

NON-LINEAR RESPONSE AND STRUCTURAL DUCTILITY  
OF REINFORCED CONCRETE BUILDINGS  
UNDER STRONG SEISMIC ACTIONS

by

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Synopsis

A simple and general method for the non-linear analysis of building structures is presented. This, called the equivalent cantilever method, basically consists in modelling a cantilever with bending, shear and base rotation deformability, with dynamic characteristics similar to those of the actual structure. Bi-linear moment-curvature and shear-rotation diagrams with parabolic transitions simulate the different yielding mechanisms liable to occur during an earthquake. The seismic response of the equivalent cantilever is obtained by means of a step-by-step numerical integration.

The method has been applied in the study of reinforced concrete framed, shear-wall, and combined structures under strong seismic actions. Linear and non-linear responses are compared. Yielding effects in bending and shear are analysed. Overall ductility factors defined as the maximum ratio between linear and non-linear base shear are determined for different types of yielding mechanisms.

1 - FUNDAMENTALS OF THE EQUIVALENT CANTILEVER METHOD

Although non-linear has generally been recognised as significantly affecting the structural response to strong seismic actions, the information available on this subject is still scarce not allowing its being fully considered in seismic codes. This is mainly due to the long computing time required by non-linear analysis.

The interest of research on simple models which can simulate the non-linear behaviour of building structures is therefore obvious. The equivalent cantilever method (1) was developed with this aim along the following steps:

i) modelling of a cantilever with a flexibility matrix (transverse displacements at floor levels due to horizontal unit loads) equal or very similar to the flexibility matrix of the actual structure. Cantilever

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deflections result from the bending and shear stiffnesses of its members and from base rotation. Several techniques have been used for modelling the equivalent cantilever (2);

ii) simulating of the different possible yielding mechanisms that can occur in the actual structure by means of bi-linear (with parabolic transitions) moment-curvature and shear-rotation diagrams of the equivalent cantilever members (1, 3). Non-linear bending and shear behaviours can thus be independently considered or associated;

iii) computing the seismic response of the equivalent cantilever by means of step-by-step numerical integration (4).

## 2 - CHARACTERISTICS OF THE STRUCTURES STUDIED

The equivalent cantilever method has recently been used in the seismic study of framed, shear-wall, and combined reinforced concrete structures of 8 to 20 story buildings, with the dynamic properties indicated in Table I.

TABLE I - Dynamic properties of buildings Bd.1 to Bd.10.

Bd.No.	Type of structure	No. of stories	Period (s)			Mode factor		
			1st mode	2nd mode	3rd mode	1st mode	2nd mode	3rd mode
1	framed	12	1.00	0.38	0.22	1.43	0.62	0.35
2		16	1.33	0.55	0.34	1.46	0.60	0.35
3		20	2.00	0.82	0.50	1.54	0.74	0.46
4	shear-wall	8	0.39	0.07	0.04	1.43	0.58	0.26
5		12	0.67	0.12	0.05	1.41	0.56	0.26
6		16	1.03	0.18	0.08	1.45	0.63	0.30
7		20	1.50	0.26	0.10	1.43	0.59	0.28
8	combined (frame + shear-wall)	12	0.67	0.19	0.10	1.41	0.66	0.36
9		16	1.00	0.32	0.18	1.43	0.59	0.30
10		20	1.43	0.50	0.26	1.50	0.66	0.29

A 3m floor height and a uniform mass distribution was considered in all cases. 5% viscous damping was ascribed to the first three modes.

## 3 - GROUND MOTION INPUT

Each structure was subjected to computer-generated accelerograms with the following characteristics:

- stationary with 20 s duration;
- Gaussian distribution of amplitudes, with a power spectral

density of acceleration  $S(f) - \text{gal}^2 \text{ Hz}^{-1}$  concentrated on the frequency range  $0 < f < 5 \text{ Hz}$  given by

$$S(f) = \frac{420 (1 + f^2/1.94)}{(1 - f^2/6.6)^2 + f^2/3.51} \approx \frac{420 (1 + 0.5 f^2)}{1 + 0.0225 f^4} \quad \dots 2)$$

The maximum ground acceleration of such an accelerogram amounts to about 0.20 g.

#### 4 - LINEAR AND NON-LINEAR RESPONSES

The structures were first subjected to linear analysis, just by keeping the values of yielding moments and shears high enough not to be attained. The maximum values of displacements, forces, moments, and shears were related to the dynamic properties of the structures (1). It was clear in all cases, that the maximum top displacements are governed by the fundamental mode and that the influence of higher modes upon base shear  $V$  can be related with the fundamental frequency  $f(\text{Hz})$  by

$$\begin{aligned} \frac{V}{V_1} &= 1.5 - \frac{f}{5} && \text{for } f \leq 2.5 \text{ Hz} \\ &&& \dots \dots \dots 3) \\ \frac{V}{V_1} &= 1.0 && \text{for } f > 2.5 \text{ Hz} \end{aligned}$$

where  $V_1$  is the base shear when the first mode alone is considered.

For studying the non-linear seismic response the following yielding patterns were considered.

Yielding in bending only was considered in shear-wall structures to simulate the effects of the very high seismic axial forces which develop in cantilever-type structures.

Yielding due to shear alone was considered in framed structures to simulate the effects of a slight yielding in the columns only.

Yielding predominantly due to bending associated with shear yielding was considered in shear-wall and combined structures.

Yielding predominantly due to shear associated with bending yielding was considered for the three types of structures.

Finally a balanced association of bending and shear yielding was considered in the analysis of the combined structures.

The structures were then subjected to series of analysis with the same ground motion input and with yielding limits that were lowered after each analysis. Displacements and loads computed at each run were compared with those obtained in the linear analysis. A series of computer runs was completed when instability of the displacements was detected.

The overall ductility factor, which expresses the influence of non-

-linear behaviour on the seismic response, can therefore be defined as the maximum value of the ratio of linear to non-linear base shears for which maximum non-linear displacements are similar to those obtained in linear analyses.

The following main conclusions were derived from this study.

Structures where shear yielding predominates are very stable in the sense that maximum displacements do not exceed those obtained in linear analyses even for low shear yielding limits. The overall ductility is limited only by the ductilities of the members and thus can reach values of 3 to 4 in reinforced concrete frames.

Almost the opposite occurs for cantilever-type structures where even a small yielding can give rise to large displacements. Overall ductility factors ranging from 1.5 to 2 were obtained in this case.

When shear and bending yielding occur simultaneously the former always exerted a stabilizing effect on the response, and ductility factors ranged between the limits indicated above.

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