TORSIONAL ANALYSIS OF CORE WALL STRUCTURES

by

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SYNOPSIS

An approximate theory is presented to analyse core wall structures subjected to applied torques. The theory assumed in-plane diaphragm action of the floors. The out-of-plane stiffness of the floors around the core walls are approximated by equivalent connecting beams. Expressions to obtain the rotational deformation, the vertical shears in the connecting beams, and the warping stresses in the core walls are obtained explicitly. Design curves are presented for the load cases of a concentrated torque at the top and triangularly distributed torques along the height of the structure.

THEORY

In many high rise buildings, the lateral resistance of the structure is provided by the central core which houses the elevators and the service ducts. In addition to study the flexural behaviour [1] it is necessary to study the torsional behaviour of such structures subjected to torques caused by the lateral loading [2]. In this paper, the deformation and stresses expressions are obtained explicitly for a core wall structure subjected to applied torques.

Consider a core wall structure of plan dimensions a x b and height H consists of two walls of channel cross-sectional shape connected by floors. It is subjected to applied torques Q_t distributed along the height of the structure and the structure is assumed to be built on a rigid foundation. Representing the effect of floors by equivalent floor beams connecting the two core walls, a plan view of the core structure is shown on Fig. 1. The analysis is based on the following assumptions. (i) Diaphragm action of the floors is assumed so that the whole structural assembly moves as a rigid body in the horizontal direction. (ii) The core walls can be treated as open thin-walled beams and Vlasov's theory for thin-walled beam holds [3]. (iii) The connecting beams at each floor are replaced by a uniform distribution of independently acting laminae with equivalent stiffness properties throughout the height of the structure. Finally, (iv) the mid-points of the laminae are assumed to be points of contraflexure.

The parameters of interest in the problem consist of the rotational deformation θ (z), the shear forces induced in the connecting laminae along the height of the structure q (z), and the warping stresses induced in the walls σ (z). Based on the consideration of torque equilibrium and also compatibility conditions, the governing equation is given by [4],

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$$2EI_\omega \theta''' + 2EI_x c_x^2 \theta''' - 2E\gamma r^2 \theta' - 2GJ\theta' = Q_t \qquad (1)$$

where prime denotes derivative respective to the vertical coordinate z. I_ω and I_x are the principal sectorial inertia and moment of inertia of each wall. J is the St. Venant torsion constant and c_x is the distance of the shear centers of the wall from the origin. γ is the connecting beam stiffness factor given by $\gamma = 12I_b/[h\ell^3 + 12EI_b h\ell/GA]$ with h as the storey height; ℓ, I_b, and A as the span, moment of inertia and cross-sectional area of the connecting beam respectively. r is the plan area of the core wall given by r = ab, and Q_t is the applied torque expression.

The first term represents the warping resistance due to each individual core wall. The second term is the warping term caused by the diaphragm action of the floor. The third and fourth term represent the effect of the connecting beams and the St. Venant torsional stiffness of the core walls respectively. The boundary conditions are such that there is no rotation at the base; warping is prevented from the base; and there is no bimoment at the top of the building. Mathematically, they are

$$\theta(0) = 0, \quad \theta'(0) = 0 \text{ and } \theta''(H) = 0 \qquad (2)$$

Of particular interest to seismic design is the load cases of a concentrated torque applied at the top and a triangularly distributed torque. These two cases of loading would correspond to the equivalent static lateral load distribution suggested by the code [5]. For a concentrated torque P at the top, the solution is:

$$\theta(z) = \frac{PH^3}{2EI_\omega^*(\lambda H)^3} - \Big[\sinh(\lambda z) + \tanh(\lambda H)\{\cosh(\lambda z) - 1\} + \lambda z\Big] \qquad (3)$$

For a triangularly distributed torque with n represents the maximum loading intensity at the top, the solution is:

$$\theta(z) = \frac{nH^4}{2EI_\omega^*(\lambda H)^3}\Big[-\{1/2 - \frac{1}{(\lambda H)^2}\}\sinh(\lambda z) + \Big(\{1/2 - 1/(\lambda H)^2\}\tanh(\lambda H) + \frac{1}{\lambda H \cosh(\lambda H)}\Big)\{\cosh(\lambda z) - 1\} + \{1/2 - 1/(\lambda H)^2\}\lambda z - \lambda z^3/6H^2\Big] \qquad (4)$$

where
$$\lambda^2 = (\gamma r^2 + GJ/E)/I_\omega^* \qquad (5a)$$

$$\text{and } I_\omega^* \equiv I_\omega + c_x^2 I_x \qquad (5b)$$

Once $\theta(z)$ is known, the shear force distribution q(z) and the warping stresses $\sigma(z)$ can be obtained as

$$q(z) = E\,\gamma\, r\, \theta' \qquad (6)$$

$$\sigma(z) = E\,\Omega\,\theta' \qquad (7)$$

The warping stresses are distributed proportional to the modified sectional area Ω. The Ω diagram for the core wall structure is shown in Fig. (4).

DESIGN CHARTS

To facilitate the use of the results presented, a set of design curves are presented for each case of loading.

(i) Concentrated torque P at the top.

rotation $\theta = PH^3 K_{1C}/(6EI_\omega^*)$ (8)

shear $q = PH^2 \gamma r K_{2C}/(4I_\omega^*)$ (9)

warping stresses $\sigma = PH\Omega K_{3C}/(2I_\omega^*)$ (10)

The factors K_{1C}, K_{2C} and K_{3C} are functions of λH and x/H. Graphically, they are presented in Fig. (2a,b,c).

(ii) Triangularly distributed torque.

rotation $\theta = 11nH^4 K_{1T}/(240EI_\omega^*)$ (11)

shear $q = nH^3 \gamma r K_{2T}/(16I_\omega^*)$ (12)

warping stresses $\sigma = nH^2 \Omega K_{3T}/(6I_\omega^*)$ (13)

The factors K_{1T}, K_{2T}, and K_{3T} are also functions of λH and x/H. Graphically, they are presented in Figs. (3a,b,c). Once the value of λH is computed, the torsional deformation, shear forces in the connecting beams and the warping stresses in the walls can be obtained using eqns. (8-13).

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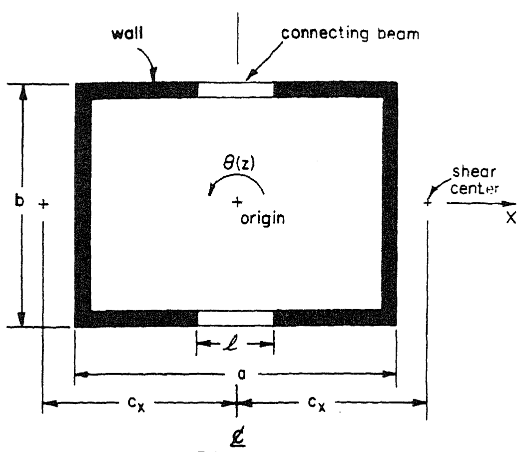


FIGURE 1.

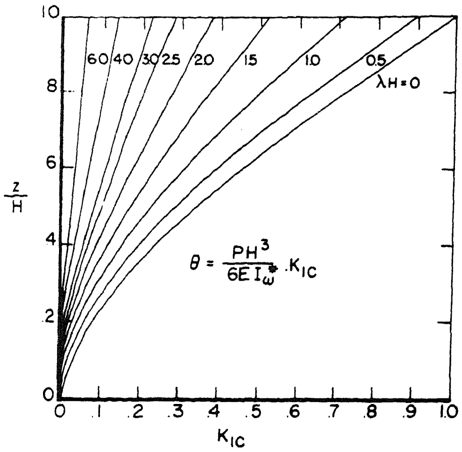


FIGURE 2a.

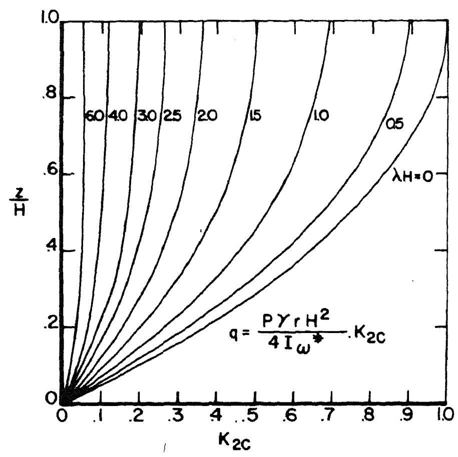


FIGURE 2b.

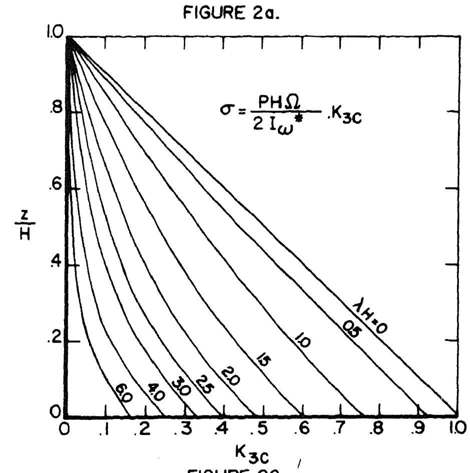


FIGURE 2C.

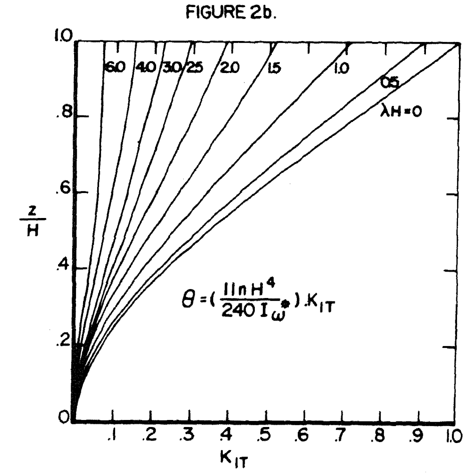


FIGURE 3a.

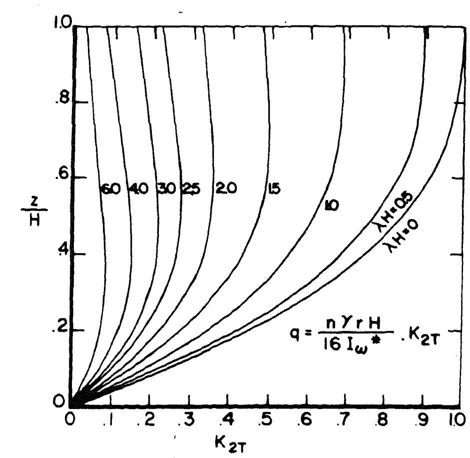


FIGURE 3b.

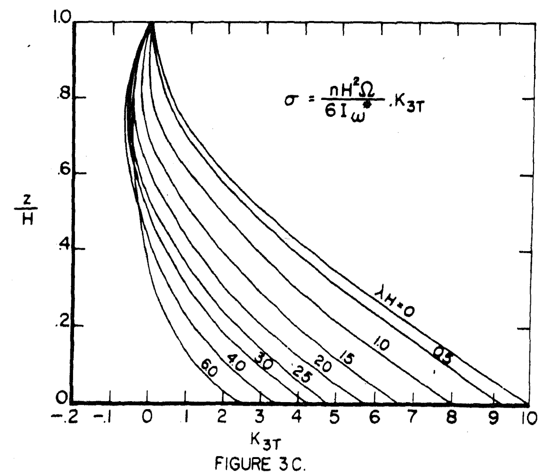
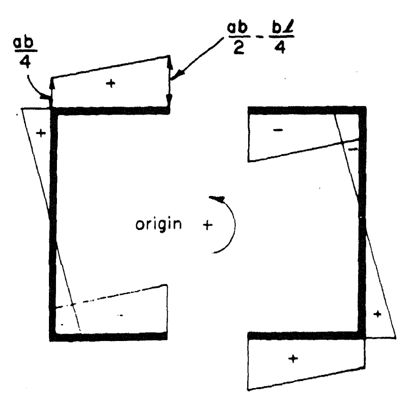


FIGURE 3C.



(+ indicates tension)

FIGURE 4