IDENTIFICATION OF DYNAMIC STRUCTURAL PARAMETERS FROM EXPERIMENTAL DATA

by

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Vibration testing is usually carried out on systems which are too complex to analyze by purely theoretical means. In some cases, theory has little basis for providing an answer (such as for estimates of structural damping). Occasionally, theoretical analysis is possible, but testing is faster and less expensive. Dams, office buildings, nuclear power plants, plant equipment, electrical distribution equipment and machine foundations have been tested. Methods of testing have used ambient, sinusoidal, impulse and other transient signals. (1, 2, 3)

Vibration testing and subsequent analysis can be carried out at various levels of complexity. Proof testing simply subjects the system to the forces anticipated during its "mission"; for example, to withstand an earthquake. This mode of testing simply tells us that the structure can or cannot withstand these loads. The test requires no dynamic model of the structure and tells us little about the dynamics of the system. Even if damage does not occur during proof testing at mission loads, the potential failure modes are of more than academic interest, and can best be predicted through the use of a model. Another disadvantage is that mission load forces are often difficult to produce. For example, in seismic certification of electrical switching gear, several simple non-seismic exciting forces are in use as opposed to earthquake time histories.(5) These signals, such as sine-beats and steady-state sines, are used because they are simpler to create with readily available test facilities. When used, they must be scaled to be equivalent, in some sense, to a seismic force. Such scaling requires the knowledge of a model of the system.

The parameters used to predict the response to mission loads need not be the mass and stiffness properties of the system. Indeed, the "eigenparameters" - mode shapes, dampings, eigenfrequencies and participation factors are all that is required. Furthermore, not all the eigenparameters are significant to the system's dynamic response. Systematic methods for finding the eigenparameters from experimental data without reference to the mass and stiffness properties are available.(1) Once found, these parameters allow calculation of the displacements and accelerations anticipated during a given excitation.

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While these responses may be sufficient in themselves, prediction of stress and failure modes requires a more detailed system description. In particular, detailed knowledge of the mass and stiffness properties must be obtained.[2,3] Experimental data does not provide a sufficient number of independent pieces of information to totally identify these properties. Assume that a finite element model of order N (N of the order of hundreds to thousands) is sufficiently accurate to describe a given structure. A reasonable experiment can only identify a few of the modes of vibration, p, (p (on the order of a few to tens). The number of unknown or free parameters in the model is on the order of N^2, while the number of independent pieces of experimental information is of the order p^2. As p^2 is much less than N^2, the structure's parameters are highly under-determined. To remove this indeterminancy, it is necessary to introduce additional constraints. Some methods for providing these are:

- To introduce a fictitious set of eigenparameters in addition to those found experimentally. These could come from a theoretical analysis.

- To assume some general property of the structure. For instance, assume that only nearest neighbor interaction occurs (close-coupled system) and hence K is tri-diagonal.

- To assume certain physical constraints on the structure. For example, the mass distribution may be specified *a priori* or the stiffness at certain points constrained within certain limits.

- To assume that some properties of the structure are known to a greater degree of confidence than are others. For example, one may seek N model parameters meeting the experimental constraints while minimizing the deviation from the expected mean of parameters in which one has more confidence.

It appears to us that one must not deal with arbitrary mass and stiffness matrices (they have too many free parameters), but rather deal with mass and stiffness matrices parameterized by a few geometrical and physical variables. For example, in the course of system identification, a cylinder in a structure should not be replaced by an element with arbitrary spacial distributions of mass and stiffness, but with another cylinder with modified length, diameter and wall thickness (only three parameters). For convenience, we refer to these types of parameters as "source parameters."

Clearly, these approaches involve considerable heuristic as well as theoretical definitions. It is required that the number of constraints introduced by these methods, when coupled with the experimental constraints, will determine the N-model source parameters. However, it is probable that they will overdetermine the source parameters. The source parameters will then have to be found in some best sense by methods similar to those discussed below.

The full identification task seeks the source parameters. Nevertheless, it is unwise to skip the eigenparameters. In our experience, they serve as a buffer between the measured response data and the source parameters. They allow the identification task to be separated into parts. First, identifying the eigenparameters from the raw test data, and then identifying the source parameters from the eigenparameters. Finally, as mentioned above, a knowledge of the eigenparameters alone gives much information about the dynamics of the structure.

Whether seeking eigenparameters or source parameters, a parameter identification scheme is required.[4] The first step is to determine a model or a mathematical concept which establishes a relationship between known information and desired information. For example, if the known information is the measured response of a structure $X(t)$, and the desired information is the value of certain eigenparameters, p, then we must predict the theoretical response as a function of these parameters $Z(t;p)$.

The second step is the selection of a criterion function by means of which the "goodness-of-fit" of the theoretical response to the actual system response can be evaluated. The criterion function is constructed such that a good fit is obtained when the function is minimized as a function of the unknown model parameters. For example, a criterion function may be chosen as

$$F(p) = \int \left\{X(t) - Z(t;p)\right\}^T B \left\{X(t) - Z(t;p)\right\} dt$$

where B is a positive definite weighting matrix. Thus F is only a function of the parameter values, and zero only if data and theory agree exactly. Heuristically if the (p) are varied to make $F(p)$ as small as possible, we feel that the model defined by these values of (p) is a good one.

In addition to minimizing the criterion function, the parameters may be subject to physical requirements (such as masses being positive!) and to mathematical requirements (such as the orthogonality of the eigenvectors of linear systems). For most applications the function is too complex to minimize analytically. Consequently, iterative procedures are used. Clearly, this same approach is applicable if the eigenparameters are the known information and the source parameters are the desired information.

So far, we have discussed only linear models. Identification of nonlinearities is a more difficult task than with linear models for several reasons. In general, closed form theoretical solutions to nonlinear equations do not exist. Some structural nonlinearities, especially those giving rise to damping, are often due to nonstructural components. This makes the creation of models of these phenomena -- let alone parameter identification -- difficult. In addition, most current testing and measurement techniques do not provide sufficient information to uniquely identify the kind of nonlinearities present. Lastly, models identified from low-level tests cannot be readily extrapolated to the levels of response anticipated during earthquakes.

Nevertheless, many of the comments made for linear systems apply to nonlinear systems as well. Reference (1) attempts to exploit some of these similarities.

Finally, we have come to realize that vibration testing is very much an art. Proper experimental technique acquired by experience and fortified by heuristic reasoning is indispensable. We do anticipate, however, that increased use of advanced computational methods and instrumentation will reduce costs of testing, and increase the reliability of the results.

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