysis technique must first be used to uncouple the equations before applying the spectral seismic analysis. Again upper bounds to the seismic hanger loads can be determined by superposing the peak responses in each mode. The results of this analysis for the same tank as mentioned previously, for the longitudinal sloshing mode leads to

Peak hanger stress (psi)	3740
Peak hanger stress from rigid model	4160
Peak base force (kips)	198
Peak base force from rigid model	256

As may be noted, the inclusion of sloshing effects into the seismic analyses reduces the loadings due to seismic inputs.

SUMMARY

The effects of sloshing on the seismic response of fluid filled tanks has been considered herein. In all cases analysed, support loadings computed on the basis of spectral analysis were lower than those obtained from the simple rigid body analyses neglecting the effects of the sloshing mass.

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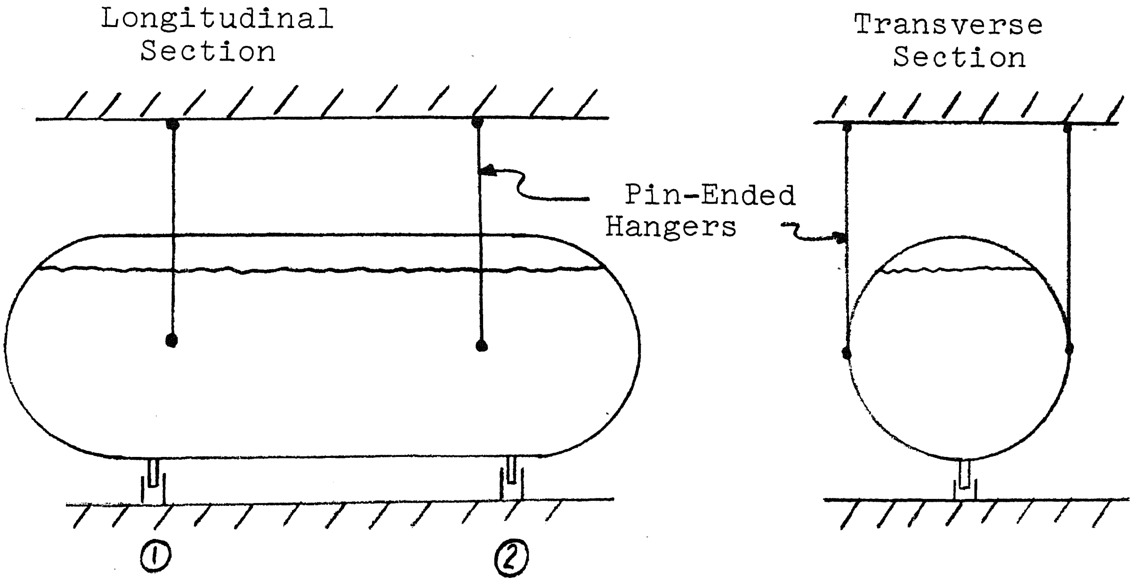
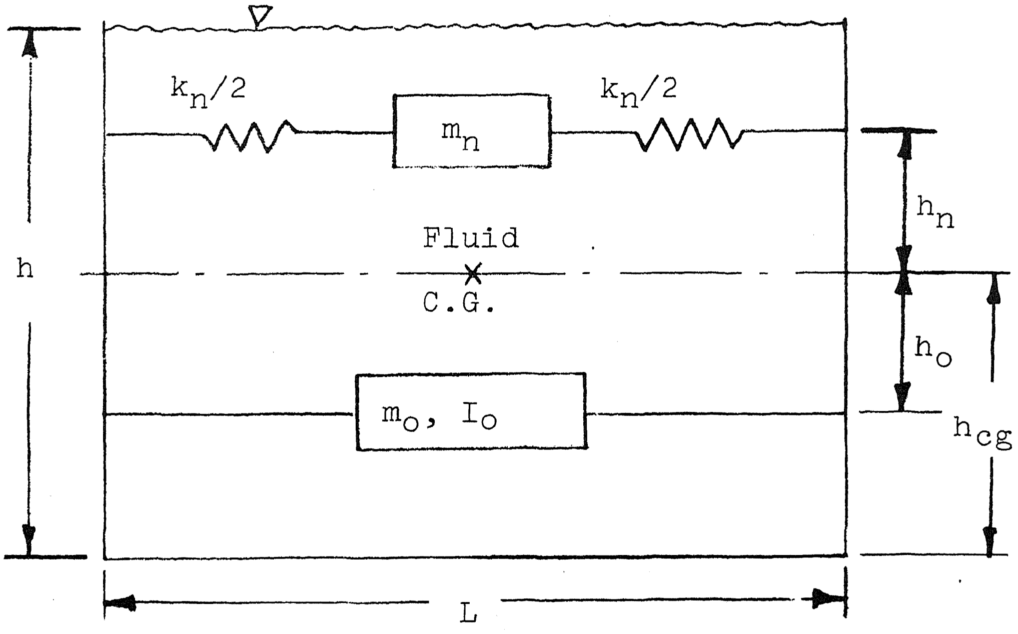
1. H.N. Abramson, "The Dynamic Behavior of Liquids in Moving Containers", NASA SP-106, 1966.



Pinned Supports 1 and 2 free in vertical direction;
Pinned Support 1 fixed in both horizontal directions;
Pinned Support 2 fixed in transverse horizontal direction;
free in longitudinal horizontal direction

Fig.1 Tank Support Configuration



Translational Mode (Circular Cross-section, Radius a):

$m_n = 2\ M_f\ (\tanh \delta_n)/[\delta_n(E_n{}^2-1)]$, M_f = **Total Fluid Mass**

$h_n/2a = (\cosh \delta_n - 1)/(\delta_n \sinh \delta_n) - 0.5772/2\ \delta_n$

$m_o = M_f - \Sigma m_n$, $h_o = \Sigma m_n\ h_n/m_o$

$I_o = (M_f a^2/8)[1 - 2\ (h/a)^2/3] - m_o\ h_o{}^2 - \Sigma m_n\ h_n{}^2$

w_n from Ref.1, $k_n = m_n\ w_n{}^2$

$\delta_n = E_n\ (h/a)$, E_n = Zeros of $J_1{}'\ (E_n)$

Longitudinal Mode (Rectangular Cross-section, h x L):

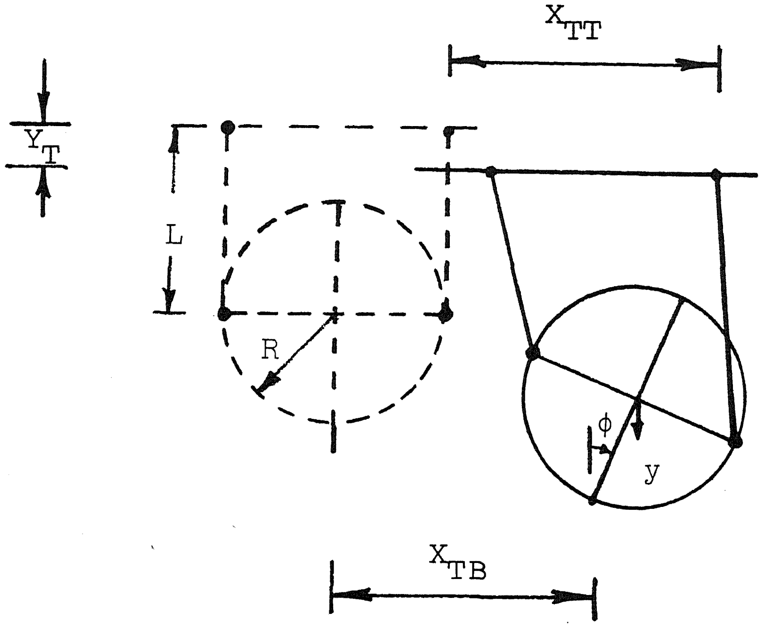
$m_n = M_f\ (8L\beta)/[\pi^3(2n - 1)\ h]$, $\beta = \tanh\ [(2n - 1)\ \pi h/L]$

$h_n = h_{cg} - 2L\beta'/(2n-1)\pi$, $\beta' = \tanh\ [(2n - 1)\ \pi h/2L]$

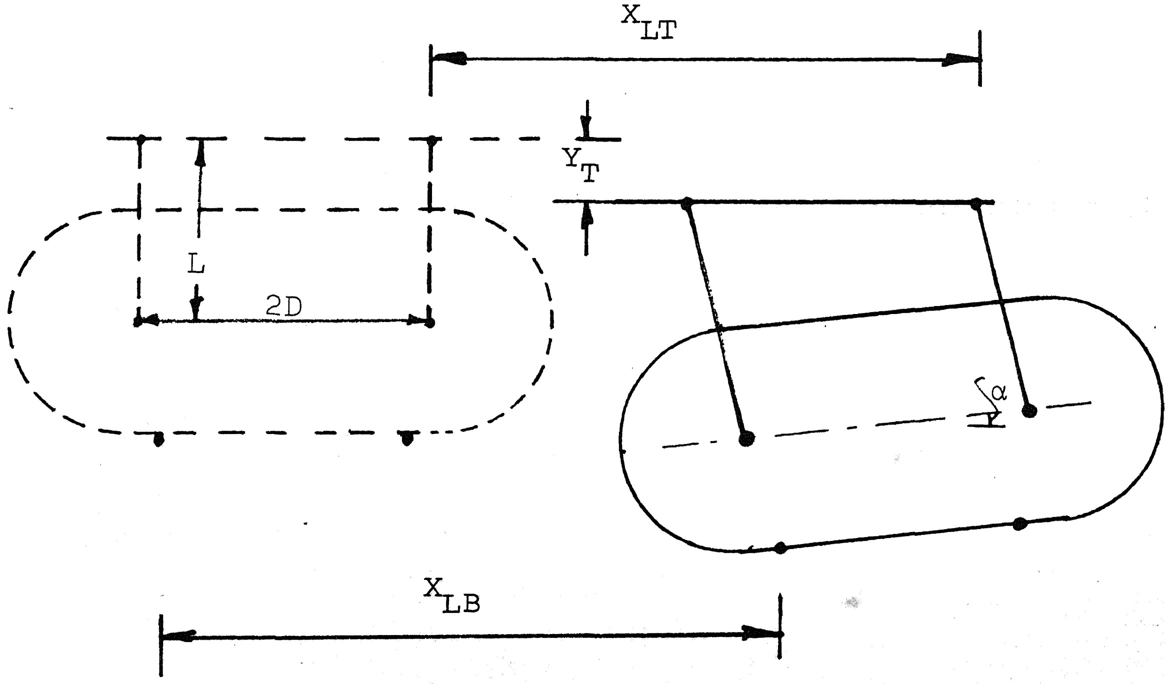
$k_n = M_f\ (4g\ \beta^2/\pi^2\ L)$, g = acceleration of gravity

I_o from Ref.1, M_f, m_o, h_o as above

Fig.2 Mechanical Model of Fluid Sloshing



(a) Translational Mode



(b) Longitudinal Mode

Fig.3 Modes Of Response for No Sloshing Assumption

Vertical Motion:

$$M_t \ddot{z} + \frac{AE}{L} z = - M_t \ddot{y}_T$$

M_t = mass of tank/fluid system, E= elastic modulus of hangers

A = total area of four hangers, $z = y - y_T$

Translatory Rocking:

$$M_1(R\ddot{\phi}) + (AE/L)(R\phi) = - [M_t(R'/R)\ \ddot{x}_{TB} + M_r\ (\ddot{x}_{TB} - \ddot{x}_{TT})/3]$$

$$F_{TB} = (M_t + M_r/3)\ \ddot{x}_{TB} + (M_t\ R' + M_r\ R/3)\ddot{\phi}$$

$$M_1 = M_t\ (R'/R)^2 + I_t/R^2 + M_r/3$$

R' = distance from tank base to c.g. of tank/fluid system

I_t = total moment of inertia about c.g. of tank/fluid system

M_r = total mass of four hanger supports

Longitudinal Rocking:

$$M_2(D\ddot{\alpha}) + (AE/L)(D\alpha) = [M_t(R'/D)\ \ddot{x}_{LB} + M_r(R/D)(\ddot{x}_{LB} - \ddot{x}_{LT})/3]$$

$$F_{LB} = (M_t + M_r/3)\ (\ddot{x}_{LB} - R'\ddot{\alpha}) - (M_r/3)\ (\ddot{x}_{LT} + (R - R')\ddot{\alpha})$$

$$M_2 = M_t\ (R'/D)^2 + (M_r/3)\ (R/D)^2 + I_L/D^2$$

I_L = total moment of inertia about c.g.

Fig. 4 Summary of Equations for No Sloshing Assumption