WAVE PROPAGATION AND LUMPED MASS ANALYSIS TECHNIQUES APPLIED TO THE DETERMINATION OF THE RESPONSE OF MULTIPLE LAYERED SYSTEMS TO SINUSOIDAL AND SEISMIC EXCITATION

by

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SYNOPSIS

Both the wave propagation(1) and the lumped mass(2) analysis techniques may be used to model the response to seismic excitation of horizontal soil layers overlying bedrock. Although the methods are based on entirely different assumptions of the mechanisms by which energy is lost from the layered system, the work described in this paper shows that under certain conditions a similar response can be obtained from either model. The particular applications in which the use of one model is more appropriate than the use of the other are indicated.

INTRODUCTION

Possible useful applications of a mathematical model simulating a layered soil system may be subdivided into three categories. In the first an earthquake record representing base motion may be modified to include the effect of one or more surface layers so that a record representative of the motion at a particular site on the ground surface is derived. In the second, a seismic record obtained at the top of one or more surface layers may be modified to remove from it the effect of the layers, thereby producing a record appropriate to the base rock motion. In the third category the motion anywhere within a layered system may be determined from the known motion at some other position and consequently the validity of a particular mathematical model may be examined using experimentally determined records at positions other than the surface or the base of the layered soil system.

Two types of mathematical model are available for the analysis of layered soil systems. Consideration of the vertical propagation of horizontally polarised shear waves through surface layers necessitates investigation of the transfer of a shear pulse across a seismic discontinuity. Partial transmission and partial reflection may be examined using considerations of continuity of displacement and compatibility of stress at each interface. In the alternative analysis procedure the method described by Seed and Idriss may be used to relate input and output at any mass point and thereby provide results to compare with those obtained from the wave propagation analysis approach.

Both the wave propagation method, previously used(1) in the second application described above, and the lumped mass technique(2), previously applied in the first and third categories, have been extended to enable all three forms of application to be undertaken by either approach(3). There is a significant difference between the analysis methods in that energy is

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considered to be lost into the sublayer in the wave propagation model, whereas it is assumed to be lost by internal viscous damping in the lumped mass case. In any real system both effects will be present. In order to eliminate the effect of a particular seismic input it is convenient to investigate the responses determined by application of each analysis technique in turn using an infinite train of constant amplitude sinusoidal waves to excite the system. By equating the responses at resonance in each vibrating mode equivalent critical viscous damping ratios may be determined for the lumped mass model so that it will lose energy equivalent to that lost by the wave propagation model.

RESPONSE TO SINUSOIDAL EXCITATION

The response to sinusoidal excitation of a multiple layered system may be determined using the equations derived by Takahasi(4) whereas a multiple lumped mass system may be analysed by normal mode theory, combining the responses of the modes to obtain the overall response. On using the site properties of the strong motion recording station at Taft(5) to set up a twenty mass model of the four layer system equivalent viscous damping requirements of 22%, 7.4%, 4.1%, 3.0% and 2.2% critical for modes 1 to 5 respectively, diminishing to about 0.001% for the twentieth mode were determined.

Considering first upward propagation, as may be seen by comparing figures 1 and 3, for periods greater than 0.24 sec. the equivalent lumped mass model gives similar amplification responses to the wave propagation one. At periods less than this the wave propagation model continues to show amplification whereas in the lumped mass case a sharp drop in response produces a filtering effect. In the downward direction (figures 2 and 4) both models produce a filtering effect at periods greater than 0.24 sec. As the period decreases this characteristic is maintained by the wave propagation model whereas the lumped mass one exhibits amplification which increases progressively to very high values.

RESPONSE TO SEISMIC EXCITATION

On applying the analysis procedures to earthquake accelerograms an additional unrepresentative component of high frequency response is introduced by the digitisation technique used in reading the original time history record(6) which will be enhanced by application of either model in that direction which results in the high frequency component being amplified. In the case of the upward wave propagation analysis numerical filtering of the high frequency component of the output can be used to eliminate this effect. For downward lumped mass analysis the in-built high frequency amplification (indicated by figure 4) extends through the lower frequencies and consequently an acceptable output cannot be obtained by filtering.

Use of the models in the directions which inherently filter the high frequency components tends to eliminate the digitisation error and it is in these directions that the analysis procedures have been applied previously. The equivalence of the two approaches can be compared most conveniently by using the models in this way. By applying to the Taft accelerogram (fig. 5) the wave propagation analysis the equivalent base time history shown in figure 7 was derived. Subsequently the lumped mass analysis method was used to produce a further surface accelerogram, shown in figure 9.

On comparing the spectra in figures 6 and 10 it can be seen that approximately the same accuracy has been achieved as could be expected from considerations of the amplification spectra. At periods greater than 0.25 sec. the frequency content of the original record has been reasonably well reconstituted while at periods less than this the calculated surface record is less than the original response. The accuracy with which figures 7 and 8 represent the Taft rock motion is critically dependent on the validity of the assumptions regarding dispersive energy loss and elastic response.

As a result of a number of similar equivalence comparisons it has been established that at low base impedance ratios, e.g. 0.35 as used above, equivalent damping factors give good agreement between the two models. As the base impedence ratio approaches unity however, the models do not give a similar response and the use of the equivalent damping factors in the lumped mass model does not provide a good representation of energy loss by dispersion.

CONCLUSIONS

The wave propagation model allows for energy loss only by dispersion into the sublayer. When the base impedence ratio is high or the material responds elastically this is the mode of the major source of energy loss. Consequently under these conditions use of this model can be expected to lead to satisfactory response predictions and it can be used with confidence to relate the motion at any two levels within the system. Numerical filtering of the high frequency components of the output is necessary to remove amplified digitisation errors when upward motion is being determined.

The lumped mass model provides for energy loss by internal viscous damping. However, when the base impedance ratio is low, equivalent viscous damping factors can be established to enable satisfactory representation of the effect of dispersion into the sublayer. This model can be readily extended to include non-linear and two dimensional considerations. It is only suitable for calculating the upper level motion from a known base motion.

BIBLIOGRAPHY

1. Kanai, K. et al.: Comparative Studies of Earthquake Motions on the Ground and Underground. Bull.Eqke.Res.Inst., 44(2),609-643, 1966.
2. Seed, H.B. and Idriss, I.M.: Influence of Soil Conditions on Ground Motions during Earthquake, Proc.A.S.C.E., 95, SM1, 99-137, 1969.
3. Travers, J.H.: The Seismic Response of Layered Ground. Unpublished thesis, University of Canterbury, N.Z., 1972.
4. Takahasi, R.: A short note on a Graphical Solution of the Spectral Response of the Ground. Bull.Eqke.Res.Inst., 33, 259-264, 1955.
5. Mathieson, R.B. et al.: Site Characteristics of Southern California Strong Motion Earthquake Stations. Report 64-15, Dept. of Eng., University of California, Los Angeles, 1964.
6. Trifunac, M.D. and Hudson, D.E.: Laboratory Evaluations and Instrument Corrections of Strong-Motion Accelerographs. EERL Report 70-04. California Institute of Technology, 1970.

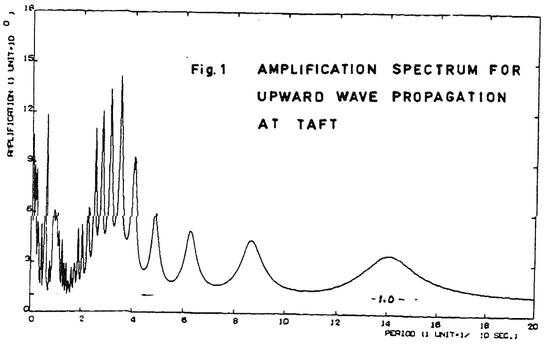

Fig. 1 AMPLIFICATION SPECTRUM FOR UPWARD WAVE PROPAGATION AT TAFT

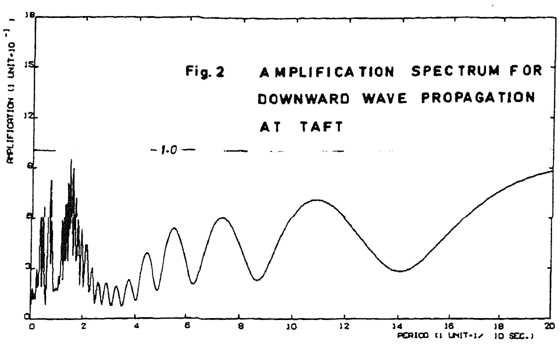

Fig. 2 AMPLIFICATION SPECTRUM FOR DOWNWARD WAVE PROPAGATION AT TAFT

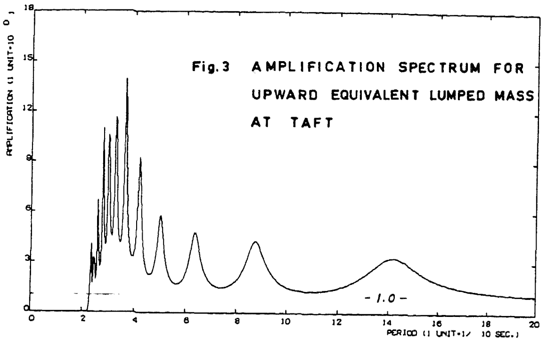

Fig. 3 AMPLIFICATION SPECTRUM FOR UPWARD EQUIVALENT LUMPED MASS AT TAFT

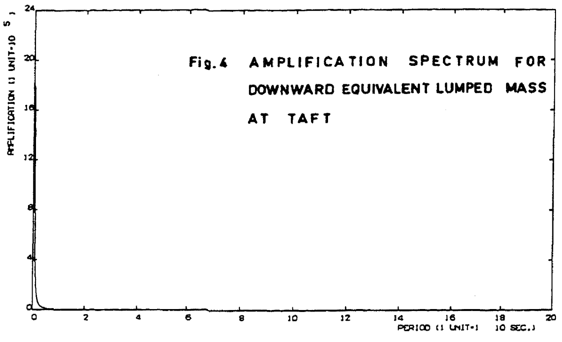

Fig. 4 AMPLIFICATION SPECTRUM FOR DOWNWARD EQUIVALENT LUMPED MASS AT TAFT

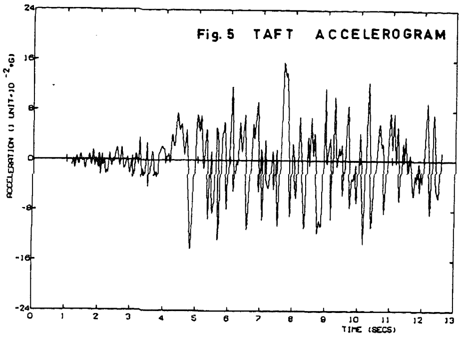

Fig. 5 TAFT ACCELEROGRAM

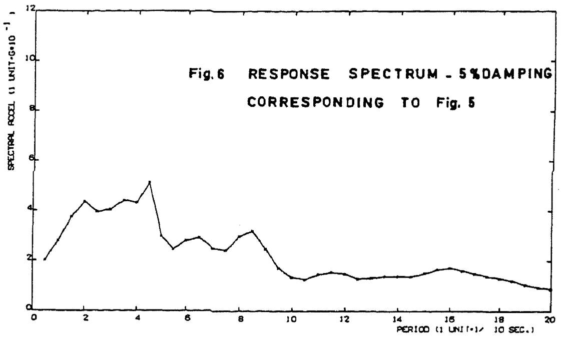

Fig. 6 RESPONSE SPECTRUM - 5% DAMPING CORRESPONDING TO Fig. 5

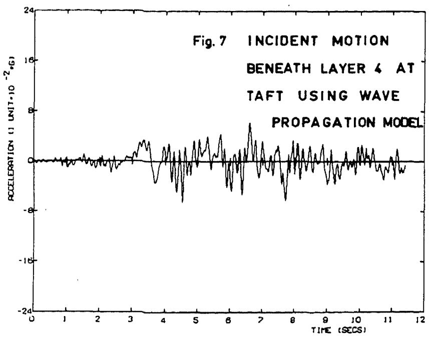

Fig. 7 INCIDENT MOTION BENEATH LAYER 4 AT TAFT USING WAVE PROPAGATION MODEL

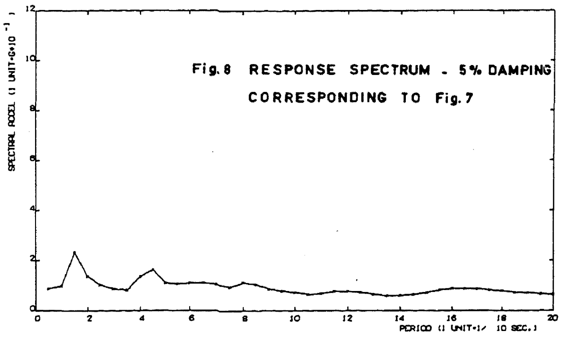

Fig. 8 RESPONSE SPECTRUM - 5% DAMPING CORRESPONDING TO Fig. 7

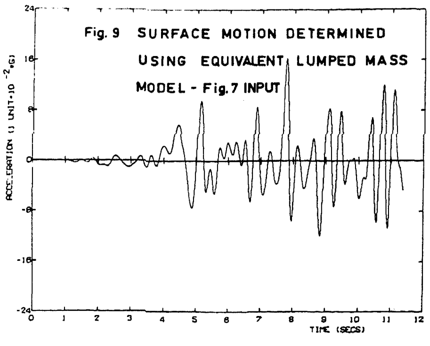

Fig. 9 SURFACE MOTION DETERMINED USING EQUIVALENT LUMPED MASS MODEL - Fig. 7 INPUT

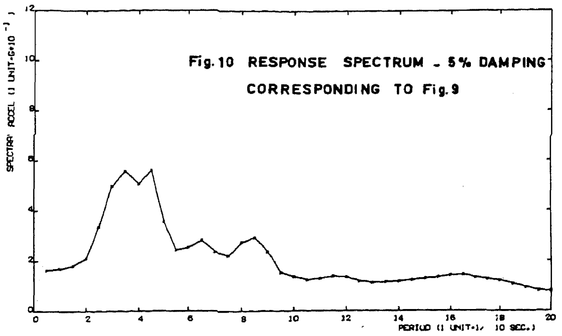

Fig. 10 RESPONSE SPECTRUM - 5% DAMPING CORRESPONDING TO Fig. 9

DISCUSSION OF "Wave Propagation and Lumped Mass Analysis Techniques Applied to the Determination of the Response of Muttiple Layered Systems to Sinusoidal and Seismic Excitation" by J H Travers and R Shepherd, paper 36, Session 1D: Dynamics of Soils and Soil Structures

by
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Radiation damping into rock can be precisely simulated in lumped-mass analysis by assuming that there is a dashpot between the rigid support that represents rock and the lowermost pair of spring and dashpot elements in the soil.(1,2) The dashpot constant is $\sqrt{\mu\rho}$ per unit area of interface, where μ and ρ are the modulus of rigidity and density of bedrock.

REFERENCES

1. E Rosenblueth and J Elorduy, "Características de los Temblores en la Arcilla de la Ciudad de México," Nabor Carrillo. El Hundimiento de la Ciudad de México y Proyecto Texcoco, contribución de Proyecto Texcoco al Séptimo Congreso Internacional de Mecánica de Suelos e Ingeniería de Cimentaciones, Secretaría de Hacienda y Crédito Público, Fiduciaria: Nacional Financiera, S.A., 1969, Mexico, 287-328.

2. N M Newmark and E Rosenblueth, Fundamentals of Earthquake Engineering, Prentice-Hall, Inc, Englewood Cliffs, N J, 1971, 77.

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