SEISMIC RESPONSE OF RETICULAR ELASTO-PLASTIC STRUCTURES

by

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SYNOPSIS

After some references to the elasto-plastic behaviour of trusses, an incremental approach for trusses submitted to dynamic or in particular to seismic actions is proposed. Plastic flow and columns instability effects are pointed out. Some numerical examples illustrate the proposed procedure.

INTRODUCTION

In the analysis of the truss seismic response is essential to consider the plastic range. Another important aspect of the problem is the columns behaviour in the post-critical field. It is well known that the vibration amplitude is reduced, when we consider the plastic range, also in the case of resonance problems, owing to the plastic dissipation of energy. This is not valid in the case of columns buckling: plasticity makes unstable the column post-critical configurations and reduces structure stiffness. In the dynamic analysis it is necessary to refer to the real elasto-plastic equilibrium conditions of the members. Moreover, owing to the variable and repeated character of the loading, members wich are in an unstable configuration may be subsequently recovered to the structure strenght, which is also connected to the loading process.

The problem, solved by following the structure behaviour step by step, leads to the solution of non-linear equations systems that can be linearized operating by sufficiently small increments. The incremental solution, because of the member constitutive law characteristics, can have some discontinuities to be taken into account in the definition of each step starting conditions.

BASIC EQUATIONS

With reference to the rectangular axes x, y, z, the geo-

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metry of the space reticular structure, whose joints are $A_i(x_i, y_i, z_i)$, is defined by the length and the axis direction-cosines of the generic member A_i, A_j :

$$\ell_{ij} = \sqrt{\Delta x_{ij}^2 + \Delta y_{ij}^2 + \Delta z_{ij}^2} \tag{1}$$

$$(\alpha_x)_{ij} = \frac{\Delta x_{ij}}{\ell_{ij}} ; \quad (\alpha_y)_{ij} = \frac{\Delta y_{ij}}{\ell_{ij}} ; \quad (\alpha_z)_{ij} = \frac{\Delta z_{ij}}{\ell_{ij}}$$

The member length variations, due to the joints displacements u_i, v_i, w_i, are given by :

$$\Delta \ell_{ij} = (\alpha_x)_{ij} \Delta u_{ij} + (\alpha_y)_{ij} \Delta v_{ij} + (\alpha_z)_{ij} \Delta w_{ij} \tag{2}$$

where $\Delta(\ )_{ij}$ is $(\ )_j - (\ )_i$

Because of the displacement increments δu_i, δv_i, δw_i, referred to the initial configuration, the following length variation increments takes place :

$$\delta(\Delta \ell_{ij}) = (\alpha_x)_{ij} \Delta \delta u_{ij} + (\alpha_y)_{ij} \Delta \delta v_{ij} + (\alpha_z)_{ij} \Delta \delta w_{ij} \tag{3}$$

The incremental joint equilibrium equations, for the load increments $\delta X_i^{(r)}$, $\delta Y_i^{(r)}$, $\delta Z_i^{(r)}$, at the instant $t^{(r)} = t^{(r-1)} + \delta t^{(r)}$, are :

$$\begin{aligned} \sum_{1}^{K}{}_j\, \delta N_{ij}^{(r)} (\alpha_x)_{ij} + \delta X_i^{(r)} - m_i\, \delta a_{x_i}^{(r)} &= 0 \\ \sum_{1}^{K}{}_j\, \delta N_{ij}^{(r)} (\alpha_y)_{ij} + \delta Y_i^{(r)} - m_i\, \delta a_{y_i}^{(r)} &= 0 \\ \sum_{1}^{K}{}_j\, \delta N_{ij}^{(r)} (\alpha_z)_{ij} + \delta Z_i^{(r)} - m_i\, \delta a_{z_i}^{(r)} &= 0 \end{aligned} \tag{4}$$

where $N_{ij}^{(r)} = N_{ij}^{(r-1)} + \delta N_{ij}^{(r)}$ represent the axial forces of the K members connected by the i joint, being the tensile forces positive; m_i is the mass concentrated in the i joint; a_{x_i}, a_{y_i}, a_{z_i} are the acceleration components of the mass m_i.

The kinematic quantities at the instant $t^{(r)}$ can be expressed, as shown in [1] , by:

$$\begin{aligned} s_i^{(r)} &= s_i^{(r-1)} + \delta s_i^{(r)} \\ v_i^{(r)} &= 3\, \frac{\delta s_i^{(r)}}{\delta t^{(r)}} - \frac{1}{2}\, a_i^{(r-1)} \delta t^{(r)} - 2\, v_i^{(r-1)} \\ a_i^{(r)} &= 6 \left(\frac{\delta s_i^{(r)}}{(\delta t^{(r)})^2} - \frac{v_i^{(r-1)}}{\delta t^{(r)}} \right) - 2\, a_i^{(r-1)} \\ \delta a_i^{(r)} &= 3 \left[2 \left(\frac{\delta s_i^{(r)}}{(\delta t^{(r)})^2} - \frac{v_i^{(r-1)}}{\delta t^{(r)}} \right) - a_i^{(r-1)} \right] \end{aligned} \tag{5}$$

where $s_i^{(r)}$, $v_i^{(r)}$, $a_i^{(r)}$, $\delta a_i^{(r)}$ represent respectively the displacement, velocity, acceleration, acceleration increment of joint i, at the instant $t^{(r)}$, with the following components:

$$s_i = \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} ; \quad v_i = \begin{bmatrix} v_{xi} \\ v_{yi} \\ v_{zi} \end{bmatrix} ; \quad a_i = \begin{bmatrix} a_{xi} \\ a_{yi} \\ a_{zi} \end{bmatrix} ; \quad \delta a_i = \begin{bmatrix} \delta a_{xi} \\ \delta a_{yi} \\ \delta a_{zi} \end{bmatrix} \tag{6}$$

The member force elongation relationships for the various phases have been given in [2], [3], [4] under the following simplifying hypotheses:

a) members hinged at the joints;
b) bilateral indefinite bending moment-curvature diagram;
c) linearized M, N dominium;
d) member deformability concentrated in the middle;
e) equilibrium biforcation owing to the stability linear theory.

These relationships, with reference to the various curves of fig. 1, are specified by the following equations.

- curve (a) (elastic behaviour)
  For $N_{Eij} < N_{ij} < N_{oij}$ $\quad (N_{Eij} \geq - N_{oij})$
  we have:

$$\delta N_{ij} = \frac{EA_{ij}}{\ell_{ij}} \, \delta(\Delta \ell_{ij}) \tag{7}$$

  where N_{Eij} is the eulerian critical load, N_{oij} the ultimate normal tensile force, A_{ij} the cross-section area of the member ij.

- curve (b) (plastic behaviour)
  For $N_{ij} = N_{oij}$, $\quad \delta(\Delta \ell_{ij}) \geq 0$
  we have: $\quad \delta N_{ij} = 0$ (8)

- curve (a') (elastic behaviour)
  The first step into the (a') curve (point B) takes place when:

$$N_{ij} = N_{oij} , \qquad \delta(\Delta \ell_{ij}) < 0$$

  and we have:

$$\delta N_{ij} = \frac{EA_{ij}}{\ell_{ij}} \, \delta(\Delta \ell_{ij}) \tag{9}$$

- curve (c) (elastic behaviour)
  For: $N_{ij} = N_{Eij}$, $\qquad \Delta \ell_{ij} > (\Delta \ell_{ij})_D$

we have: $\delta N_{ij} = 0$ (10)

with: $$(\Delta \ell_{ij})_D = \ell_{ij}\, \varepsilon_o \frac{N_{ij}}{N_{oij}} - 2\, \frac{d_{ij}^2}{\ell_{ij}} \left(1 - \frac{N_{oij}}{N_{Eij}}\right)^2$$

being ε_o the maximum elastic strain and $d_{ij} = |M_{oij}/N_{oij}|$ the ratio between the ultimate values of the bending moment and the axial force

- curve (d) (plastic behaviour)
The first step into the curve (d) (point D) corresponds to the condition:

$$\Delta \ell_{ij} = (\Delta \ell_{ij})_D$$

If we have: $$\left(\frac{d(\Delta \ell_{ij})}{d N_{ij}}\right)_D < 0$$

the constitutive law is given by:

$$\delta N_{ij} = \frac{N_{oij}}{\varepsilon_o \ell_{ij} + 4\, \frac{d_{ij}^2}{\ell_{ij}}\, \frac{1 + N_{ij}/N_{oij}}{(N_{ij}/N_{oij})^3}}\, \delta(\Delta \ell_{ij}) \qquad (11)$$

under the condition: $\delta(\Delta \ell_{ij}) \leq 0$.
But in the case of:
$$\left(\frac{d(\Delta \ell_{ij})}{d N_{ij}}\right)_D \geq 0$$
it is not possible to follow the (d) curve in the tract starting from the point D: at such a tract do not correspond indeed possible incremental equilibrium configurations of the structure.
A possible equilibrium configuration takes place at the point of the (d) curve defined by the intersection with the straight line of equation $\Delta \ell_{ij} = (\Delta \ell_{ij})_D$. We have therefore a sudden variation of the member force. Consequently in the next step it is necessary to take into account the inertia forces increments in order to re-establish the dynamic equilibrium of the joints at the extremities.

- curve (e) (elastic behaviour).
If at some point F of the (d) curve we have $\Delta \ell_{ij} > 0$ the member returns into the elastic range and the constitutive law is given by:

$$\delta N_{ij} = \frac{N_{oij}}{\varepsilon_o \ell_{ij} + 4\, \frac{d_{ij}^2}{\ell_{ij}} \left(1 + \frac{N_{oij}}{(N_{ij})_F}\right)^2 \frac{((N_{ij})_F - N_{Eij})^2}{(N_{ij} - N_{Eij})^3}\, N_{oij}}\, \delta(\Delta \ell_{ij}) \qquad (12)$$

under the condition : $N_{ij} < (N_{ij})_H$.

- curve (f) (plastic behaviour).
  When following the (e) curve we have $N_{ij} = (N_{ij})_H$ the constitutive law becomes:

$$\delta N_{ij} = \frac{N_{oij}}{\varepsilon_o \ell_{ij} + 4 \frac{d_{ij}^2}{\ell_{ij}} \frac{1 - N_{ij}/N_{oij}}{(N_{ij}/N_{oij})^3}} \delta(\Delta \ell_{ij}) \tag{13}$$

  under the conditions: $\delta(\Delta \ell_{ij}) \geq 0$ and $N_{ij} < N_{oij}$.

- curve (g) (elastic behaviour).
  If in a point I of the (f) curve we have $\delta(\Delta \ell_{ij}) < 0$ the member returns into the elastic range and the constitutive law is given by:

$$\delta N_{ij} = \frac{N_{oij}}{\varepsilon_o \ell_{ij} + 4 \frac{d_{ij}^2}{\ell_{ij}} \left(1 - \frac{N_{oij}}{(N_{ij})_I}\right)^2 \frac{\left((N_{ij})_I - N_{\varepsilon ij}\right)^2}{\left(N_{ij} - N_{\varepsilon ij}\right)^3} N_{oij}} \delta(\Delta \ell_{ij}) \tag{14}$$

In the case we have plastic tensile strain $(\Delta \ell_{ij})_r$ the constitutive law curves translate into the direction of the abscissa axis of the quantity $\overline{AB} = (\Delta \ell_{ij})_r$ (fig. 1).

Therefore the incremental relationships do not change while the limit values of the elongations are modified of the quantity $(\Delta \ell_{ij})_r$.

NUMERICAL SOLUTION

The numerical solution of the equation system (1) is obtained through the classical procedure of the incremental solutions. At each step the validity of the solution is checked both regarding the effective correspondence between the constitutive law assumed for each member and its instantaneous forces and elongations values, and regarding the required approximation degree.

SOME EXAMPLES

Some numerical applications are developed referring to the structure of fig. 2. In an initial phase we consider a vertical loading statically applied given by the weight forces. In a subsequent dynamic phase we suppose to submit the structure supports to equal horizontal displacements following the law: $u(t) = u_o \sin \omega t$.

In fig. 3 is reported the time-displacements diagram relative to the (1) and (2) joints.

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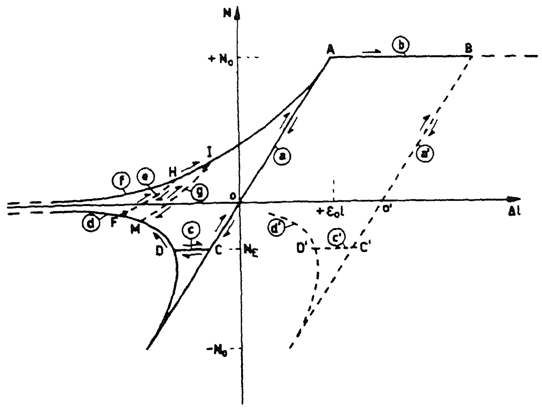
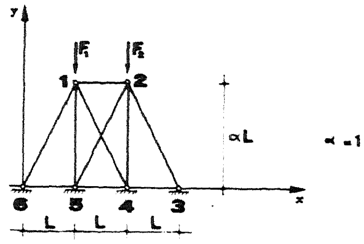


fig.1



	A (cm²)	I (cm⁴)	W (cm³)
1-2	1.02	0.276	0.42
1-4	1.84	1.37	1.37
1-5	1.84	1.37	1.37
1-6	1.84	1.37	1.37
2-3	2.79	3.29	2.5
2-4	2.79	3.29	2.5
2-5	2.79	3.29	2.5

$L = 100$ cm
$\sigma_s = 2400$ kg/cm²
$E = 2.1 \cdot 10^6$ kg/cm²
$F_1 = 1000$ kg
$F_2 = 2000$ kg
$m_1 = F_1/g$
$m_2 = F_2/g$

fig. 2

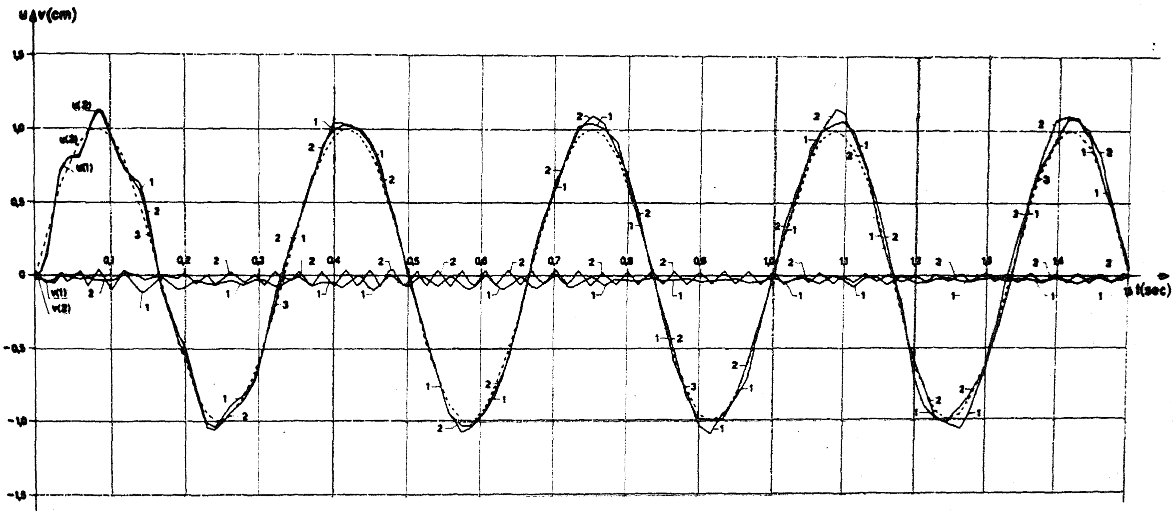


fig. 3