DYNAMIC INSTABILITY AND ULTIMATE CAPACITY OF PARAMETRICALLY EXCITED STRUCTURES

by

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SYNOPSIS

The effect of parametric instability on dynamic response of frameworks has been studied by using the displacement method and the fourth order Runge-Kutta technique. The longitudinal and lateral excitations may be caused by ground motions or applied forces. A digital computer program has been developed for finding natural frequencies, static buckling loads, dynamic instability regions and responses of elastic and inelastic structures. It is found that the structure becomes dynamically unstable when a certain frequency of vertical motion is present and that the growth of the vibrating amplitude may possibly cause lateral or lateral-torsional collapse of a structure.

METHOD OF ANALYSIS

The presentation involves the dynamic parametric instability analysis and dynamic responses of different types of structures based on the following considerations: a) the displacement method for consistent mass and lumped mass formulation (1) with bending deformation only; b) the reduced iteration method used for eigenvalues (1) and the fourth order Runge-Kutta method for numerical integration; c) the dynamic instability region based on axial pulsating load having periodic function cos θt; d) the structural meterial (2) assumed to be composed of elastic component (stiffness $s_1=ps$) and elasto-plastic component (stiffness $s_2=qs$, s=total stiffness=s_1+s_2); e) the inclusion of axial-flexural interaction; f) the iteration process employed for checking the member end moments no greater than the reduced plastic moments; g) the reduced plastic moment determined according to the AISC steel design manual for strong axis bending of short column; h) the correction of axial loads in columns due to the shears transmitted from girders; and i) the linear and nonlinear moment curvature relationship determined from the direction of the plastic hinge rotation of the elasto-plastic component.

The differential equations of motion of a vibrating system subjected to vertical and horizontal forces can be written as

$$[M]\{\ddot{q}\} + ([ASA^T] - \alpha P^*[S_s] - \beta P^*[S_t])\{q\} = \{F(t)\} \ldots \quad (1)$$

For vertical and horizontal earthquake motions, the motion equations become

$$[M]\{\ddot{u}\} + ([ASA^T] - \gamma[M][S_s] - \tau[M][S_g])\{u\} = -[M]\{1\}\ddot{X}_g \ldots \quad (2)$$

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The notations in Eqs. 1 and 2 are similar to Cheng's previous work (1) and may be defined as: [A]=equilibrium matrix; [S]=static stiffness matrix; [M]=consistent or lumped mass matrix; α and β=percentage of the static buckling load P*; $[S_s]$=geometric matrix due to static axial loads, $[S_t]$= geometric matrix due to dynamic axial load having periodic function cos θt; $[S_g]$=geometric matrix due to vertical earthquake accelerations; γ and τ=percentage of gravity load; $\{\ddot{q}\}$ and {q}=acceleration and displacement of generalized coordinates, respectively; $\{\ddot{u}\}$ and {u}=acceleration and displacement of relative coordinates, respectively; {F(t)}=transverse dynamic excitations; $\ddot{X}_g$=horizontal earthquake accelerations.

Applying the fourth order Runge -Kutta method, one may find the displacements from Eq. 1 or 2 and then the internal forces of all constituent members may be obtained. Let {F(t)}=0, then Eq. 1 yields the dynamic instability region of first order described by α, β, θ and ω. The natural frequency ω and the static buckling load P* are evaluated from $[M]\{\ddot{q}\} + [ASA^T]\{q\} = 0$ and $([ASA^T] - \alpha P^*[S_s])\{q\} = 0$, respectively.

EXAMPLES AND CONCLUSIONS

The steel beam of Fig. 1a having I_1=192 in^4, I_2=96 in^4, A_1=30.24 in^2, and A_2=24 in^2, is subjected to $P(t)=\alpha P^*+\beta P^*\cos\theta t$ and F(t) varying from t=0 to 0.24 sec. as shown in Fig. 1b. It has been found that the buckling load P*=2975k and the fundamental frequency ω=28.95 cps. Consequently, the instability region and the displacement response are obtained from Eq. 1 and shown in Figs. 1c and 3, respectively. The rigid frame of Fig. 2a has been also analyzed and found that P*=2003k, ω=1.6cps. The instability region for $P(t)=1/2\alpha P^*+1/2\beta P^*\cos\theta t$ is given in Fig. 2b and displacement response at Y_1 due to the ground acceleration $\ddot{X}_g=-8\pi^2\sin 4\pi t$ in/sec^2 is sketched in Fig. 4. Other numerical calculations have been performed on a number of selected frames subjected to various vertical and horizontal excitations such as harmonic forces, impulsive loads, initial disturbances, and earthquakes (1940 El Centro). The general conclusions are that a) the effect of longitudinal excitation on the dynamic response depends on the value of θ/ω. When θ/ω is closer to the instability region, the effect becomes more significant; b) although no definite value of θ/ω may be obtained from earthquake motions, earthquakes may excite certain structures dynamically unstable; c) the comparison between consistent mass method and lumped mass method indicates each of these two only suitable to certain types of structures; and d) the displacement response of inelastic structures is less than that of elastic systems primarily because of energy absorption and constant change of dynamic characteristics.

ACKNOWLEDGEMENT

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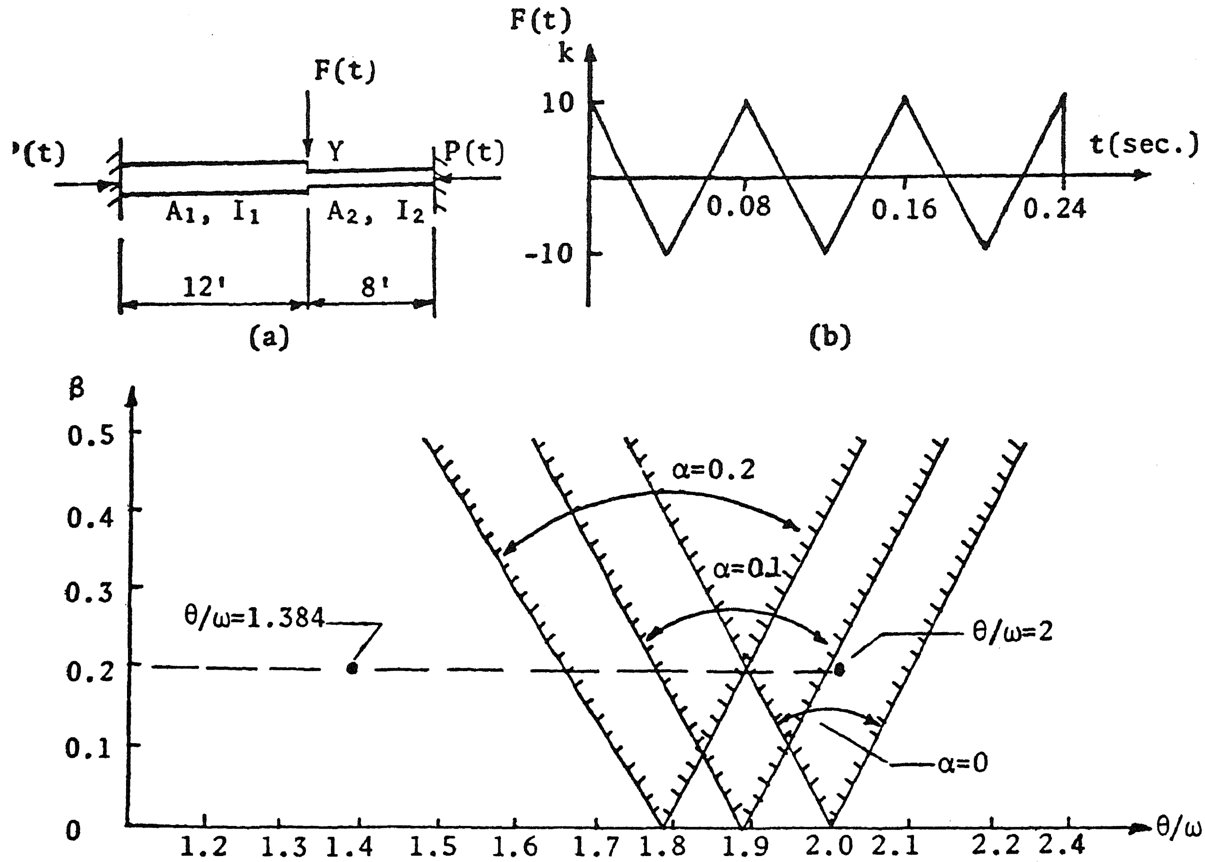

FIG. 1 STEPPED BEAM WITH BOTH ENDS FIXED

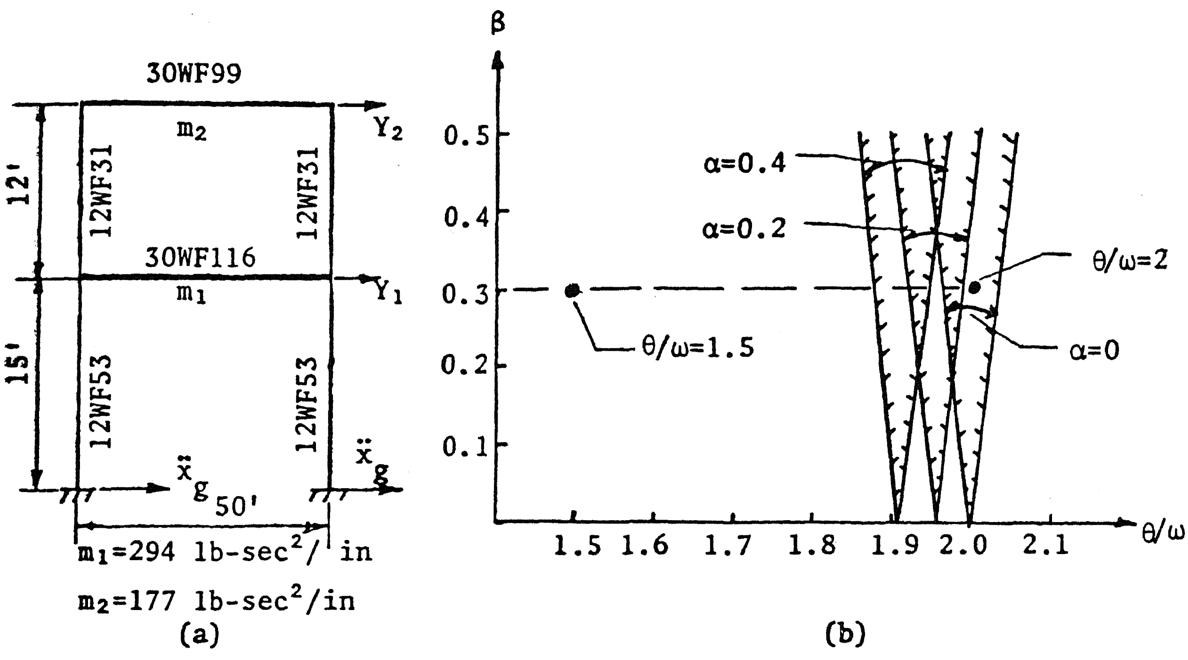

FIG. 2 RIGID FRAME

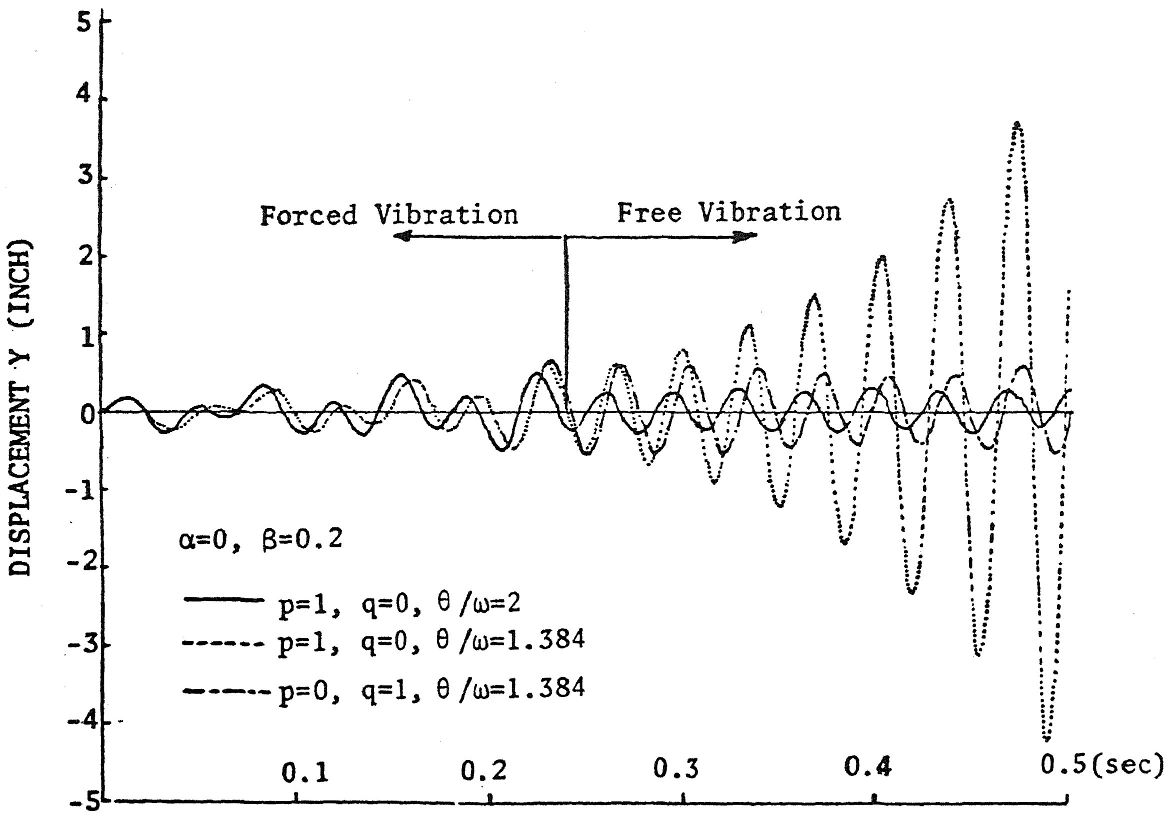

FIG. 3 DISPLACEMENT RESPONSE OF FIG. 1

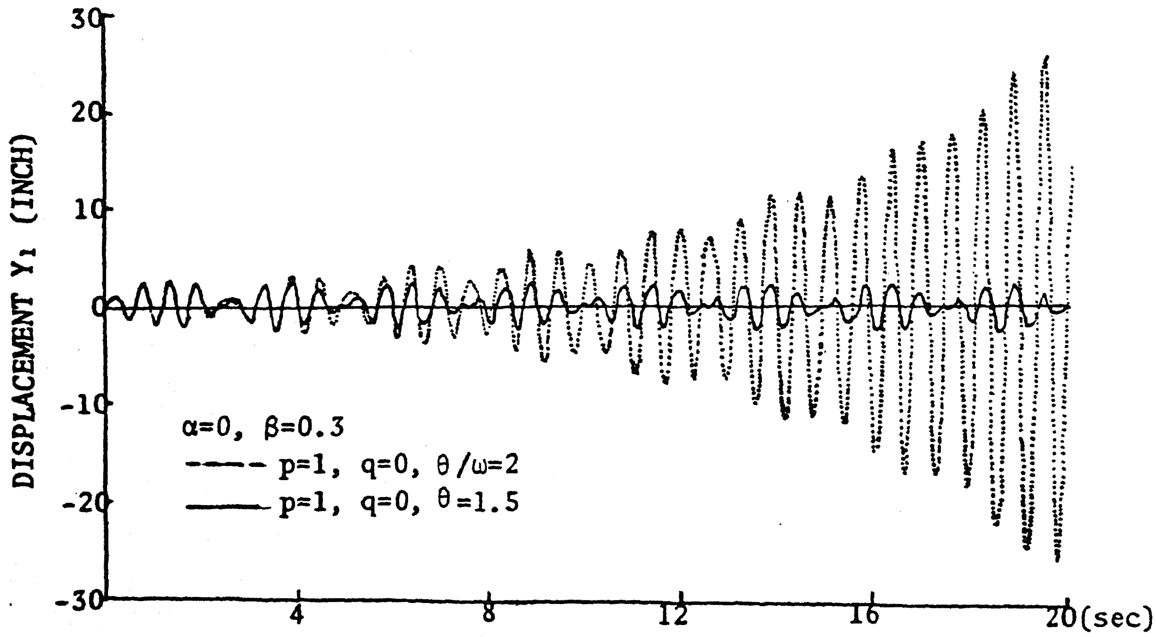

FIG. 4 DISPLACEMENT RESPONSE OF FIG. 2