A FINITE ELEMENT FOR THE INTERACTION OF FRAMES AND SHEAR WALLS

by

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SYNOPSIS

A new type of structural element has been developed to handle the behavior of shear walls interacting with rigid frames under the action of lateral loads. This element preserves the elastic characteristics of shear walls in a scheme based on the finite element method, referring them to a set of nodal displacements chosen to enable the practical use of this element in standard routines of structural analysis developed for the solution of rigid frames. All the characteristics of this shear wall element are contained in its stiffness matrix, derived in this presentation. Also, a flow chart incorporating this element into standard methods of frame analysis is included.

INTRODUCTION

In the analysis of multistory buildings subjected to lateral forces caused by wind gusts or earthquake motions, designers are faced with the need of reliable and practical procedures for the analysis of structures formed by the combined action of rigid frames and shear walls.

In fact, various methods have been developed for the approximate analysis of this type of structures.

In this paper, a new, automatic procedure is presented (1) based on the concept of finite elements, that serve to evaluate the properties of the shear wall and to assemble it into the rigid frame using standard procedures of stiffness methods of analysis of structures formed by slender elements.

For this purpose, it has to be recognized that in the stiffness analysis of a rigid frame subjected to lateral loads, the typical displacement degrees of freedom are the vertical displacement and rotation of each joint, plus one lateral displacement at each floor level. These considerations are taken into account

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in the development of a new structural element used to represent the shear wall in a scheme that allows the application of standard techniques for the automatic analysis of framed structures.

DESCRIPTION OF THE ELEMENT

The basic considerations in the development of a shear wall element are:

a) Represent the structural interaction of framed structures and shear walls under lateral loads,

b) maintain the same degrees of freedom used in frame analysis, to ease the structural assemblage and the use of existing methods of analysis, and

c) simulate plane stress behavior.

Therefore, a rectangular structural element of the characteristics shown in figure 1 was developed following the basic principles of the finite element method of structural mechanics (2). This element was designed so that it will represent the complete shear wall in each story. It was constructed using beam-type functions as displacement patterns associated with the ten degrees of freedom of adjacent beams and lateral story distortions, as illustrated in figure 1.

The selection of these degrees of freedom to define the structural behavior of the element implies some general hypotheses that are consistent with usual assumptions considered in the analysis of framed structures, namely: a) there are no horizontal strains at floor levels (floor systems are assumed to be infinitely rigid), and b) there are no shear strains in the corners of the element (joints are assumed to be rigid).

Using a standard finite element approach, the element stiffness matrix is developed for a plane stress state, and included in figure 2. This stiffness matrix for the shear wall element has been developed in a completely general manner so that, when performing a standard structural analysis, this new element may be handled in the same way as beams and columns that are, similarly, represented by their own stiffness matrices.

TEST APPLICATIONS

In order to test the results obtained with these element, several applications were carried out analyzing the response of a single shear wall, eight stories high, subjected to lateral forces, as shown in figure 3. These results are compared with those obtained for a cantilever beam analyzed by the simple theory of elasticity. The comparisons are made for various values of the panel ratio θ, that for typical cases takes on values between 0.4 and 0.8. It was observed that the comparison of aproximate results versus "exact" values was almost independent of Poisson's ratio. However, due to the fact that lateral translation is forced to take place without horizontal strains, for comparison purposes the "exact" results must be amplified by a factor of $(1 - \nu^2)$ that takes into account this fact. For all practical purposes, it is concluded that these results are satisfactorily acceptable; thus, the structural element developed for the analysis of shear walls can be reliably accepted for the analysis of the interaction between shear walls and rigid frames under the action of lateral loads.

STRUCTURAL ASSEMBLAGE

In order to illustrate the practical way to handle the shear wall element just described, an application to the evaluation of the lateral stiffness of a combined structure, as shown in figure 4, is described, following a modified formulation of the tridiagonal matrix method (3), i.e., a particular application of the stiffness approach of structural analysis.

In this formulation there are essentially three different types of elements, namely, columns, beams and shear walls.

Columns are elements with six degrees of freedom, beams have four degrees of freedom and shear walls have ten. The element stiffness matrices are assembled together, story by story, in a scheme that allows the sequential condensation of vertical displacements and rotations at every joint, thus retaining only lateral displacements as degrees of freedom.

The contribution of each story to the total stiffness matrix is obtained by a direct stiffness procedure, arrainging the degrees of freedom in the following sequence:

a) Vertical displacements and rotations of each lower joint u^b,

b) vertical displacements and rotations of each joint in the top chord u^t, and

c) lateral displacements, ordered from bottom to top.

The story stiffness matrix takes the form

$$\begin{bmatrix} K^b & C & E^b \\ C^T & K^t & E^t \\ {E^b}^T & {E^t}^T & K^r \end{bmatrix}$$

where:

K^b = stiffness associated with the degrees of freedom of the bottom chord. Depends on column properties in diagonal terms and on shear wall properties in banded terms within a band width of 4.

C = coupling stiffness between bottom and top degrees of freedom. Has a similar internal structures as K^b.

K^t = stiffness associated with the degrees of freedom of the top chord. Depends on column properties in diagonal terms and on beam properties as well as shear wall properties in banded terms within a band width of 4.

E^b and E^t = coupling stiffness with joint degrees of freedom and lateral displacements. Depends on column and shear wall properties.

K^r = stiffness associated with lateral displacements.

The total stiffness matrix can be obtained as an assemblage of story stiffness matrices in a direct stiffness procedure. This process is performed in such a way that the joint degrees of freedom are condensend by a typical Gauss elimination during the assemblage; thus, ending up with a stiffness matrix associated only to lateral displacements. Therefore this becomes the lateral stiffness matrix of the structure.

The purpose of describing this typical process for the evaluation of the lateral stiffness of framed structures is to indicate the ease with which shear wall elements can be incorporated in the procedure,afecting only the generation of some elements in the stiffness matrices. Figure 5 shows a flow chart designed for the evaluation of the lateral stiffness of framed structures interacting with shear walls. For this analysis, columns and beams are represented by their standard stiffness matrices (3) and shear wall elements are represented by their own stiffness matrix as shown in figure 2.

REFERENCES

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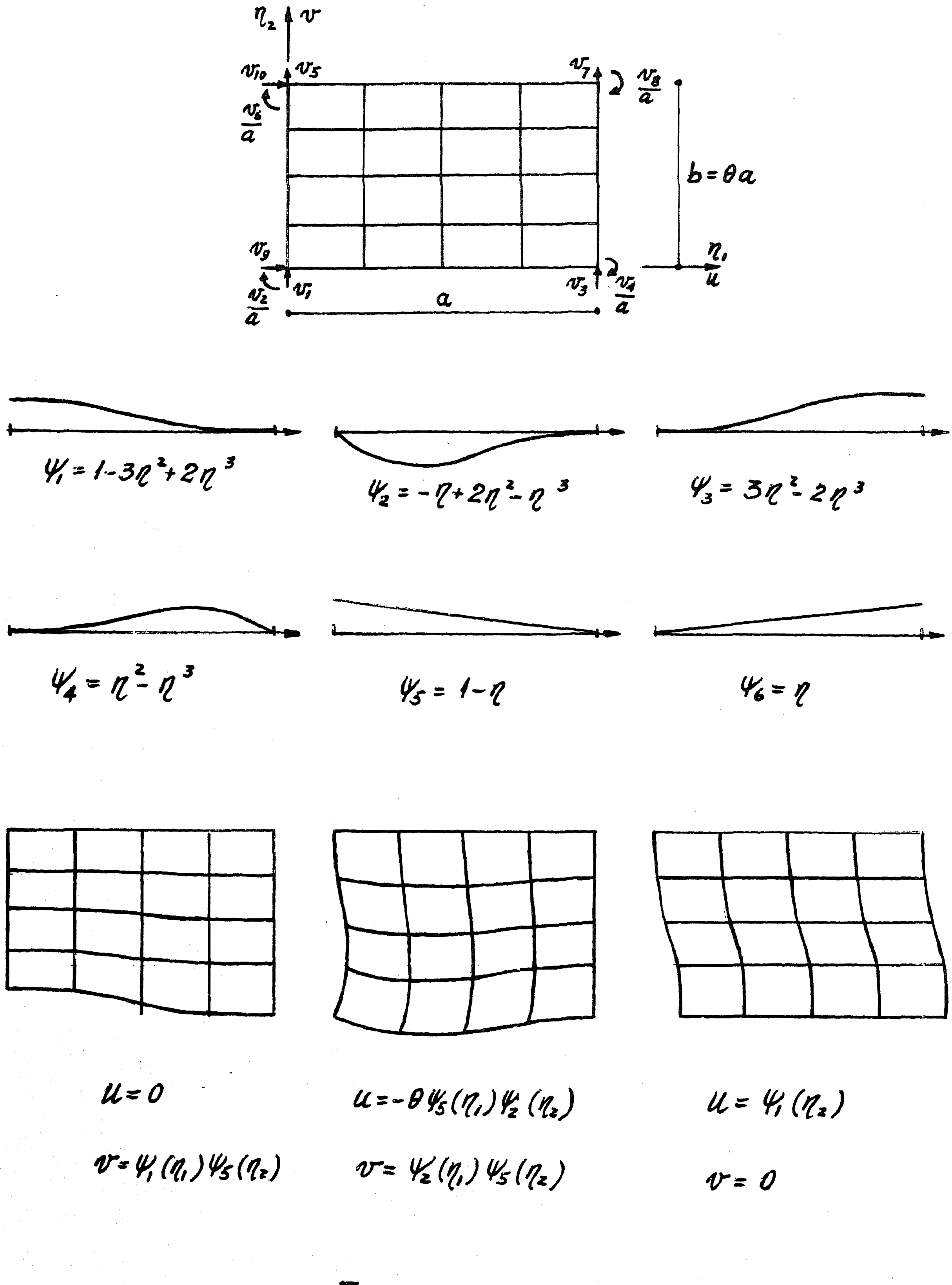

η2 v
v10 v5
v7 v8/a
v6/a
b = θa
v9
η1
u
v2/a v1
v3 v4/a
a
ψ1 = 1-3η² + 2η³
ψ2 = -η + 2η² - η³
ψ3 = 3η² - 2η³
ψ4 = η² - η³
ψ5 = 1 - η
ψ6 = η
u = 0
v = ψ1(η1) ψ5(η2)
u = -θ ψ5(η1) ψ2(η2)
v = ψ2(η1) ψ5(η2)
u = ψ1(η2)
v = 0

Figure 1

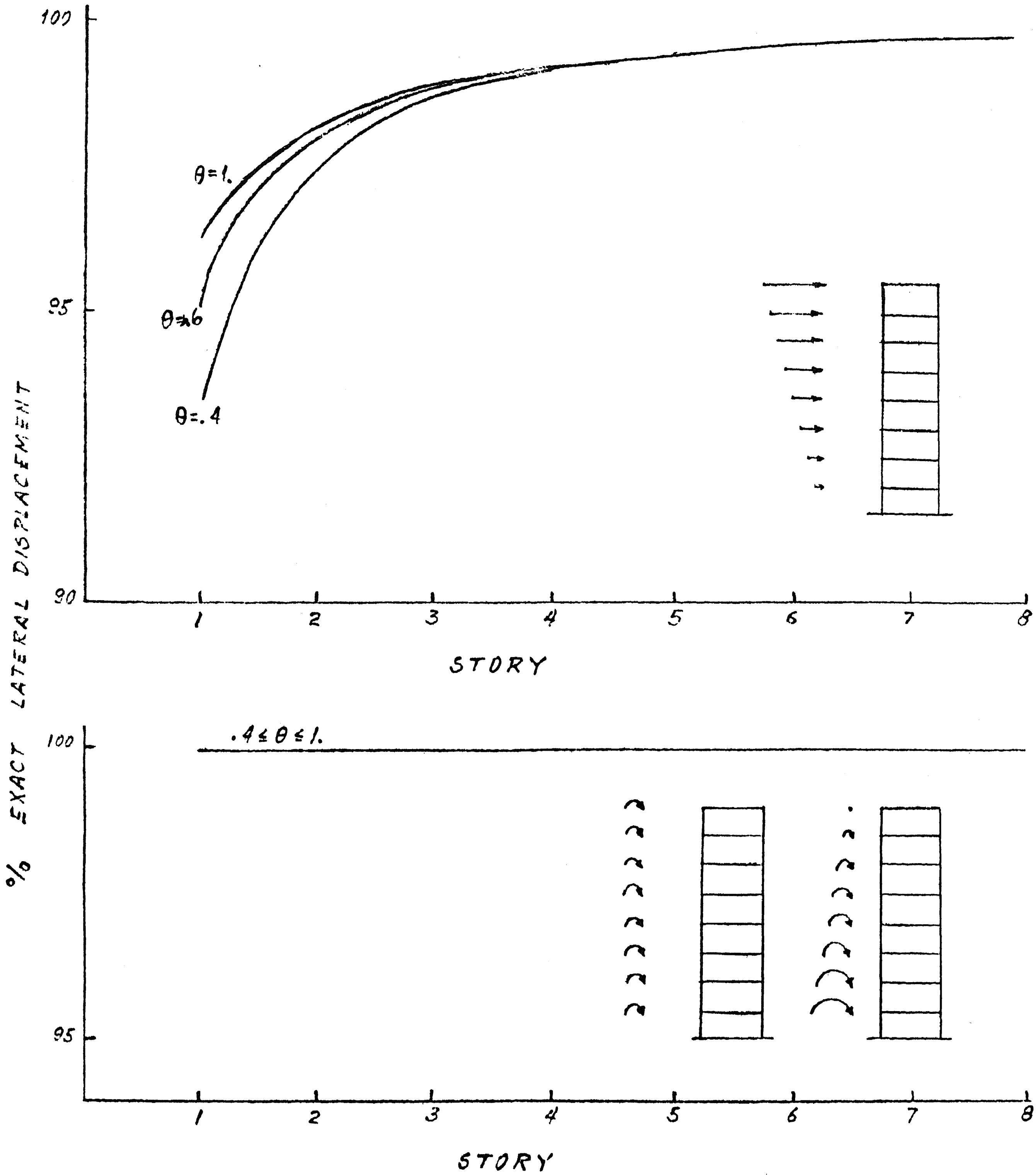
$$[k] = \frac{Et\theta}{1-\nu^2}\left[\begin{array}{cccccc}
\frac{1}{5}-\frac{1}{5}\nu+\frac{13}{35}\frac{1}{\theta^2} & & & & & \\
-\frac{3}{80}+\frac{19}{240}\nu-\frac{11}{210}\frac{1}{\theta^2} & \frac{3}{80}-\frac{37}{720}\nu+\frac{1}{105}\theta^2+\frac{1}{105}\frac{1}{\theta^2} & & & \text{Symmetric} & \\
-\frac{1}{5}+\frac{1}{5}\nu+\frac{9}{70}\frac{1}{\theta^2} & \frac{3}{80}-\frac{19}{240}\nu-\frac{13}{420}\frac{1}{\theta^2} & \frac{1}{5}-\frac{1}{5}\nu+\frac{13}{35}\frac{1}{\theta^2} & & & \\
-\frac{3}{80}+\frac{19}{240}\nu+\frac{13}{420}\frac{1}{\theta^2} & \frac{1}{80}+\frac{1}{720}\nu-\frac{1}{105}\theta^2-\frac{1}{140}\frac{1}{\theta^2} & \frac{3}{80}-\frac{19}{240}\nu+\frac{11}{210}\frac{1}{\theta^2} & \frac{3}{80}-\frac{37}{720}\nu+\frac{1}{105}\theta^2+\frac{1}{105}\frac{1}{\theta^2} & & \\
\frac{1}{10}-\frac{1}{10}\nu-\frac{13}{35}\frac{1}{\theta^2} & \frac{1}{80}+\frac{7}{240}\nu+\frac{11}{210}\frac{1}{\theta^2} & -\frac{1}{10}+\frac{1}{10}\nu-\frac{9}{70}\frac{1}{\theta^2} & \frac{1}{80}+\frac{7}{240}\nu-\frac{13}{420}\frac{1}{\theta^2} & \frac{1}{5}-\frac{1}{5}\nu+\frac{13}{35}\frac{1}{\theta^2} & \\
\frac{1}{80}+\frac{7}{240}\nu+\frac{11}{210}\frac{1}{\theta^2} & \frac{1}{80}+\frac{1}{720}\nu-\frac{1}{140}\theta^2-\frac{1}{105}\frac{1}{\theta^2} & -\frac{1}{80}-\frac{7}{240}\nu+\frac{13}{420}\frac{1}{\theta^2} & -\frac{1}{80}-\frac{1}{720}\nu+\frac{1}{140}\theta^2+\frac{1}{140}\frac{1}{\theta^2} & -\frac{3}{80}+\frac{19}{240}\nu-\frac{11}{210}\frac{1}{\theta^2} & \cdots \\
-\frac{1}{10}+\frac{1}{10}\nu-\frac{9}{70}\frac{1}{\theta^2} & -\frac{1}{80}-\frac{7}{240}\nu+\frac{13}{420}\frac{1}{\theta^2} & \frac{1}{10}-\frac{1}{10}\nu-\frac{13}{35}\frac{1}{\theta^2} & -\frac{1}{80}-\frac{7}{240}\nu-\frac{11}{210}\frac{1}{\theta^2} & -\frac{1}{5}+\frac{1}{5}\nu+\frac{9}{70}\frac{1}{\theta^2} & \cdots \\
\frac{1}{80}+\frac{7}{240}\nu-\frac{13}{420}\frac{1}{\theta^2} & -\frac{1}{80}-\frac{1}{720}\nu+\frac{1}{140}\theta^2+\frac{1}{140}\frac{1}{\theta^2} & -\frac{1}{80}-\frac{7}{240}\nu-\frac{11}{210}\frac{1}{\theta^2} & \frac{1}{80}+\frac{1}{720}\nu-\frac{1}{140}\theta^2-\frac{1}{105}\frac{1}{\theta^2} & -\frac{3}{80}+\frac{19}{240}\nu+\frac{13}{420}\frac{1}{\theta^2} & \cdots \\
\frac{1}{4}\frac{1-\nu}{\theta} & \frac{1}{40}\frac{1-\nu}{\theta} & -\frac{1}{4}\frac{1-\nu}{\theta} & \frac{1}{40}\frac{1-\nu}{\theta} & \frac{1}{4}\frac{1-\nu}{\theta} & \cdots \\
-\frac{1}{4}\frac{1-\nu}{\theta} & -\frac{1}{40}\frac{1-\nu}{\theta} & \frac{1}{4}\frac{1-\nu}{\theta} & -\frac{1}{40}\frac{1-\nu}{\theta} & -\frac{1}{4}\frac{1-\nu}{\theta} & \cdots
\end{array}\right.$$

Figure 2

Symmetric

$$
\begin{bmatrix}
\cdots & \frac{3}{80}-\frac{37}{720}\nu+\frac{1}{105}\theta^2+\frac{1}{105}\frac{1}{\theta^2} & & & & \\
\cdots & \frac{3}{80}-\frac{19}{240}\nu-\frac{13}{420}\frac{1}{\theta^2} & \frac{1}{5}-\frac{1}{5}\nu+\frac{13}{35}\frac{1}{\theta^2} & & & \\
\cdots & \frac{1}{80}+\frac{1}{720}\nu-\frac{1}{105}\theta^2-\frac{1}{140}\frac{1}{\theta^2} & \frac{3}{80}-\frac{19}{240}\nu+\frac{11}{210}\frac{1}{\theta^2} & \frac{3}{80}-\frac{37}{720}\nu+\frac{1}{105}\theta^2+\frac{1}{105}\frac{1}{\theta^2} & & \\
\cdots & \frac{1}{40}\frac{1-\nu}{\theta} & -\frac{1}{4}\frac{1-\nu}{\theta} & \frac{1}{40}\frac{1-\nu}{\theta} & \frac{3}{5}\frac{1-\nu}{\theta^2} & \\
\cdots & -\frac{1}{40}\frac{1-\nu}{\theta} & \frac{1}{4}\frac{1-\nu}{\theta} & -\frac{1}{40}\frac{1-\nu}{\theta} & -\frac{3}{5}\frac{1-\nu}{\theta^2} & \frac{3}{5}\frac{1-\nu}{\theta^2}
\end{bmatrix}
$$

Figure 2 (cont.)



% EXACT LATERAL DISPLACEMENT
100
95
90
θ=1.
θ=.6
θ=.4
1
2
3
4
5
6
7
8
STORY
100
95
.4≤θ≤1.
1
2
3
4
5
6
7
8
STORY

Figure 3

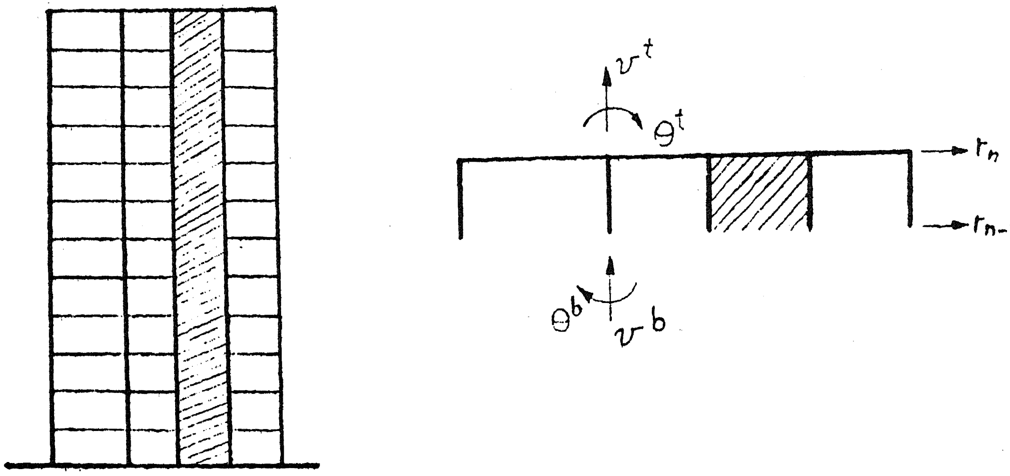


Figure 4

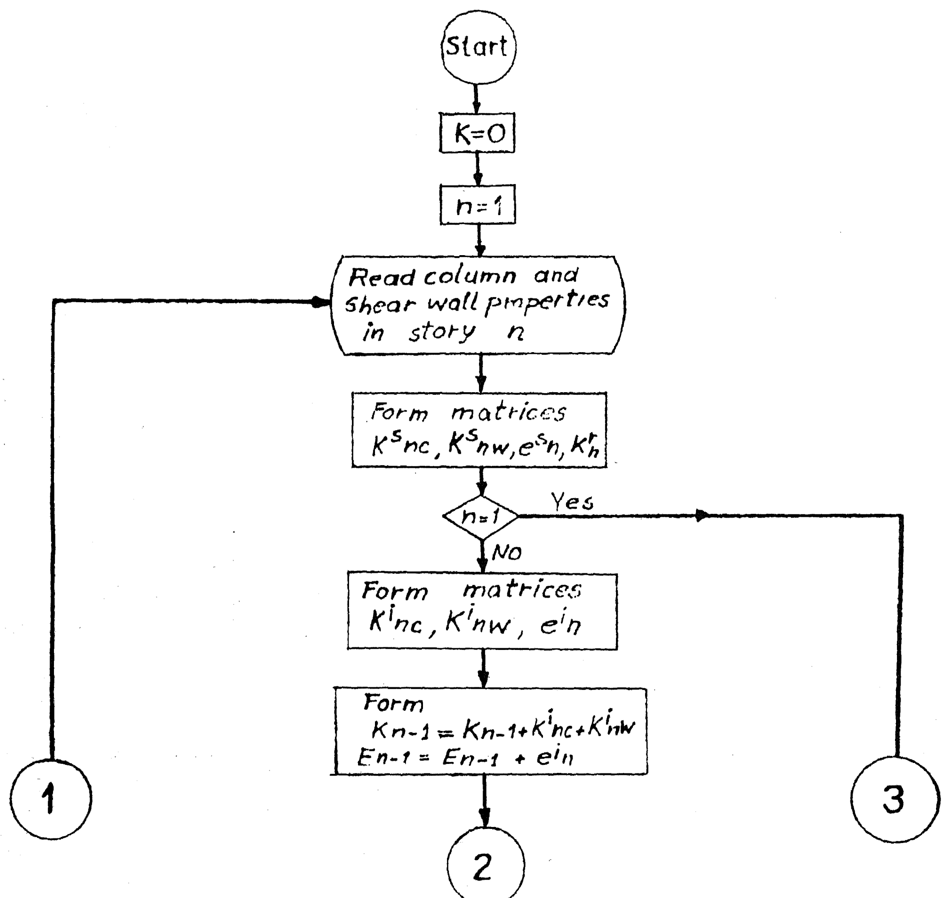


Figure 5

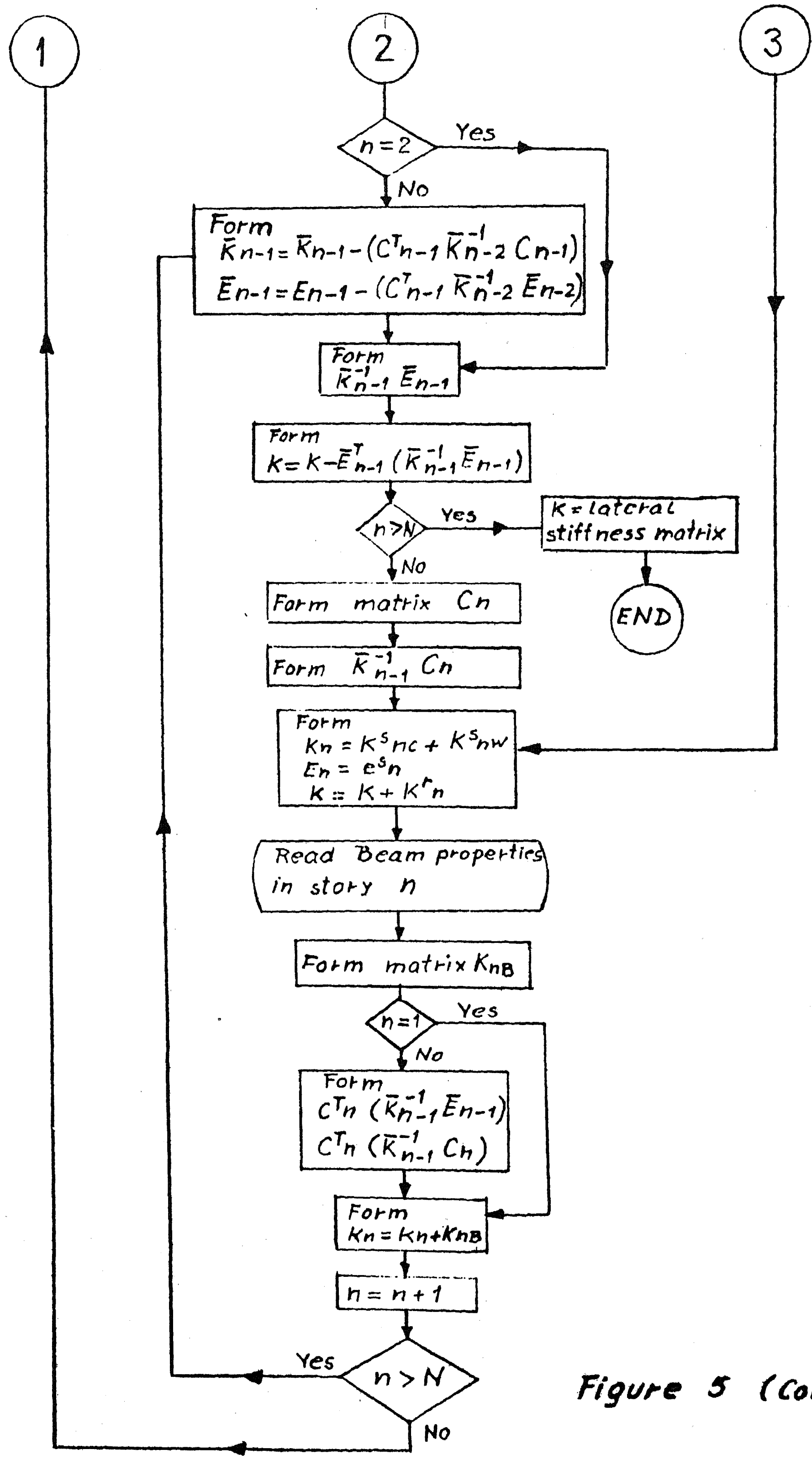


Figure 5 (Cont.)