A THEORETICAL SIMULATION OF EARTHQUAKE ACCELERATION SPECTRA

by

D. Özgür[I] and A. Gürpınar[II]

SYNOPSIS

Propagation of plane harmonic waves in a layered, elastic half space is considered. Using transfer matrices, the spectral density matrices are obtained. Then, a band-limited white noise spectral density function is assumed for the longitudinal and transverse particle velocities at the source. Spectral density matrices are obtained at the surface for various configurations of the layers.

INTRODUCTION

Spectral analysis of strong motion earthquakes and their simulation has been a widely studied subject in the past two decades. Usually by a spectrum, a response spectrum is meant which is the relationship between the period of a single degree of freedom system with the maximum response of this system to a given excitation. Response spectra are very useful in the design of earthquake resistant structures if the design criterion is the maximum value of a response quantity such as deflection or stress. When the design criterion is the total energy absorption capacity, however, response spectra fail to serve as quidelines to the design engineer. Here, the total energy input of the earthquake is required. To be able to obtain an estimate of this energy, the spectral density function is necessary. The spectral density function of the earthquake record depends on the spectral density function at the source and the frequency response function of the path on which the seismic waves propagate. Therefore, two basic idealizations are necessary to solve the problem. The basic assumption on the spectral density function of the source is that the shape of this function is the same as the probability density function of the frequency content of the motion. The simplest of these functions is a band-limited white noise assumption, i.e.

$$S(\omega) = S_o \quad \text{for} \quad \omega_1 < \omega < \omega_2 \quad \text{and} \quad S(\omega) = 0 \quad \text{otherwise} \qquad 1.1$$

It is also assumed that $S(\omega)$ is time independent, in other words the

[I] Assistant Professor of Engineering Sciences, Middle East Technical University, Ankara, Turkey

[II] Assistant Professor of Engineering Sciences, Middle East Technical University, Ankara, Turkey

source mechanism produces a process which is at least weakly stationary in time. Although the stationarity assumption is not strictly valid since the motion is of finite duration, the fact that the motion dies and does not indefinitely increase allows for this stability assumption at least on a physical basis.

The path characteristics on the other hand require assumptions about the medium. Basically a horizontally layered medium on top of an elastic half space is considered. These layers are arbitrary in number and thickness and are elastic, homogeneous and isotropic within themselves. This idealization may be inadequate from a practical standpoint however, because it requires the Lamé constants and thicknesses of all the layers from the focus of the earthquake to the surface. Here, it is convenient to separate the problem into two parts: the response of the underlying, thicker, high velocity layers (i.e. response at the bedrock level) and the response of the thinner surface layers. The bedrock response can be considered to be equal within a given geological region where the properties of the underlying layers are obtained from geophysical studies and the probable focal depth is known. Using this spectral density function at the bedrock level spectral density functions for different smaller regions can be obtained by the local soil characteristics which in general are known more precisely (Fig. 1). In this figure, the following relationships will generally hold:

$$c_1^{(i)}(i=1,m) > c_1^{(i)}(i=m+1,m+n),\quad c_2^{(i)}(i=1,m) > c_2^{(i)}(i=m+1,m+n),$$

$$d^{(i)}(i=1,m) > d^{(i)}(i=m+1,m+n)$$

where $c_1^{(i)}$, $c_2^{(2)}$ and $d^{(i)}$ are longitudinal and transverse wave phase velocities and thickness for the i-th layer respectively.

FORMULATION

Field equations of an elastic, isotropic medium are

$$(\lambda+\mu)\, u_{l,lk} + \mu\, u_{k,ll} = \rho \ddot{u}_k \tag{2.1}$$

where λ, μ are Lamé constants, ρ is the mass density and u_i are the components of the displacement vector. Body forces are assumed to be zero. Repeated indices denote the rule of summation convention, a comma indicates partial differentiation with respect to space variables and a dot over a letter represents time rate.

For harmonic wave propagation problems, field equations (2.1) may be transformed into a convenient form by introducing the Helmholtz decomposition of theparticle velocity vector, which is defined as

$$v_i \equiv \phi_{,i} + e_{ijk}\, \psi_{kj} \tag{2.2}$$

where e_{ijk} is the permutation symbol, ϕ and Ψ_k are scalar and vectorial potentials respectively. Substitution of this decomposition into the field equations (2.1) yields the following wave equations

$$c_1^2 \phi_{,ii} = \ddot{\phi} \tag{2.3}$$

$$c_2^2 \Psi_{k,jj} = \ddot{\Psi}_k \tag{2.4}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho} \quad , \quad c_2^2 = \frac{\mu}{\rho} \tag{2.5}$$

In this paper a layered elastic half space is considered in two dimensions (Fig. 2). Potentials for an arbitrary layer are assumed to be

$$\phi = \phi' \exp i[kn_1x_1 + kn_3x_3 - \omega t] + \phi'' \exp i[kn_1x_1 - kn_3x_3 - \omega t] \tag{2.6}$$

$$\Psi = \Psi' \exp i[\kappa n_1x_1 + \kappa n_3x_3 - \omega t] + \Psi'' \exp i[\kappa n_1x_1 - \kappa n_3x_3 - \omega t] \tag{2.7}$$

where ω is the angular frequency, n_i are the components of the unit wave normal, k and κ are the wave numbers.

Continuity of the displacements

$$u_1 = \frac{i}{\omega}(\phi_{,1} - \Psi_{,3}) \quad , \quad u_3 = \frac{i}{\omega}(\phi_{,3} + \Psi_{,1}) \tag{2.9}$$

and the stresses

$$t_{13} = \mu(u_{1,3} + u_{3,1}) \quad , \quad t_{33} = \lambda(u_{1,1} + u_{3,3}) + 2\mu u_{3,3} \tag{2.10}$$

along the interface of the layers requires

$$\frac{\sin\theta_{incident}}{c_{incident}} = \frac{\sin\theta}{c_1} = \frac{\sin\gamma}{c_2} \tag{2.11}$$

With this formula, directions of reflected and refracted waves can be expressed in terms of the incident wave characteristics.

The velocity and the stress components at the upper and lower surfaces of the k-th layer can be related to each other by a matrix as

$$Z^{(k)} = [a_{ij}] Z^{(k-1)} \tag{2.12}$$

where $Z^{(k)} = [v_1^{(k)}, v_3^{(k)}, t_{33}^{(k)}, \frac{1}{2\mu} t_{13}^{(k)}]$ is a column vector. The matrix $\| a_{ij} \|$ is obtained by the substitution of (2.6) and (2.7) into (2.9) and (2.10) and the explicit forms of a_{ij}'s are given in reference [1] . By successive applications of (2.12) a matrix which connects velocities and stresses at $x_3=0$ to their corresponding values at the free surface can be obtained.

$$\underset{\sim}{z}^{(n+m)} = [A_{ij}] \; \underset{\sim}{z}^{(1)} \tag{2.13}$$

In the present work a computer program was employed to obtain numerical values for A_{ij}'s.

Imposing the boundary conditions,

$$v_1^{(1)} = v_1^{*}, \quad v_3^{(1)} = v_3^{*}, \quad t_{33}^{(n)} = t_{13}^{(n)} = 0 \tag{2.14}$$

Equation (2.12) takes the form

$$\begin{Bmatrix} v_1 \\ v_3 \\ 0 \\ 0 \end{Bmatrix}^{(n)} = \begin{bmatrix} & & & \\ & A_{ij} & & \\ & & & \end{bmatrix} \begin{Bmatrix} v_1^{*} \\ v_3^{*} \\ t_{33} \\ \frac{1}{2\mu} t_{13} \end{Bmatrix}^{(1)} \tag{2.15}$$

By using matrix algebra $[A_{ij}]$ can be reduced to a 2x2 matrix,

$$\begin{Bmatrix} v_1 \\ v_3 \end{Bmatrix}^{(n)} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{Bmatrix} v_1^{*} \\ v_3^{*} \end{Bmatrix}^{(1)} \tag{2.16}$$

where,

$$\begin{aligned} w_{11} &= A_{11} - A_{31}(A_{13}A'_{33} + A_{14}A'_{43}) - A_{41}(A_{13}A'_{34} + A_{14}A'_{44}) \\ w_{12} &= A_{12} - A_{32}(A_{13}A'_{33} + A_{14}A'_{43}) - A_{42}(A_{13}A'_{34} + A_{14}A'_{44}) \\ w_{21} &= A_{21} - A_{31}(A_{23}A'_{33} + A_{24}A'_{43}) - A_{41}(A_{23}A'_{34} + A_{24}A'_{44}) \\ w_{22} &= A_{22} - A_{32}(A_{23}A'_{33} + A_{24}A'_{43}) - A_{42}(A_{23}A'_{34} + A_{24}A'_{44}) \end{aligned} \tag{2.17}$$

and

$$\begin{bmatrix} A'_{33} & A'_{34} \\ A'_{43} & A'_{44} \end{bmatrix} = \begin{bmatrix} A_{33} & A_{34} \\ A_{43} & A_{44} \end{bmatrix}^{-1} \tag{2.18}$$

Now using the usual definition for the frequency response function, i.e.

$$\begin{aligned} v_1^{*} &= A e^{i\omega t} \\ v_1^{(n)} &= H_{11}(\omega) A e^{i\omega t} \end{aligned} \tag{2.19}$$

Hence,

$$[H_{ij}] = [w_{ij}] \tag{2.20}$$

Then, the velocity spectral density matrix $[S_{sp}]^{(1)}$ can be related to $[S_{sp}]^{(m+n)}$

$$[S_{sp}]^{(m+n)} = [H_{sp}(i\omega)][H_{sp}(i\omega)]^{*} [S_{sp}]^{(1)} \tag{2.21}$$

Here $*$ denotes complex conjugate, subscripts s and p refer to transverse and longitudinal waves. S_p and S_s are the spectral density functions for the P and S waves respectively and S_{ps} and S_{sp} are cross-spectral density functions.

RESULTS AND DISCUSSION

The spectral density functions of the P and S wave acceleration were assumed to be a band limited white noise of unit intensity with the frequency ranging from 0 to 50 rad/sec at the source. Three layers were considered with the following properties,

$$\lambda^{(A)} = 2.88\times10^{5}\ kg/cm^2 \quad \lambda^{(B)} = 2.86\times10^{5}\ kg/cm^2 \quad \lambda^{(C)} = 2.14\times10^{5}\ kg/cm^2$$
$$\mu^{(A)} = 1.92\times10^{5}\ '' \quad \mu^{(B)} = 0.71\times10^{5}\ '' \quad \mu^{(C)} = 0.53\times10^{5}\ ''$$
$$c_1^{(A)} = 5.0\times10^{5}\ cm/sec \quad c_1^{(B)} = 4.59\times10^{5}\ cm/sec \quad c_1^{(C)} = 4.0\times10^{5}\ cm/sec$$
$$c_2^{(A)} = 2.68\times10^{5}\ '' \quad c_2^{(B)} = 1.89\times10^{5}\ '' \quad c_2^{(C)} = 1.62\times10^{5}\ ''$$

For different layering configurations and thickness ratio r, P and S wave velocity spectral density functions have been obtained. The configuration referred to in figures 3-8 are indicated in Table 1. For configuration 5, thinner layers have been used to give the effect of surface layering. The material properties are given below.

$$\lambda^{(D)} = 5.61\times10^{5}\ kg/cm^2 \quad \lambda^{(E)} = 0.54\times10^{5}\ kg/cm^2 \quad \lambda^{(F)} = 2.0\times10^{5}\ kg/cm^2$$
$$\mu^{(D)} = 1.77\times10^{5}\ '' \quad \mu^{(E)} = 1.89\times10^{5}\ '' \quad \mu^{(F)} = 1.5\times10^{5}\ ''$$
$$c_1^{(D)} = 6.0\times10^{5}\ cm/sec \quad c_1^{(E)} = 4.6\times10^{5}\ cm/sec \quad c_1^{(F)} = 5.4\times10^{5}\ cm/sec$$
$$c_2^{(D)} = 2.6\times10^{5}\ '' \quad c_2^{(E)} = 3.1\times10^{5}\ '' \quad c_2^{(F)} = 2.9\times10^{5}\ ''$$

It can be observed from figures 5-8 that both the ordering of the layers and thickness ratios influence the spectral density functions significantly. Influence of thickness ratios increase for higher frequencies although its effect is not negligible even for low frequencies. The cross-spectral density function (Fig.4) is important since it is an indication of the coupling between transverse and longitudinal spectra. Figure 3 shows that at low frequencies thinner layers are very sensitive to changes in frequency.

Numerical solutions for a large number of layers is straightforward but rather time consuming. For this reason an alternative approach based on one of the author's previous work [2] is under progress in which the layered half space is replaced by an homogeneous, anisotropic polar medium.

ACKNOWLEDGEMENT

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REFERENCES

[1] Özgür, D., A. Gürpınar, "Deprem İvme Spektrumunun Modellendirilmesi", TBTAK Araştırma Raporu No. MAG-302, Nisan, 1973.

[2] Özgür, D., T.R. Tauchert, "Propagation of Plane Waves in Anisotropic Micropolar Elastic Media", Research Report No. AMS-936, Princeton University, February, 1970.

NOMENCLATURE

- A_{ij} : Components of transfer matrix
- $c_1^{(i)}, c_2^{(i)}$: P and S wave velocities of i-th layer
- $d^{(i)}$: Thickness of i-th layer
- e_{ijk} : Permutation symbol
- H_{ij} : Components of frequency response matrix
- k, K : Wave numbers
- n_i : Components of unit wave normal
- S_{ij} : Components of spectral density matrix
- t_{ij} : Components of stress tensor
- u_i : Components of displacement vector
- v_i : Components of velocity vector
- x_i : Components of coordinate system
- λ, μ : Lame constants
- ρ : Mass density
- ϕ : Scalar potential
- $\underset{\sim}{\Psi}$: Vectorial potential
- ω : Angular frequency
- θ_i : Angle of incidence

TABLE 1

LAYERING CONFIGURATIONS

Configuration No.	θ	d_2 (m)	Material at d_2	d_3 (m)	Material at d_3	d_4 (m)	Material at d_4
1	30°	600	A	300	B	300r	C
2	45°	300	A	600	B	600r	C
3	30°	600	A	600r	C	300	B
4	45°	300	C	600	B	600r	A
5	45°	40	D	30	E	30	F

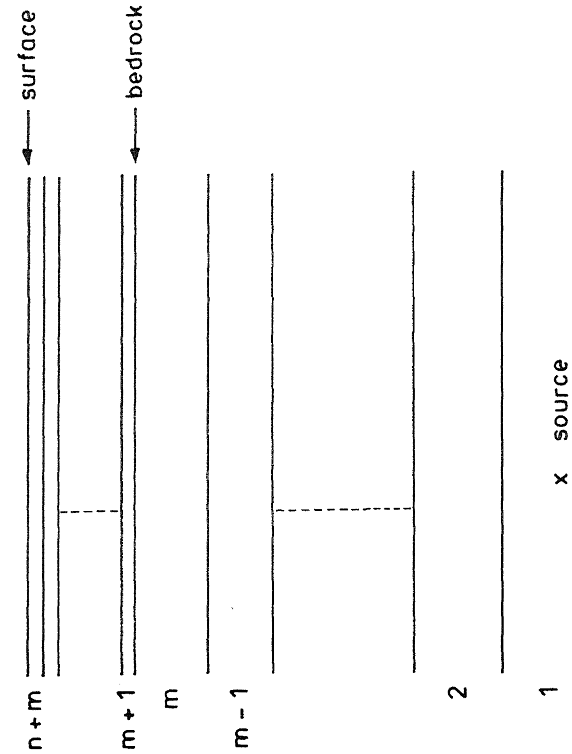

Figure 1. Idealization of the Layering

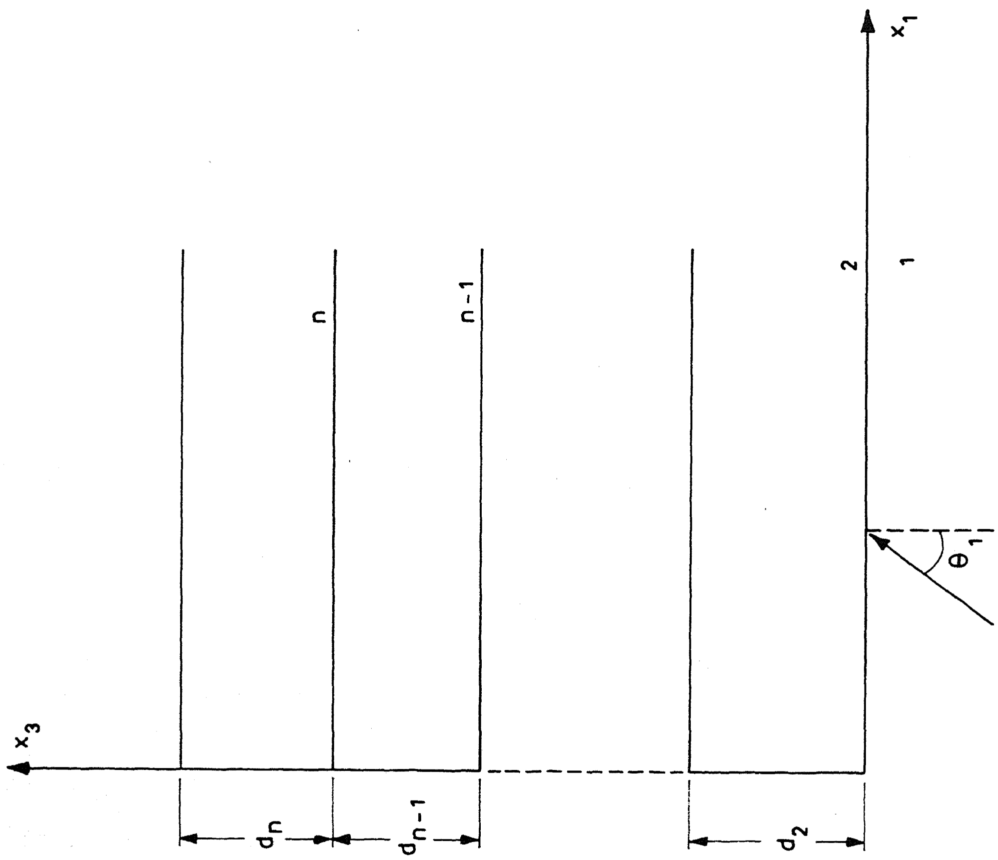

Figure 2. Layering of the half Space and the Coordinate System.

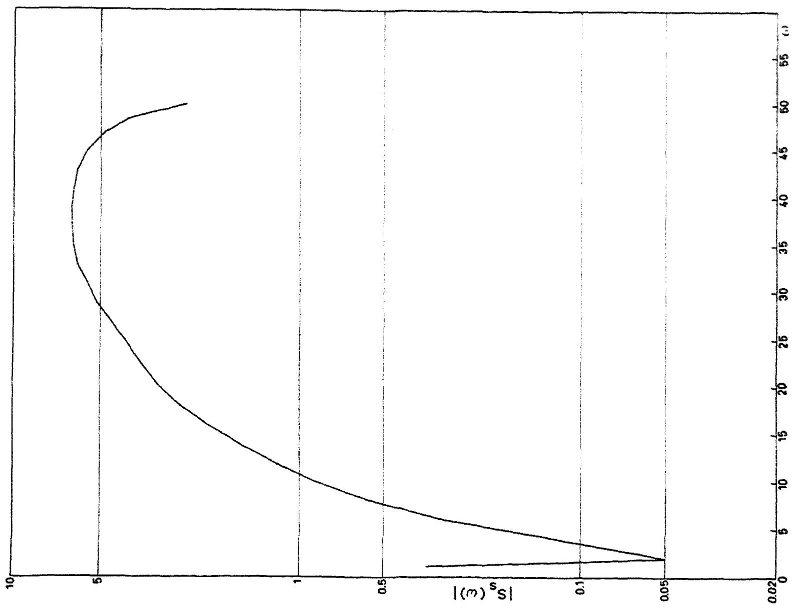

Figure 3. S - Spectral Density Function for Configuration (2)

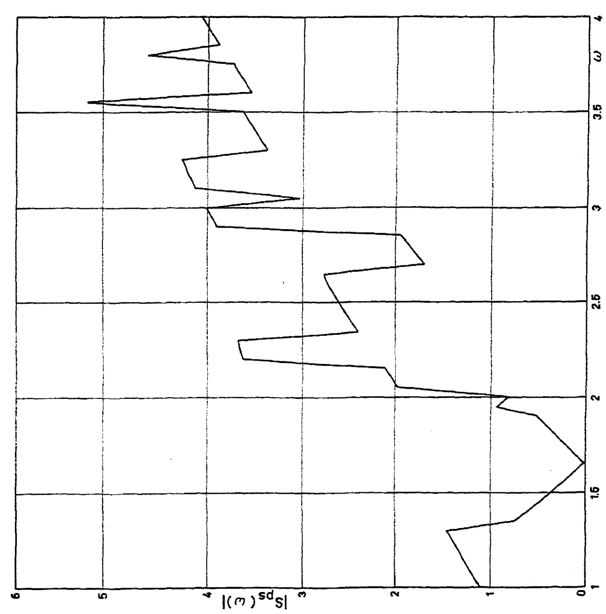

Figure 4. Cross - Spectral Density Function for Configuration (5) for Low Frequencies.

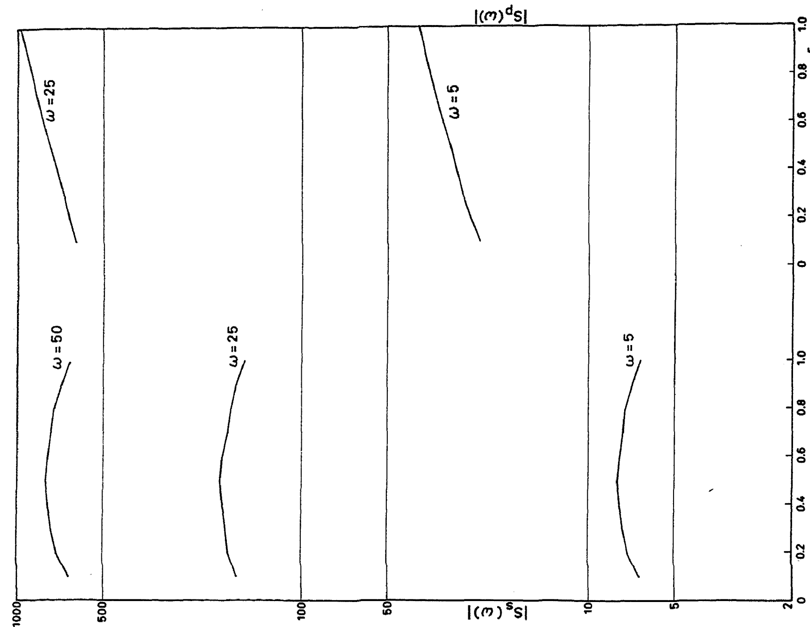

Figure 5. **Spectral Density Function for Configuration (1) for Varying Layer Thickness.**

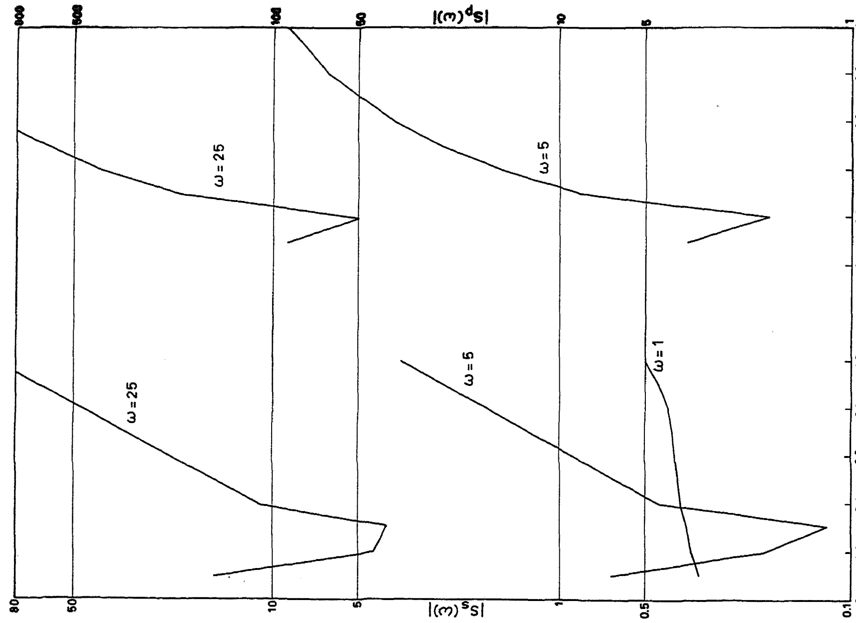

Figure 6. **Spectral Density Function for Configuration (2) for Varying Layer Thickness.**

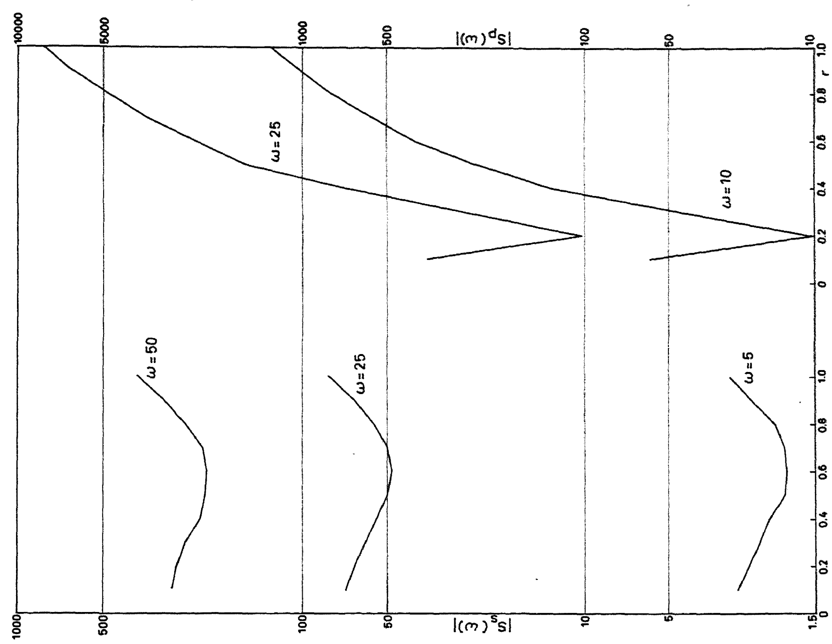

Figure 7. Spectral Density Function for Configuration (3) for Varying Layer Thickness.

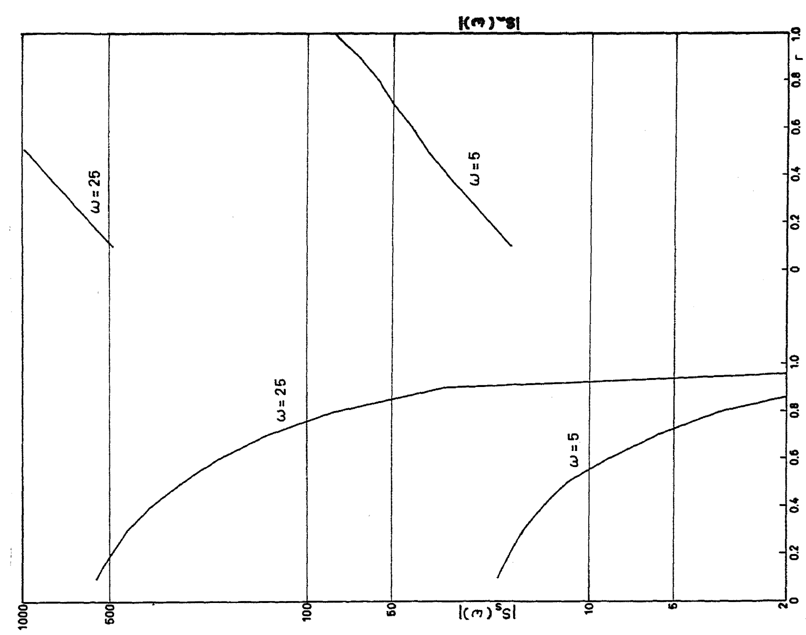

Figure 8. Spectral Density Function for Configuration (4) for Varying Layer Thickness.