DETERMINATION OF COMPOSITE DAMPING MATRICES

by

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SYNOPSIS

The composite damping matrix for a structure formed by substructures with different damping characteristics is developed. The proposed method is based on the "fixed-base" damping properties of the substructures and the damping values for rigid-body motions. This damping information is set up in the related coordinate system (formed of the fixed-base modes of vibration together with rigid-body degrees of freedom) and then transformed to the physical coordinate system (formed of all the degrees of freedom relative to the free field). The transformation matrix is obtained from the relationship between the two coordinate systems.

INTRODUCTION

When soil-structure interaction is important or when substructures of a complex system possess dissimilar damping properties, an "equivalent" modal analysis would necessarily overestimate or underestimate significantly the response of one or more important feature. In such cases, there is a need to develop a more appropriate damping matrix.

Equations of motion, having tensorial character, are invariant under any linear transformation. The equations representing equilibrium of forces in the physical coordinates,

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{P\}$$

can be transformed by the linear relations,

$$\{X\} = [T]\{Z\} \text{ and } \{Z\} = [T^{-1}]\{X\}$$

to represent the equilibrium of forces in the transformed coordinates,

$$[M_T]\{\ddot{Z}\} + [C_T]\{\dot{Z}\} + [K_T]\{Z\} = \{P_T\}$$

where $$[M_T, C_T, K_T] = [T]'[M, C, K][T] \text{ and } \{P_T\} = [T]'\{P\}$$

The mass and stiffness matrices can be set up in the physical coordinates but it is difficult to obtain the damping matrix directly. Thus, the damping matrix is set up in the transformed coordinate system and then transformed to the physical coordinate system by the relation

$$[C] = [T^{-1}]'[C_T][T^{-1}]$$

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SOLUTION OF REPRESENTATIVE CASES

CASE 1 - Consider the degrees of freedom X (relative to free-field) as shown in Fig. 1(b) and the transformed coordinates Z (related to the damping information available) as described below:

$\left[\diagdown C_n^* \diagdown\right] = \left[\diagdown 2\lambda_n \omega_n \diagdown\right]$		$\{Z_n\} = \{n^{th}$ mode of fixed-base vibration$\}$
C_H = equivalent damping: rigid-body translation	=>	$Z_{N+1} = Z_H$ = rigid-body translation
C_θ = equivalent damping: rigid-body rocking		$Z_{N+2} = Z_\theta$ = rigid-body rocking

Thus, assembling the above damping information in the same sequence as the Z-coordinates, gives

$$\left[C_T\right] = \begin{bmatrix} \left[\diagdown 2\lambda_n \omega_n \diagdown\right] & & 0 \\ & C_H & \\ 0 & & C_\theta \end{bmatrix}$$

The proper coupling terms between rigid-body motions and between them and the structural modes of vibration can be added to C_T if available.

The transformation matrix T is developed from the relationship between the two coordinates:

$$\{X\} = \begin{Bmatrix} \{X_n\} \\ X_H \\ X_\theta \end{Bmatrix} = \begin{Bmatrix} \left\{\left(\sum_{i=1}^{N} \phi_{n,i} Z_i\right) + Z_H + h_n Z_\theta\right\} \\ Z_H \\ Z_\theta \end{Bmatrix} = \begin{bmatrix} \left[\phi_{\text{fixed-base}}\right] & \{1\} & \{h_n\} \\ \langle\text{---}\ 0\ \text{---}\rangle & 1 & 0 \\ \langle\text{---}\ 0\ \text{---}\rangle & 0 & 1 \end{bmatrix} \begin{Bmatrix} \{Z_n\} \\ Z_H \\ Z_\theta \end{Bmatrix} = [T]\{Z\}$$

CASE 2 - Described below are the coordinate systems X and Z for the example shown in Fig. 2:

$$X = \begin{Bmatrix} \{X_n\} \\ \{Y_j\} \\ \{V_s\} \\ \{W_r\} \\ X_H \\ X_\theta \end{Bmatrix} \qquad Z = \begin{Bmatrix} \{n^{th} \text{ mode, fixed at base, isolated from branches}\} \\ \{j^{th} \text{ mode, fixed at base, isolated from branches}\} \\ \{s^{th} \text{ mode, fixed at ends}\} \\ \{r^{th} \text{ mode, fixed at its base}\} \\ \text{rigid-body translation} \\ \text{rigid-body rocking} \end{Bmatrix}$$

The damping matrix in Z-coordinates will be,

$$C_T = \begin{bmatrix} \left[2\lambda_n \omega_n \right] & & & & & \\ & \left[2\lambda_j \omega_j \right] & & 0 & & \\ & & \left[2\lambda_s \omega_s \right] & & & \\ & & & \left[2\lambda_r \omega_r \right] & & \\ & 0 & & & C_H & \\ & & & & & C_\theta \end{bmatrix}$$

The relation between the two coordinates is given by

$$\left\{ X_n \right\} = \left\{ \left(\sum_{i=1}^{N} \phi_{ni} \; Z_i \right) + Z_H + h_n Z_\theta \right\}; \quad \left\{ Y_j \right\} = \left\{ \left(\sum_{i=1}^{J} \phi_{ji} \; Z_i \right) + Z_H + h_j Z_\theta \right\} ;$$

$$\left\{ W_r \right\} = \left\{ \left(\sum_{i=1}^{R} \phi_{ri} \; Z_i \right) + \left(\sum_{i=1}^{J} \phi_{ji} \; Z_i \right) + Z_H + h_r Z_\theta \right\};$$

$$\left\{ V_s \right\} = \left\{ \left(\sum_{i=1}^{S} \phi_{si} \; Z_i \right) + \left(\sum_{i=1}^{N} \alpha_i \, \phi_{ni} \; Z_i \right) + \left(\sum_{i=1}^{J} \beta_i \, \phi_{ji} Z_i \right) + Z_H + h_s Z_\theta \right\} ;$$

$$X_H = Z_H; \quad X_\theta = Z_\theta$$

where α_i is the deflection at node s if the pipe would be held at node j and excited at node n by a unit harmonic displacement with frequency ω_i; while β_i is the deflection at node s if the pipe would be held at node n and excited at node j by a unit harmonic displacement with frequency ω_i.

Thus, the transformation matrix is obtained as:

$$T = \begin{bmatrix} \left[\Phi_n \right] & \left[0 \right] & \left[0 \right] & \left[0 \right] & \{1\} & \{h_n\} \\ \left[0 \right] & \left[\Phi_j \right] & \left[0 \right] & \left[0 \right] & \{1\} & \{h_j\} \\ \left[\langle \alpha_1 \phi_{n1} \;|\; \alpha_N \phi_{nN} \rangle \right] & \left[\langle \beta_1 \; \phi_{j1} \;|\; \beta_J \; \phi_{jJ} \rangle \right] & \left[\Phi_s \right] & \left[0 \right] & \{1\} & \{h_s\} \\ \left[0 \right] & \begin{bmatrix} \langle \phi_{j1} & \text{---} & \phi_{jJ} \rangle \\ & | & \\ \langle \phi_{j1} & \text{---} & \phi_{jJ} \rangle \end{bmatrix} & \left[0 \right] & \left[\Phi_r \right] & \{1\} & \{h_r\} \\ \langle \text{---} \; 0 \; \text{---} \rangle & \langle \text{---} \; 0 \; \text{---} \rangle & \langle - 0 - \rangle & \langle - 0 - \rangle & 1 & 0 \\ \langle \text{---} \; 0 \; \text{---} \rangle & \langle \text{---} \; 0 \; \text{---} \rangle & \langle - 0 - \rangle & \langle - 0 - \rangle & 0 & 1 \end{bmatrix}$$

OBSERVATIONS AND REMARKS

1. The solution in the relative coordinates U, shown in Fig. 1(c), obtained both by the use of the proper transformation and by direct formulation check against each other and against the solution obtained in X coordinates. The damping matrix in U coordinates is,

$$\left[C_u\right] = \left[T_u^{-1}\right]' \left[C_T\right]\left[T_u^{-1}\right] \qquad \text{where} \left\{u\right\} = \left[T_u\right]\left\{Z\right\}$$

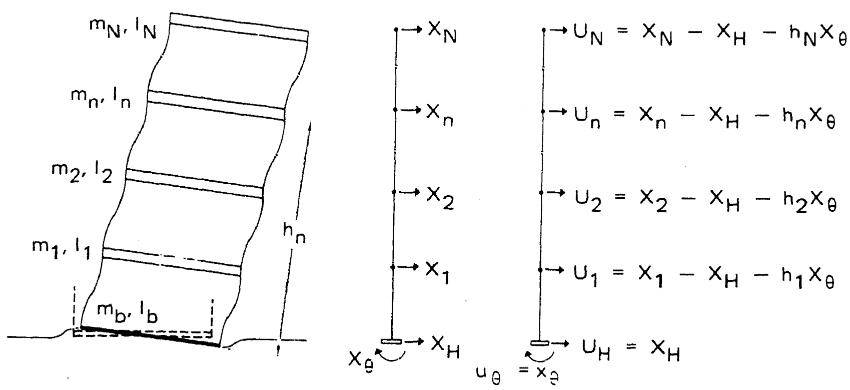
and T_u is obtained in a similar fashion as T for Case 1.

2. It is simpler to set up the stiffness matrix through the transformation T rather than by the usual direct methods.

3. The damping matrix for a free-free model can be obtained from C by assigning zero damping to rigid-body motions.

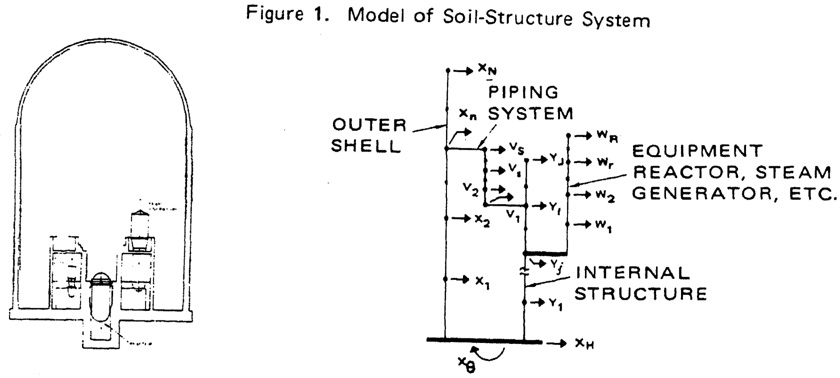
4. In Figs. 1 & 2, the substructures are symbolically represented by lumped-mass stick models with translational degrees of freedom only. However, they may be represented by more complete models if so desired. The effect of rigid-body motion on rotational degrees of freedom would obviously be Z_θ.

5. If a simplified solution using fewer modes is desired, the solution can be obtained in Z-coordinates with the number of equations truncated to include as few of the fixed-base modes as necessary and the results transformed to the physical coordinates.



(a) (b) (c)

Figure 1. Model of Soil-Structure System



(a) (b)

Figure 2. Model of PWR Containment Structure and Internal System