ON THE RESPONSE ANALYSIS OF THE STRUCTURE SYSTEM SUBJECTED TO MULTI-SEISMIC MOTIONS

by
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SYNOPSIS

This paper is concerning with the response analysis of the mechanical equipment system subjected to two seismic excitations, which is basic study for the aseismic design of the general mechanical equipment structure having several input ends. At first the response properties of the equipment system appended to building which is simulated by lumped mass system are investigated. Then the basic study of response behaviour of a simple system which is subjected to two seismic excitations with some amount of time delay interval with each other is conducted. The analysis by means of random process simulating earthquake motion is also made for this system. Finally the response of two inputs system with nonlinear elasto-plastic deformation characteristic is dealt with. The response as for relative displacement is given by taking the ductility factor as parameter. This analysis is also carried out for the asymmetrical system in which the stiffness at individual input end is made different.

RESPONSE ANALYSIS FOR B-M MODEL [1),2),3)]

As the simplest model of structure subjected to two seismic excitations, a single-degree-of-freedom system shown in Fig. 1 (a) is considered. Fixed ends 1 and 2 are attached to the building structure or directly to the ground. The mechanical equipment is often installed on the building structure. Then as a typical model for the analysis the integrated building-machine structure model, which would be called as B-M model below, is proposed in Fig. 2. Fixed ends 1 and 2 of two-degrees-of-freedom equipment model are connected with the lower and upper stories of the building structure.

Basic equations of motion for the simplest two inputs system shown in Fig. 1 (a) can be given in terms of relative motion between the mass and the respective input end. k_1 and k_2 are the spring constant, and c_1 and c_2 are the viscous damping coefficient in the figure.

$$\begin{aligned}(\ddot{X}-\ddot{Y}_1)+2h_m\omega_m(\dot{X}-\dot{Y}_1)+\omega_m^2(X-Y_1)&=-\ddot{Y}_1+h_m\omega_m(\dot{Y}_2-\dot{Y}_1)+\frac{\omega_m^2}{2}(Y_2-Y_1)\\(\ddot{X}-\ddot{Y}_2)+2h_m\omega_m(\dot{X}-\dot{Y}_2)+\omega_m^2(X-Y_2)&=-\ddot{Y}_2+h_m\omega_m(\dot{Y}_1-\dot{Y}_2)+\frac{\omega_m^2}{2}(Y_1-Y_2)\end{aligned} \tag{1}$$

where X is the absolute displacement of the mass, Y_1 and Y_2 are those of the input ends. For simplicity the spring constant and the damping coefficient are made equal in Eq. (1). h_m and ω_m are given as $h_m = c/(2\sqrt{mk})$ and $\omega_m = \sqrt{k/m}$.

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Equations of motion for the integrated B-M model shown in Fig. 2 can be developed from these. Putting parameters as follows $m_1 = m_2 = m$, $c_1 = c_2 = c_3 = c$, $k_1 = k_2 = k_3 = k$ and $M_1 = M_2 = M$ so as to simplify the system, equations for the building structure system to the ground motion are given as

$$
\begin{aligned}
&(\ddot{Y}_1-\ddot{Y})+\left(\frac{C_1}{M}+\frac{C_2}{M}\right)(\dot{Y}_1-\dot{Y})+\left(\frac{K_1}{M}+\frac{K_2}{M}\right)(Y_1-Y)-\frac{C_2}{M}(\dot{Y}_2-\dot{Y})-\frac{K_2}{M}(Y_2-Y)\\
&\quad=-\ddot{Y}-\gamma\frac{c}{m}(\dot{X}_1-\dot{Y}_1)-\gamma\frac{k}{m}(X_1-Y_1)\\
&(\ddot{Y}_2-\ddot{Y})+\frac{C_2}{M}(\dot{Y}_2-\dot{Y})+\frac{K_2}{M}(Y_2-Y)-\frac{C_2}{M}(\dot{Y}_1-\dot{Y})-\frac{K_2}{M}(Y_1-Y)\\
&\quad=-\ddot{Y}+\gamma\frac{c}{m}(\dot{X}_2-\dot{Y}_2)+\gamma\frac{k}{m}(X_2-Y_2)
\end{aligned}
\tag{2}
$$

and those of the equipment model to each boundary motion as for the lower mass are described by referring Eq. (1) as

$$
\begin{aligned}
&(\ddot{X}_1-\ddot{Y}_1)+\frac{2c}{m}(\dot{X}_1-\dot{Y}_1)+\frac{2k}{m}(X_1-Y_1)-\frac{c}{m}(\dot{X}_2-\dot{Y}_1)-\frac{k}{m}(X_2-Y_1)=-\ddot{Y}_1\\
&(\ddot{X}_1-\ddot{Y}_2)+\frac{2c}{m}(\dot{X}_1-\dot{Y}_2)+\frac{2k}{m}(X_1-Y_2)-\frac{c}{m}(\dot{X}_2-\dot{Y}_2)-\frac{k}{m}(X_2-Y_2)\\
&\quad=-\ddot{Y}_2+\frac{c}{m}(\dot{Y}_1-\dot{Y}_2)+\frac{k}{m}(Y_1-Y_2)
\end{aligned}
\tag{3}
$$

Equations as for the upper mass are obtained similarly. γ means the mass ratio of the equipment to that of the building, that is, $\gamma = m/M$. As earthquake input motions El Centro (May, 1940) and some others are used.

Parameters used for the response analysis are given as follows. Damping ratio for the building structure $h_b = 0.05$ and that for the mechanical equipment system h_m is set as 0.007, 0.02, 0.1 and 0.2 taking the instrumented value for piping system installed in the steam power plant into consideration. In this analysis the mass ratio γ plays important role because it relates to the terms of the interactive effect between the equipment and the building structure. γ = 0, 0.01 and 0.1 are taken for the analysis. The first natural period of the equipment T_m is selected about 0.1 ~ 2.0 s in this analysis.

These descriptions of the equations of motion have the following advantages. (1) It is possible to obtain the vibration characteristic of the building and the equipment system respectively and to integrate the whole B-M system taking reciprocal interactive effect into acount. The effect can be gained by evaluating γ and the relative motion between the mass of the equipment and that of the building. (2) Analysis of the vibration characteristic of the system makes it possible to describe the motion of the respective system by the normal co-ordinate about the principal modes concerning with the frequency band width of the earthquake motion.

According to this approach the modal analysis method can be applicable for the response spectrum analysis[4)] of the multi-degree-of-freedom system or the continuous system. For example the equations of motion of the equipment system are written as

$$\ddot{\xi}_{mI}+\frac{c}{m}\dot{\xi}_{mI}+\frac{k}{m}\xi_{mI}=\alpha_{mI}\left\{-2\ddot{Y}_1-\frac{c}{m}(\dot{Y}_1-\dot{Y}_2)-\frac{k}{m}(Y_1-Y_2)\right\}$$
$$\ddot{\xi}_{mII}+\frac{3c}{m}\dot{\xi}_{mII}+\frac{3k}{m}\xi_{mII}=\alpha_{mII}\left\{\frac{c}{m}(\dot{Y}_1-\dot{Y}_2)+\frac{k}{m}(Y_1-Y_2)\right\} \quad (4)$$

where ξ_{mI} and ξ_{mII} are displacement component in the normal co-ordinate for the first and the second natural frequency respectively and α_{mI} and α_{mII} are so called participating factors for the respective mode.

A response spectrum of the equipment varies its natural period T_m for a natural period of the building T_b. The similar spectrum is obtained for various T_b. Figure 3 shows an example of the acceleration response spectrum for El Centro earthquake. The same tendency as that of the spectrum for the single input system is found. As for the acceleration response spectrum the extreme taking place at every $T_m = T_{bI}$ reaches the maximum for rather short period. The displacement response spectrum has this maximum in long period range. Table 1 shows the ratio of the acceleration amplification factor of the equipment at the resonant state for the second natural period to that for the first one. It tells us that the influence of the resonance to the amplification factor at higher order natural period may be slight.

Since the system is analyzed as the integrated one in the aforementioned study, the interactive terms between the building and the equipment are all considered. The floor response method which utilizes the acceleration at the floors as the direct inputs to the equipment is frequently adopted in the practical design procedure. This means that the response properties may be computed by taking $\ddot{Y}_1$ and $\ddot{Y}_2$ in Eq. (3) or Eq. (4) as the input to the equipment. This approach may make the response analysis of the equipment subjected to two seismic motions from the building easier. However it should be kept in mind that the interactive effect between the equipment and the building, that is, the terms associated with $(\dot{Y}_1 - \dot{Y}_2)$ and $(Y_1 - Y_2)$ are neglected.

The applicability of this method is investigated in Tab. 2. The ratio of the acceleration amplification factor by the exact system to that by the simplified system are shown. It is concluded that this simplified approach is applicable for the equipment system with the natural period longer than 0.6 s, because the ratio is 1.10 ~ 1.15 at most. For the system having rather short natural period the effect of the term concerning $2\omega_m h_m(\dot{Y}_1 - \dot{Y}_2)$ and $\omega_m^2(Y_1 - Y_2)$ increases, so that this approach does not seem adequate.

RESPONSE OF THE STRUCTURE SUBJECTED TO TWO MOTIONS WITH TIME DELAY INTERVAL

When the dynamic behaviour of the structure due to multi-seismic exciations are investigated, some types of correlational relations are to be considered between multi-seismic motions. In this study two inputs are assumed to have same wave form. This would be the simplest simulation for the response of the long span structures such as bridge, piping, crane and so on subjected to propagating seismic motion.

Investigating the response behaviour of the structure for time delay T_l, Eq. (1) are used and relations are assumed as follows

$$Y_2(t)=Y_1(t+T_l),\ \dot{Y}_2(t)=\dot{Y}_1(t+T_l)\ \text{and}\ \ddot{Y}_2(t)=\ddot{Y}_1(t+T_l) \tag{5}$$

Schematic flow chart of this computation is shown in Fig. 4. In this procedure the velocity and the displacement motions as well as that of acceleration are required as the input. Then it is attempted to obtain the velocity and the displacement motions by means of an approximate integral operation. This can be carried out by the analog computer conveniently. The approximate integral is performed through the operational circuit of adequately damped single-degree-of-freedom system with sufficiently long natural period. The integrated velocity and displacement are obtained by cutting off the component of the natural period through high pass filter. The system parameters such as the natural period and the damping ratio of the approximate integral and the time constant of the high pass filter should be chosen so that the essential component and phase relation of the original earthquake motion may be kept.

Figure 5 is an example of the wave forms of the velocity and the displacement obtained as above mentioned. The acceleration at Akita City for Niigata earthquake(June, 1964) is used. The solid curves are the integrated wave which passed through the high pass filter. In Tab. 3, the computed maximum velocity and displacement for some earthquakes are shown. Here the maximum acceleration value is assumed to be 0.30 g for all earthquakes.

The response analysis is made due to the flow chart shown in Fig. 4 using not only the acceleration, but also the velocity and the displacement. Time delay T_l is generated by the apparatus utilizing magnetic tape. Two distinct characteristics are obtained from this analysis. (1) The acceleration amplification factor shows maximum at $T_l = 0$ and have the wavy shape as T_l increases, which is close to $|\cos(\pi T_l/T_m)| + A$, where T_m is the natural period of the structure and A is constant. (2) On the other hand the relative displacement stands minimum at $T_l = 0$ and it shows slightly wavy behaviour after it monotonously increases as T_l becomes large.

The statistical approach which adopts the stationary random vibration as an artificial earthquake to the system is attempted. The amplification factor of the response acceleration and the response displacement to the artificial earthquake in terms of the respective standard deviation is obtained. The criterion for this factor is based on Tajimi's study.[5] Equation (1) is transformed into

$$\begin{aligned}&\ddot{Z}+h_m\omega_m(1+\alpha)\dot{Z}+(\omega_m/2)^2(1+\beta)Z\\&=-\frac{1}{2}\left\{(\ddot{Y}_1+\ddot{Y}_2)-h_m\omega_m(1-\alpha)(\dot{Y}_1-\dot{Y}_2)-(\omega_m/2)^2(Y_1-Y_2)\right\}\end{aligned} \tag{6}$$

where $Z = X-(Y_1+Y_2)/2$, $\alpha = c_2/c_1$ and $\beta = k_2/k_1$ are used.

After some mathematical operations the square of the acceleration amplification factor $\lambda(T_l)$ is obtained as follows

$$\lambda^2(T_l) = \left\{ \int_0^\infty |H_t(s)|^2 d\omega \right\} / \left\{ \int_0^\infty |H_g(s)|^2 d\omega \right\} \tag{7}$$

where

$$|H_t(s)|^2 = 2|H_m(s)|^2 |H_g(s)|^2 \left[1 + \left\{ \exp(T_l s) + \exp(-T_l s) \right\} / 2 \right]$$
$$|H_m(s)|^2 = \left\{ -2(h_m\omega_m)^2 s + \omega_m^2 \right\} / \left\{ s^4 + 2\omega_m^2(1-h_m^2)s^2 + (\omega_m^4/2) \right\} \tag{8}$$

$H_g(s)$ is transfer function of the ground formulated by Tajimi based on Kanai's formula[6] describes ground predominant period. Describing the relation of the time delay, the expression $Y_2(s) = \exp(T_l s)Y_1(s)$ can be used. $Y_1(s)$ and $Y_2(s)$ are the Laplace transform of the motion of the individual end. s is Laplace operater. Equation (7) is introduced for the simplest case $\alpha = \beta = 1$, that is, the system is symmetric. The denominator and the numerator in Eq. (7) is carried out by the residue integral.[7]

Similarly the amplification factor of the relative displacement to the input acceleration $d(T_l)$ is derived from

$$d^2(T_l) = \left\{ \int_0^\infty |H_d(s)|^2 |H_g(s)|^2 d\omega \right\} / \left\{ \int_0^\infty |H_g(s)|^2 d\omega \right\} \tag{9}$$

where

$$H_d(s) = -1 + \frac{1}{2} H_m(s) \left\{ 1 + \exp(T_l s) \right\} \tag{10}$$

Figure 6 compares the theoretical acceleration response behaviour due to Eq. (7) with that computed by the method shown in Fig. 4 using El Centro. It can be said that both spectra agree fairly well. Figure 7 gives an example of the theoretical characteristic of displacement response taking T_l as abscissa. The damping ratio of the structure system is set larger than that for Fig. 6, $h_m = 0.2$. Comparison of Fig. 6 with Fig. 7 makes it obvious that at the delay time where the acceleration becomes minimum, the displacement stands maximum as T_l increases. This clearly suggests that the behaviour of the displacement response should be taken into consideration as well as that of the acceleration for the dynamical aseismic design of the structure system subjected to multi-inputs due to propagating seismic motions.

RESPONSE ANALYSIS OF NONLINEAR ELASTO-PLASTIC SYSTEM

Since the authors have been interested in the response characteristic of the structure having elasto-plastic deformation, the investigation on the behaviour of the structure system with nonlinear element exposed to two inputs as shown in Fig. 1 (b). The equations of motion for this model can be described in terms of the relative displacement as for the individual input end.

$$m\ddot{U}+(c_1+c_2)\dot{U}+f(U,V,t)=-m\ddot{Y}_1+c_2(\dot{Y}_2-\dot{Y}_1)$$
$$m\ddot{V}+(c_1+c_2)\dot{V}+f(U,V,t)=-m\ddot{Y}_2+c_1(\dot{Y}_1-\dot{Y}_2) \qquad (11)$$

where $U = X-Y_1$ and $V = X-Y_2$ are relative displacement from each input end and f(U, V, t) is time dependent nonlinear function of restoring force, which is reduced to $k_1U + k_2V$ in linear system.

When the complete elasto-plastic characteristic the yield force of which are given as F_1 and F_2 is attributed to the each element, the static property of the whole system is composed as shown in Fig. 8, in which $F_2 > F_1$ is assumed. In this hysteresis loop the region denoted by 1 stays in linear elastic system. The system behaves as elasto-plastic one in the regions of 2, 4, 5, 6, 8 and in the region 3 and 7 only the plastic deformation keeps going on.

In the elasto-plastic response special interest is directed to the relative displacement which would play an important role on the design of the structure system. As the model of the system that in Fig. 9 is considered. The equipment is subjected to two different motions, that is, the building and the ground motions. Damages in actual system are often reported for this sort of the structure. The following parameters $\kappa_1 = k_1/(k_1 + k_2)$ and $\kappa_2 = k_2/(k_1 + k_2) = 1-\kappa_1$ are introduced, which denote the asymmetricity of the stiffness of the system. The natural period of the equipment is kept constant through the change of κ_1 and κ_2. $\kappa_1 = \kappa_2 = 0.5$ means the symmetrical state. The mass of the equipment is assumed small enough comparing with that of the building.

Figure 10 (a) shows the case for $T_b = T_m = 0.5$ s, which means that the system is in the resonance. The ductility factor μ is taken as the parameter. When the system is close to linear, the characteristic becomes the maximum at $\kappa_1 = 0.5$. The value at $\kappa_1 = 0.5$ gives the sharp valley as μ increases. This suggests that asymmetricity of the two inputs system causes large displacement response especially in case that the system shows the elasto-plastic behaviour. Figure 10 (b) is the results for $T_b = 1.0$ s and $T_m = 0.5$ s. The effect of the asymmetricity is scarecely seen, for small μ. Comparing U_{max} with V_{max} , V_{max} which is responded to larger displacement input than that from the ground motion indicates larger value. The effect of the asymmetricity appears as μ gets large. The increase of the upper side stiffness, which is exposed to large displacement input, makes the response displacement increase. This tendency can be analogyzed by the response characteristic of the elasto-plastic system for the single input. It should be mentioned that for both cases the stiffness asymmetricity conspicuously affects the displacement response as μ increases.

CONCLUSIONS AND ACKNOWLEDGMENT

1. The response analysis on the mechanical equipment system which is appended to building structure and is subjected to multi-seismic inputs is made. It is desirable to evade the coincidence of the first order natural

periods of the both system, which causes very large response of the acceleration and the displacement depending on the period. This is similar to the case of the single input appended system.

2. The analysis suggests that the response behaviour of the displacement as well as the acceleration should be taken into account, corresponding to the time delay between two inputs and not only the nature of the ground acceleration, but also that of the ground velocity and displacement give influence on the response.

3. The displacement response for the simple two-inputs system with elasto-plastic deformation is markedly affected by the asymmetricity of the stiffness especially for the system with large ductility factor. Some basic characteristics obtained for the single input system is applicable.

In closing, the authors would like to express their sincere thanks to Professors A. Watari, S. Fujii and H. Shibata at University of Tokyo for their constructive discussions. They are also grateful to Professor H. Umemura at University of Tokyo who kindly allowed to use earthquake records of SERAC.

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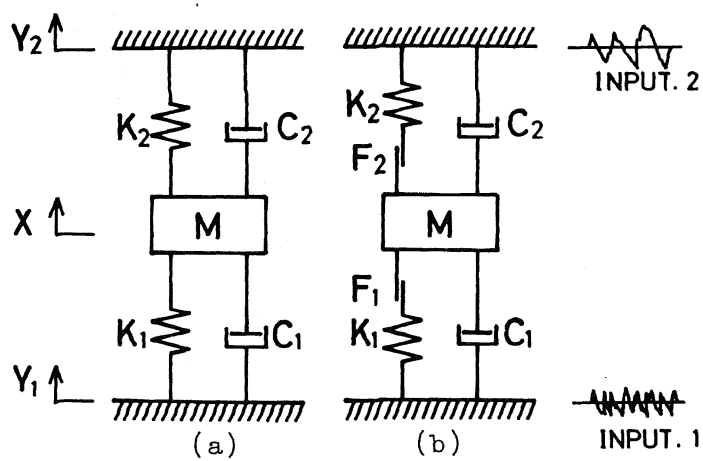


Linear Model Nonlinear Model

Fig. 1 Simple Two Inputs Model

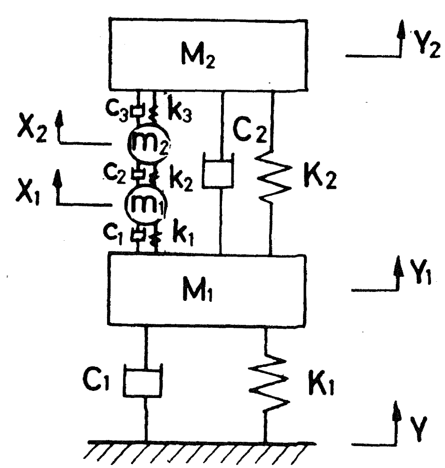


Fig. 2 An Integrated B-M Model

Tm \ Tb / hm	0.4		0.6		1.0	
	0.02	0.05	0.02	0.05	0.02	0.05
0.2	3.08	2.68	1.68	1.48	1.53	1.56
0.4	1.20	2.17	1.72	1.38	1.46	1.57
0.6	1.06	1.05	1.02	1.05	1.08	1.03
0.8	1.02	1.08	1.03	1.04	1.11	1.12
1.0	1.10	1.10	1.00	1.00	1.05	1.04
1.4	1.06	1.03	1.00	1.00	1.00	1.00
2.0	1.00	1.00	1.00	1.00	1.00	1.00

Table 2 Investigation of the Response Amplification Factor by the Floor Response Method

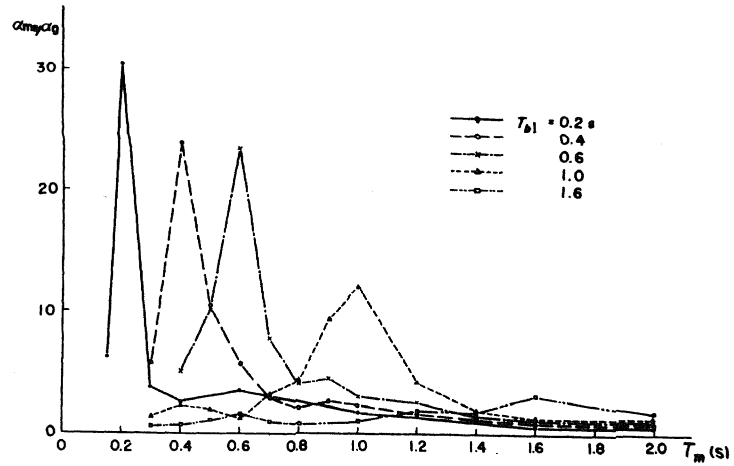


Fig. 3 Acceleration Response Spectrum for Equipment Model (El Centro, h_b = 0.05, γ = 0)

h_m \ T_{b1}(s)	Acceleration Amplification Factor $d(T_m=T_{bII})/d(T_m=T_{bI})$		Relative Displacement Response $d(T_m=T_{bII})/d(T_m=T_{bI})$	
	0.02	0.05	0.02	0.05
0.4	0.53	0.62	0.42	0.51
0.6	0.43	0.52	0.17	0.28
1.0	0.37	0.56	0.19	0.45
1.6	0.83	0.75	0.13	0.27

Table 1 Comparison of Response Properties at the First Resonance Period with Those at the Second One (El Centro, h_b = 0.05, γ = 0)

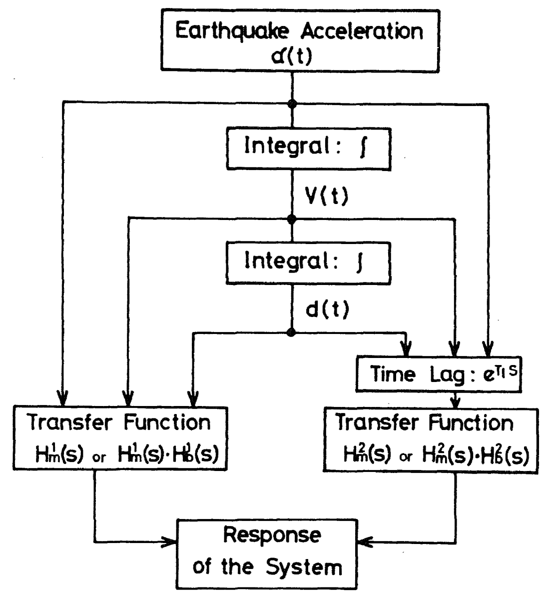


Fig. 4 Flow Chart of the Analysis of the Structure to Two Motions with Time Delay T_l

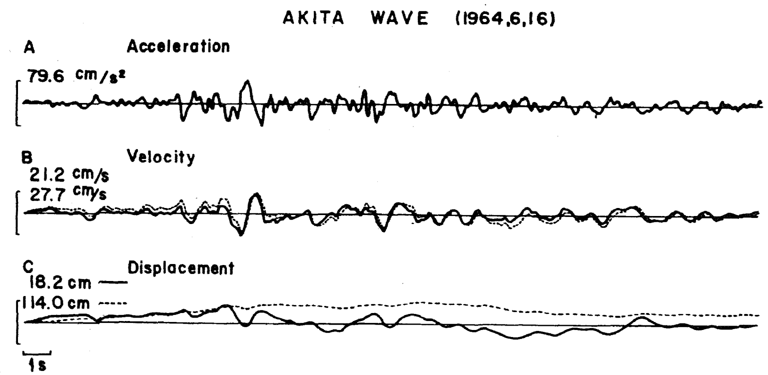


Fig. 5 Time Histories of Earthquake Acceleration and Integrated Velocity and Displacement

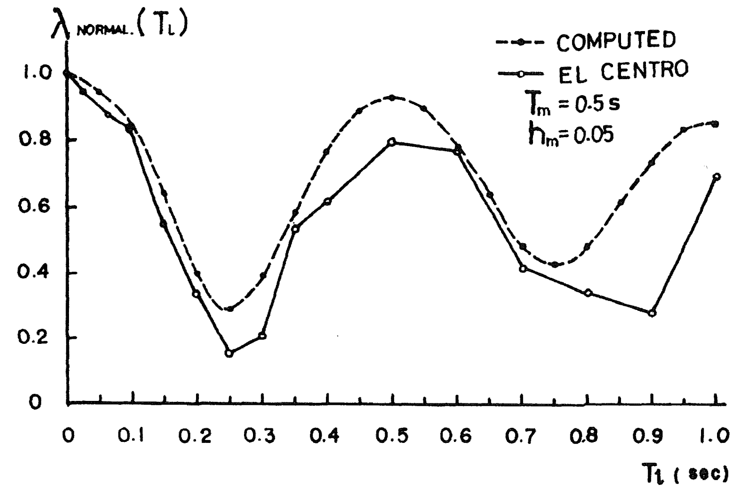


Fig. 6 Acceleration Response Behaviour as for Time Delay T_l

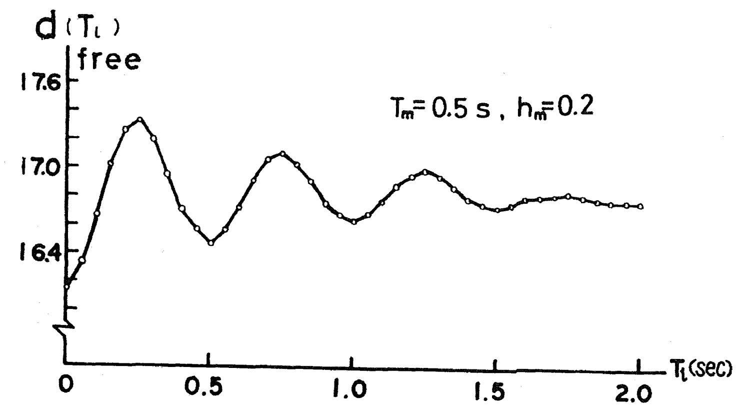


Fig. 7 Displacement Response Behaviour as for T_l by Statistical Method of Estimation

	Maximum Velocity (kine)	Maximum Displacement (cm)
Akita NS (June 1964)	70.0	14.0
Niigata NS (June 1964)	90.0	78.0
Kushiro NS EW (Dec. 1961)	21.0 18.3	3.5 4.2
Saitama NS (Feb. 1956)	32.2	6.3
Echizen-oki NS (March 1963)	63.0	17.7
Shizuoka EW (April 1965)	39.0	9.7
Matsushiro EW (Oct. 1968)	33.5	8.1
Taft NS (July 1952)	48.8	10.9
El Centro NS (May 1940)	57.0	12.7

Maximum Acceleration; 0.30 g

Table 3 Computed Maximum Velocity and Displacement of Some Famous Earthquakes

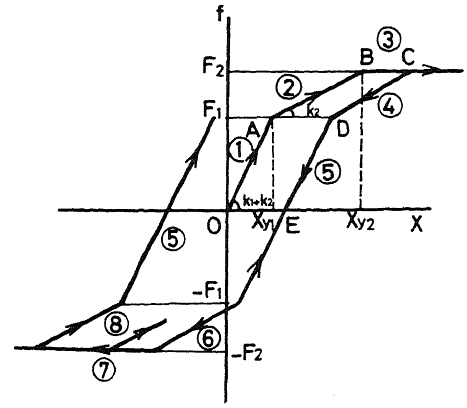


Fig. 8 Hysteresis Loop for the Two Inputs System with Elasto-Plastic Deformation

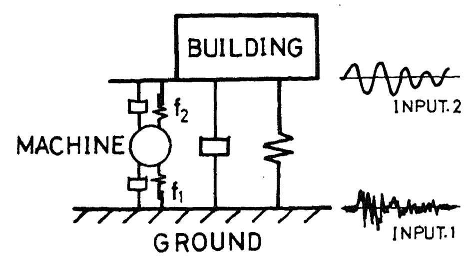
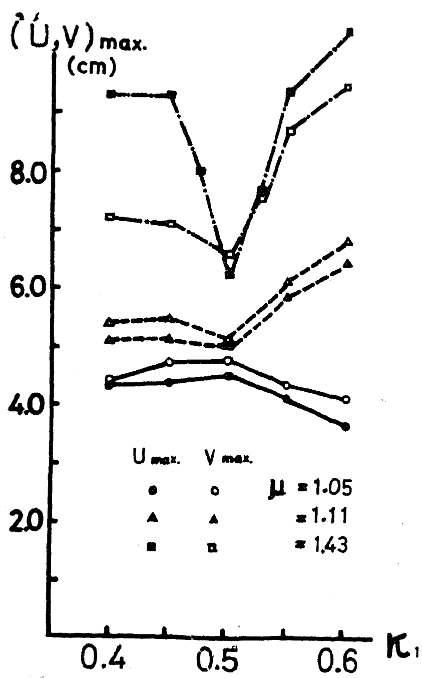
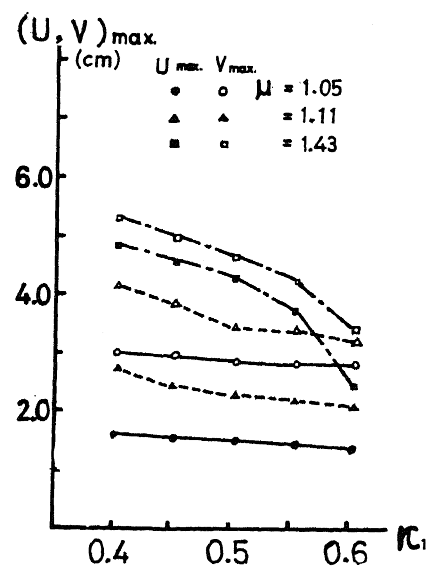


Fig. 9 Nonlinear Equipment Model Subjected to Building and Ground



(a) for resonant case ($T_b = T_m = 0.5$ s)



(b) for nonresonant case (T_b = 1.0 s, T_m = 0.5 s)

Fig. 10 Maximum Relative Displacement Response of the Nonlinear Two Inputs Model