PROBABILISTIC ANALYSIS OF NONLINEAR SEISMIC RESPONSE OF STRATIFIED SOIL DEPOSITS

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SYNOPSIS

A frequency-domain, stochastic method is presented for calculating the spectral response of stratified soil deposits exhibiting Ramberg-Osgood nonlinear behaviour. The method is based on the assumption that the nonlinear response can be approximately represented by a modal combination of the responses of several simple equivalent viscoelastic systems. For a chosen stratigraphy the results are compared with those obtained by direct numerical simulation and are shown to provide a close approximation.

INTRODUCTION

Recent soil dynamics investigations have repeatedly stressed the need of using nonlinear constitutive laws in strong-motion seismic response studies. For practical purposes, the actual behaviour of a soil element under simple shear cyclic load can be closely approximated by the Ramberg-Osgood hysteretic model.[1] This material description has been applied either in direct numerical simulation[2,3] or through equivalent, strain-dependent shear moduli and damping factors.[3] In both cases the step-by-step calculation of the surface response of a deposit to a prescribed bedrock input may become a rather time-consuming process, since a representative average response is needed and this implies repetition of the calculations for several different inputs. Actually, many important applications do not require the time-history of the response. In order to obtain spectral ordinates, for example, the power spectral density (PSD) constitutes an adequate measure of the response and for its obtention a frequency-domain approach is obviously more appropriate. The method presented herein leads to the determination of the PSD of acceleration response of horizontally stratified deposits of Ramberg-Osgood soil materials excited at the base by ensambles of random stationary SH signals with prescribed PSD. The basic idea of the method consists of finding several equivalent simple viscoelastic systems whose combined frequency response to the prescribed random excitation approximates the response of the hysteretic system under an equivalent harmonic excitation.

TREATMENT OF HYSTERETIC RESPONSE

Let a given soil profile be discretized into a system of n masses interconnected with springs defined by nonlinear Ramberg-Osgood relations. The bottom mass is connected to the base through a piston in order to account for radiation effects.[4] The motion of the system is governed by the equations

$$m_i \ddot{x}_i + p_{i-1}(x_i - x_{i-1}) + p_{i+1}(x_i - x_{i+1}) = 0 \qquad i = 1, \ldots, n-1$$

$$m_n \ddot{x}_n + p_{n-1}(x_n - x_{n-1}) + \mu \times (\dot{x}_n - \dot{x}_o) = 0 \qquad i = n \qquad (1)$$

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where

$p(.)$ is the nonlinear stiffness of the i-th spring
x_i is the absolute horizontal displacement of mass m_i
x_o is the prescribed base motion
μ is the viscosity of the base piston (equal to the seismic impedance of bedrock).

If sinusoidal excitation of frequency ω is considered and only first-order terms in the Fourier expansion of the response are retained, it can be shown that steady-state motions of the system are approximately described by an equivalent set of equations where each term $p_i\ (x_{i+1} - x_i)$ is replaced by

$$m_i(\omega_{o,i}^2/\omega)\ S_i\ \times (x_{i+1} - x_i) + m_i\omega_{o,i}^2\ C_i\ \times (x_{i+1} - x_i) \qquad (2)$$

where $\omega_{o,i} = k_{o,i}/m_i$ and $k_{o,i}$ is the initial tangent stiffness of i-th spring. The functions S_i/ω and C_i, which depend on the amplitude of oscillation and the parameters of the hysteresis cycle, are equivalent nondimensional damping and stiffness as defined in References 1 and 5. Given a sinusoidal motion of assigned amplitude and frequency acting on mass 1, the equivalent set can be sequentially solved from the top to the bottom. This yields amplitude and phase angle of the oscillation of each mass and also of the base motion (taken as if no soil deposit existed above). If a given amplitude of relative displacement (RD) between top and base is prescribed, an iterative procedure has to be applied whereby the excitation amplitude is varied until the resulting RD equals the assigned one.

RESPONSE TO SEISMIC EXCITATION

The general flow chart of the method is illustrated in Fig. 2. Only its salient points will be described here. The essence of the approach consists in determining at any new iteration the properties of each modal equivalent linear system in such a manner that the variance of the RD response of the hysteretic system to harmonic excitation in resonance with it is equal to the variance of the RD response of the equivalent modal system to the random excitation computed in the previous iteration. The starting point for the iterative procedure is conveniently provided by the fundamental frequencies and the corresponding peaks of the RD amplification curve resulting from a system with initial tangent stiffnesses and arbitrary damping factors. In any iteration, if y denotes the peak RD amplification yielded by the hysteretic system for a given mode, the damping factor ζ of the corresponding 1 d.o.f. modal system is simply taken as $\zeta = 1/2y$. The PSD of the input acceleration is taken in the well-known from

$$S_o(\omega) = \text{const}\ (1 + \omega^2/147.8)/[(1 - \omega^2/242)^2 + \omega^2/147.8] \qquad (3)$$

and is used to excite separately each modal system. The method can be applied to any other sufficiently smooth PSD function. The RD response variance is simply calculated as

$$\sigma_{RD}^2 = \int_0^\infty \left(S_o(\omega)/[(\omega_o^2 - \omega^2)^2 + 4\,\zeta^2\omega_o^2\omega^2] \right)\ d\omega \qquad (4)$$

Convergence is checked on both fundamental frequency and damping factor of each modal system. Once it is achieved the resultant RD transfer funcion (TF) is taken to be

$$H_{RD}(\omega) = \Sigma_i \gamma_i H_i (\omega) \quad (5)$$

where H_i are the modal RD TFs and γ_i participation factors computed with an approximate formula for shear beams.[6] In order to obtain the desired PSD of surface acceleration response, the necessary TF for acceleration, H_a, is computed as follows

$$H_a(\omega) = 1 + \omega^2 \; H_{RD}(\omega) \quad (6)$$

After obtaining the response PSD as $S_1(\omega) = |H_a(\omega)|^2 \; S_0(\omega)$, the undamped relative velocity spectrum is calculated from the approximate formula[1]

$$S_v^o(\omega) = 1.15 \sqrt{\pi S_1(\omega) \; t} \quad (7)$$

where t is the assumed duration of the acceleration response.

A case problem was studied, with the soil profile illustrated in Fig 1. This profile was discretized into twenty masses. The a parameters for the Ramberg-Osgood description were obtained from best fittings of experimental data, whereas $r = 3$ was assumed for all springs. The average intensity of excitation was taken equal to 0.45 m/s^2. Two modes were considered and convergence was achieved in 5 and 7 cycles for the first and second mode respectively. The resulting transfer function and response spectrum are shown in Figs. 3 and 4. The influence of nonlinearity on dominant frequencies and amplification factors is evident. In order to check the validity of the results, the assigned case problem was also analyzed by direct numerical simulation[2] using the same number of masses, soil parameters, etc. Eight independent artificial accelerograms,[1] obtained from the PSD function (2), were considered. Surface acceleration histories and response spectra were obtained for each input and then arithmetically averaged. These average curves are shown in Figs. 3 and 4 and the agreement with the results obtained by the proposed method is seen to be rather satisfactory. If only the first mode is taken into account, the computer time needed to compute the response spectrum by the present method is about 1/6 of that required by direct simulation averaging on 8 samples. With two modes, it is of about 1/4.

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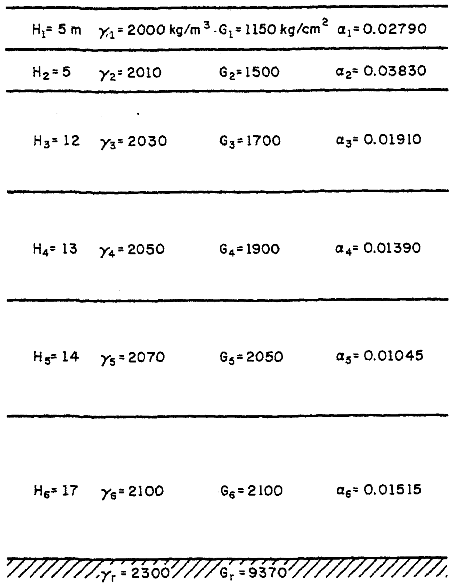

Fig 1 Stratigraphy of case problem

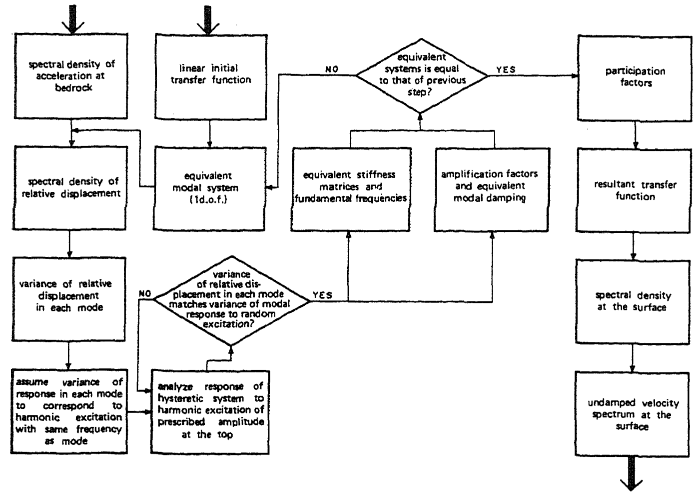

Fig 2 General flow chart

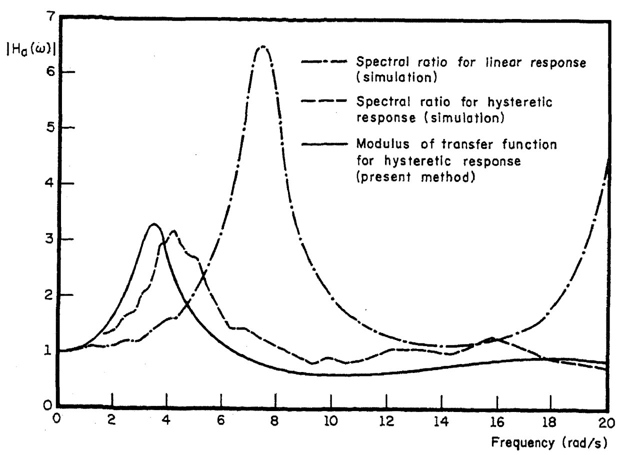

Fig 3 Amplification functions

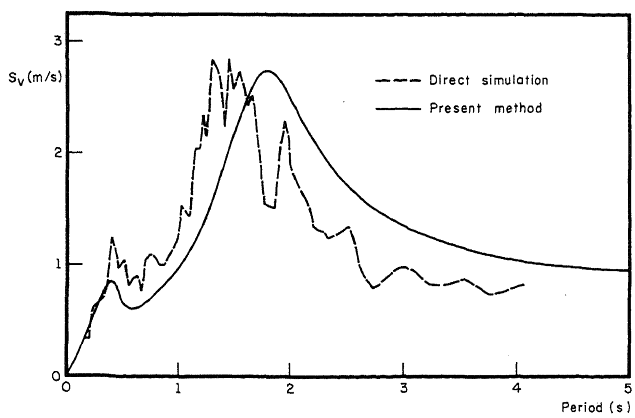

Fig 4 Velocity spectra